



AP[®] Calculus AB 2004 Scoring Guidelines Form B

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AP[®] CALCULUS AB
2004 SCORING GUIDELINES (Form B)

Question 1

Let R be the region enclosed by the graph of $y = \sqrt{x-1}$, the vertical line $x = 10$, and the x -axis.

- (a) Find the area of R .
 (b) Find the volume of the solid generated when R is revolved about the horizontal line $y = 3$.
 (c) Find the volume of the solid generated when R is revolved about the vertical line $x = 10$.

(a) Area = $\int_1^{10} \sqrt{x-1} \, dx = 18$

3 : $\left\{ \begin{array}{l} 1 : \text{limits} \\ 1 : \text{integrand} \\ 1 : \text{answer} \end{array} \right.$

(b) Volume = $\pi \int_1^{10} (9 - (3 - \sqrt{x-1})^2) \, dx$
 = 212.057 or 212.058

3 : $\left\{ \begin{array}{l} 1 : \text{limits and constant} \\ 1 : \text{integrand} \\ 1 : \text{answer} \end{array} \right.$

(c) Volume = $\pi \int_0^3 (10 - (y^2 + 1))^2 \, dy$
 = 407.150

3 : $\left\{ \begin{array}{l} 1 : \text{limits and constant} \\ 1 : \text{integrand} \\ 1 : \text{answer} \end{array} \right.$

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Question 2

For $0 \leq t \leq 31$, the rate of change of the number of mosquitoes on Tropical Island at time t days is modeled by $R(t) = 5\sqrt{t} \cos\left(\frac{t}{5}\right)$ mosquitoes per day. There are 1000 mosquitoes on Tropical Island at time $t = 0$.

- (a) Show that the number of mosquitoes is increasing at time $t = 6$.
 (b) At time $t = 6$, is the number of mosquitoes increasing at an increasing rate, or is the number of mosquitoes increasing at a decreasing rate? Give a reason for your answer.
 (c) According to the model, how many mosquitoes will be on the island at time $t = 31$? Round your answer to the nearest whole number.
 (d) To the nearest whole number, what is the maximum number of mosquitoes for $0 \leq t \leq 31$? Show the analysis that leads to your conclusion.

(a) Since $R(6) = 4.438 > 0$, the number of mosquitoes is increasing at $t = 6$.

1 : shows that $R(6) > 0$

(b) $R'(6) = -1.913$
 Since $R'(6) < 0$, the number of mosquitoes is increasing at a decreasing rate at $t = 6$.

2 : $\left\{ \begin{array}{l} 1 : \text{considers } R'(6) \\ 1 : \text{answer with reason} \end{array} \right.$

(c) $1000 + \int_0^{31} R(t) dt = 964.335$
 To the nearest whole number, there are 964 mosquitoes.

2 : $\left\{ \begin{array}{l} 1 : \text{integral} \\ 1 : \text{answer} \end{array} \right.$

(d) $R(t) = 0$ when $t = 0$, $t = 2.5\pi$, or $t = 7.5\pi$
 $R(t) > 0$ on $0 < t < 2.5\pi$
 $R(t) < 0$ on $2.5\pi < t < 7.5\pi$
 $R(t) > 0$ on $7.5\pi < t < 31$
 The absolute maximum number of mosquitoes occurs at $t = 2.5\pi$ or at $t = 31$.

4 : $\left\{ \begin{array}{l} 2 : \text{absolute maximum value} \\ 1 : \text{integral} \\ 1 : \text{answer} \\ 2 : \text{analysis} \\ 1 : \text{computes interior critical points} \\ 1 : \text{completes analysis} \end{array} \right.$

$$1000 + \int_0^{2.5\pi} R(t) dt = 1039.357,$$

There are 964 mosquitoes at $t = 31$, so the maximum number of mosquitoes is 1039, to the nearest whole number.

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Question 3

A test plane flies in a straight line with positive velocity $v(t)$, in miles per minute at time t minutes, where v is a differentiable function of t . Selected values of $v(t)$ for $0 \leq t \leq 40$ are shown in the table above.

| | | | | | | | | | |
|--------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| t (min) | 0 | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 |
| $v(t)$ (mpm) | 7.0 | 9.2 | 9.5 | 7.0 | 4.5 | 2.4 | 2.4 | 4.3 | 7.3 |

- (a) Use a midpoint Riemann sum with four subintervals of equal length and values from the table to approximate $\int_0^{40} v(t) dt$. Show the computations that lead to your answer. Using correct units, explain the meaning of $\int_0^{40} v(t) dt$ in terms of the plane's flight.
- (b) Based on the values in the table, what is the smallest number of instances at which the acceleration of the plane could equal zero on the open interval $0 < t < 40$? Justify your answer.
- (c) The function f , defined by $f(t) = 6 + \cos\left(\frac{t}{10}\right) + 3\sin\left(\frac{7t}{40}\right)$, is used to model the velocity of the plane, in miles per minute, for $0 \leq t \leq 40$. According to this model, what is the acceleration of the plane at $t = 23$? Indicate units of measure.
- (d) According to the model f , given in part (c), what is the average velocity of the plane, in miles per minute, over the time interval $0 \leq t \leq 40$?

(a) Midpoint Riemann sum is
 $10 \cdot [v(5) + v(15) + v(25) + v(35)]$
 $= 10 \cdot [9.2 + 7.0 + 2.4 + 4.3] = 229$
 The integral gives the total distance in miles that the plane flies during the 40 minutes.

3 : $\begin{cases} 1 : v(5) + v(15) + v(25) + v(35) \\ 1 : \text{answer} \\ 1 : \text{meaning with units} \end{cases}$

(b) By the Mean Value Theorem, $v'(t) = 0$ somewhere in the interval $(0, 15)$ and somewhere in the interval $(25, 30)$. Therefore the acceleration will equal 0 for at least two values of t .

2 : $\begin{cases} 1 : \text{two instances} \\ 1 : \text{justification} \end{cases}$

(c) $f'(23) = -0.407$ or -0.408 miles per minute²

1 : answer with units

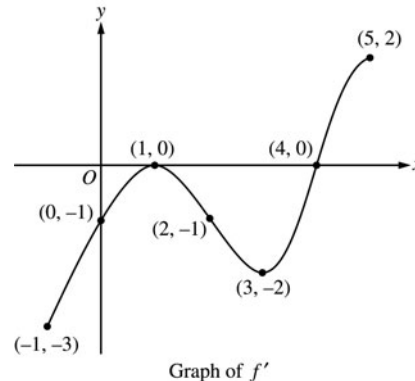
(d) Average velocity $= \frac{1}{40} \int_0^{40} f(t) dt$
 $= 5.916$ miles per minute

3 : $\begin{cases} 1 : \text{limits} \\ 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

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Question 4

The figure above shows the graph of f' , the derivative of the function f , on the closed interval $-1 \leq x \leq 5$. The graph of f' has horizontal tangent lines at $x = 1$ and $x = 3$. The function f is twice differentiable with $f(2) = 6$.



- (a) Find the x -coordinate of each of the points of inflection of the graph of f . Give a reason for your answer.
- (b) At what value of x does f attain its absolute minimum value on the closed interval $-1 \leq x \leq 5$? At what value of x does f attain its absolute maximum value on the closed interval $-1 \leq x \leq 5$? Show the analysis that leads to your answers.
- (c) Let g be the function defined by $g(x) = xf(x)$. Find an equation for the line tangent to the graph of g at $x = 2$.

- (a) $x = 1$ and $x = 3$ because the graph of f' changes from increasing to decreasing at $x = 1$, and changes from decreasing to increasing at $x = 3$.

2 : $\begin{cases} 1 : x = 1, x = 3 \\ 1 : \text{reason} \end{cases}$

- (b) The function f decreases from $x = -1$ to $x = 4$, then increases from $x = 4$ to $x = 5$. Therefore, the absolute minimum value for f is at $x = 4$. The absolute maximum value must occur at $x = -1$ or at $x = 5$.

4 : $\begin{cases} 1 : \text{indicates } f \text{ decreases then increases} \\ 1 : \text{eliminates } x = 5 \text{ for maximum} \\ 1 : \text{absolute minimum at } x = 4 \\ 1 : \text{absolute maximum at } x = -1 \end{cases}$

$$f(5) - f(-1) = \int_{-1}^5 f'(t) dt < 0$$

Since $f(5) < f(-1)$, the absolute maximum value occurs at $x = -1$.

- (c) $g'(x) = f(x) + xf'(x)$
 $g'(2) = f(2) + 2f'(2) = 6 + 2(-1) = 4$
 $g(2) = 2f(2) = 12$

3 : $\begin{cases} 2 : g'(x) \\ 1 : \text{tangent line} \end{cases}$

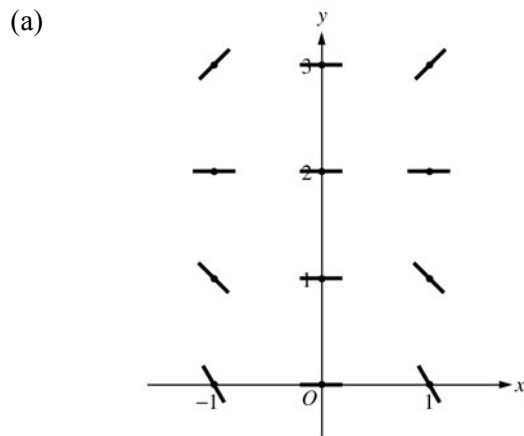
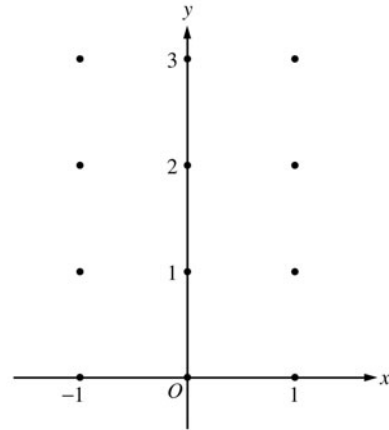
Tangent line is $y = 4(x - 2) + 12$

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Question 5

Consider the differential equation $\frac{dy}{dx} = x^4(y - 2)$.

- (a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated.
(Note: Use the axes provided in the test booklet.)
- (b) While the slope field in part (a) is drawn at only twelve points, it is defined at every point in the xy -plane. Describe all points in the xy -plane for which the slopes are negative.
- (c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = 0$.



- (b) Slopes are negative at points (x, y) where $x \neq 0$ and $y < 2$.

(c)

$$\frac{1}{y-2} dy = x^4 dx$$

$$\ln|y-2| = \frac{1}{5}x^5 + C$$

$$|y-2| = e^C e^{\frac{1}{5}x^5}$$

$$y-2 = Ke^{\frac{1}{5}x^5}, \quad K = \pm e^C$$

$$-2 = Ke^0 = K$$

$$y = 2 - 2e^{\frac{1}{5}x^5}$$

- 1 : zero slope at each point (x, y) where $x = 0$ or $y = 2$
- 2 : { positive slope at each point (x, y) where $x \neq 0$ and $y > 2$
- 1 : { negative slope at each point (x, y) where $x \neq 0$ and $y < 2$

1 : description

- 6 : { 1 : separates variables
 2 : antiderivatives
 1 : constant of integration
 1 : uses initial condition
 1 : solves for y
 0/1 if y is not exponential

Note: max 3/6 [1-2-0-0-0] if no constant of integration

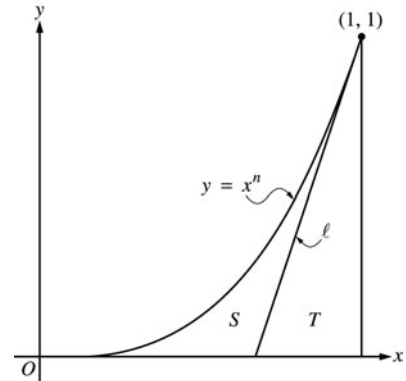
Note: 0/6 if no separation of variables

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Question 6

Let ℓ be the line tangent to the graph of $y = x^n$ at the point $(1, 1)$, where $n > 1$, as shown above.

- (a) Find $\int_0^1 x^n dx$ in terms of n .
- (b) Let T be the triangular region bounded by ℓ , the x -axis, and the line $x = 1$. Show that the area of T is $\frac{1}{2n}$.
- (c) Let S be the region bounded by the graph of $y = x^n$, the line ℓ , and the x -axis. Express the area of S in terms of n and determine the value of n that maximizes the area of S .



(a) $\int_0^1 x^n dx = \frac{x^{n+1}}{n+1} \Big|_0^1 = \frac{1}{n+1}$

2 : $\left\{ \begin{array}{l} 1 : \text{antiderivative of } x^n \\ 1 : \text{answer} \end{array} \right.$

- (b) Let b be the length of the base of triangle T .

$\frac{1}{b}$ is the slope of line ℓ , which is n

3 : $\left\{ \begin{array}{l} 1 : \text{slope of line } \ell \text{ is } n \\ 1 : \text{base of } T \text{ is } \frac{1}{n} \\ 1 : \text{shows area is } \frac{1}{2n} \end{array} \right.$

$$\text{Area}(T) = \frac{1}{2}b(1) = \frac{1}{2n}$$

(c) $\text{Area}(S) = \int_0^1 x^n dx - \text{Area}(T)$
 $= \frac{1}{n+1} - \frac{1}{2n}$

4 : $\left\{ \begin{array}{l} 1 : \text{area of } S \text{ in terms of } n \\ 1 : \text{derivative} \\ 1 : \text{sets derivative equal to } 0 \\ 1 : \text{solves for } n \end{array} \right.$

$$\frac{d}{dn} \text{Area}(S) = -\frac{1}{(n+1)^2} + \frac{1}{2n^2} = 0$$

$$2n^2 = (n+1)^2$$

$$\sqrt{2}n = (n+1)$$

$$n = \frac{1}{\sqrt{2}-1} = 1 + \sqrt{2}$$