Exploring Slope

High Ratio Mountain

Lesson 11-1 Linear Equations and Slope

Learning Targets:

- Understand the concept of slope as the ratio $\frac{change\ in\ y}{change\ in\ x}$ between any two points on a line.

- Graph proportional relationships; interpret the slope and the *y*-intercept (0, 0) of the graph.

- Use similar right triangles to develop an understanding of slope.

SUGGESTED LEARNING STRATEGIES: Create Representations, Marking The Text, Discussion Groups, Sharing and Responding, Interactive Word Wall

Misty Flipp worked odd jobs all summer long and saved her money to buy passes to the ski lift at the High Ratio Mountain Ski Resort. In August, Misty researched lift ticket prices and found several options. Since she worked so hard to earn this money, Misty carefully investigated each of her options.

High Ratio Mountain Ski Resort

Student Lift Ticket prices

Daily Lift Ticket	$30
10-Day Package	$80 upon purchase and $20 per day (up to 10 days)
Unlimited Season Pass	$390

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1. Suppose Misty purchases a daily lift ticket each time she goes skiing. Complete the table below to determine the total cost for lift tickets.

| Number of Days	0	1	2	3	4	5	6
Total Cost of Lift Tickets							

2. According to the table, what is the relationship between the cost of the lift tickets and the number of days?

My Notes

3. Let *d* represent the number of days for which Misty bought lift tickets and *C* represent Misty's total cost. Write an equation that can be used to determine the total cost of lift tickets if Misty skis for *d* days.

4. **Model with mathematics.** Plot the data from the table on the graph below. The data points appear to be *linear*. What do you think this means?

Total Cost of Lift Tickets vs. Days
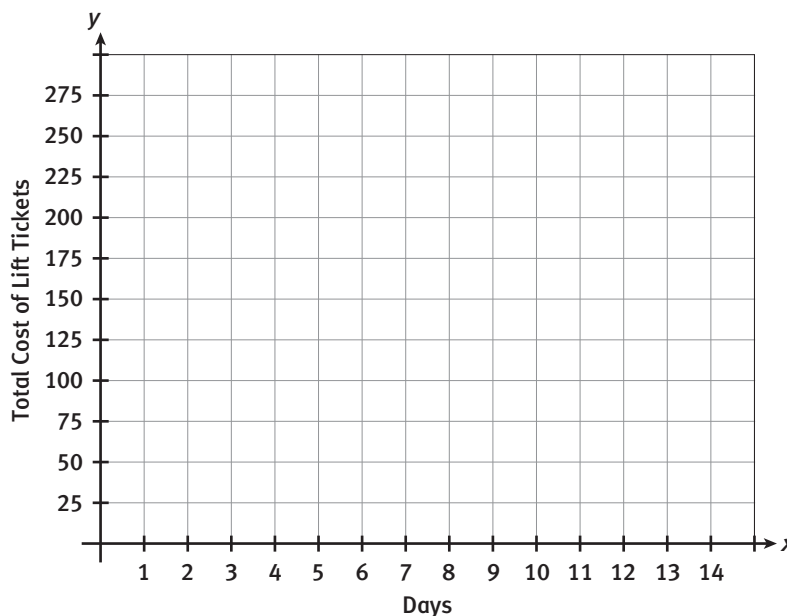
5. Label the leftmost point on the graph point *A*. Label the next 6 points, from left to right, points *B*, *C*, *D*, *E*, *F*, and *G*.

6. **Reason quantitatively.** According to the graph, what happens to the total cost of lift tickets as the number of days increases? Justify your answer.

7. Describe the movement, on the graph, from one point to another.

A to *B*: Vertical Change _____ Horizontal Change _____

B to *C*: Vertical Change _____ Horizontal Change _____

C to *D*: Vertical Change _____ Horizontal Change _____

D to *E*: Vertical Change _____ Horizontal Change _____

E to *F*: Vertical Change _____ Horizontal Change _____

F to *G*: Vertical Change _____ Horizontal Change _____

MATH TIP

Vertical change is the number of spaces moved up or down on a graph. "Up" movement is represented by a positive number. "Down" is a negative number.

Horizontal change is the number of spaces moved right or left on a graph. Movement to the right is indicated by a positive number. Movement to the left is indicated by a negative number.

8. a. The movements you traced in Item 7 can be written as ratios. Write ratios in the form $\frac{vertical\ change}{horizontal\ change}$ to describe the movement from:

A to *B*:	*B* to *C*:
C to *D*:	*D* to *E*:

b. Vertical change can also be described as the *change in y*. Similarly, the horizontal change is often referred to as the *change in x*.
Therefore, the ratio $\frac{vertical\ change}{horizontal\ change}$ can also be written as $\frac{change\ in\ y}{change\ in\ x}$. Determine the *change in y* and *change in x* from *A* to *C* in Item 4. Write the ratio as $\frac{change\ in\ y}{change\ in\ x}$.

Continue to use the data from Item 4. Determine the *change in y* and *change in x* for each movement described below. Then write the ratio $\frac{change\ in\ y}{change\ in\ x}$.

c. From *B* to *E*:   **d.** From *A* to *E*:

e. From *B* to *A*:   **f.** From *E* to *B*:

My Notes

My Notes

9. Describe the similarities and differences in the ratios written in Item 8. How are the ratios related?

10. **Make sense of problems.** What are the units of the ratios created in Item 8? Explain how the ratios and units relate to Misty's situation.

11. How do the ratios relate to the equation you wrote in Item 3?

MATH TIP

In similar triangles, corresponding angles are congruent and corresponding sides are in proportion.

12. The ratio $\dfrac{change\ in\ y}{change\ in\ x}$ between any two points on a line is constant. Use the diagram below and what you know about similar triangles to explain why the $\dfrac{change\ in\ y}{change\ in\ x}$ ratios are equivalent for the movements described.

From W to V:

From W to Z:
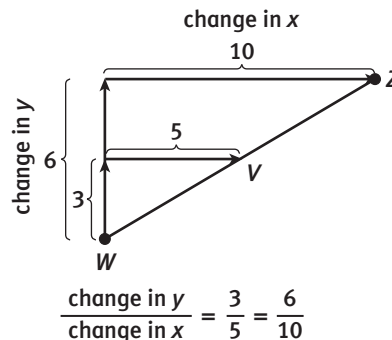
$$\frac{change\ in\ y}{change\ in\ x} = \frac{3}{5} = \frac{6}{10}$$

The **slope** of a line is determined by the ratio $\dfrac{\text{change in } y}{\text{change in } x}$ between any two points that lie on the line.

- The slope is the *constant rate of change* of a line. It is also sometimes called the *average rate of change*.
- All linear relationships have a *constant rate of change*.
- The slope of a line is what determines how steep or flat the line is.
- The *y*-intercept of a line is the point at which the line crosses the *y*-axis, $(0, y)$.

13. Draw a line through the points you graphed in Item 4. Use the graph to determine the slope and *y*-intercept of the line. How do the slope and *y*-intercept of this line relate to the equation you wrote in Item 3?

14. Complete the table to show the data points you graphed in Item 4. Use the table to indicate the ratio $\dfrac{\text{change in } y}{\text{change in } x}$ and to determine the slope of the line.

Number of Days	Total Cost of Lift Tickets
0	
1	
2	
3	
4	
5	
6	

change in *y*:   change in *x*:

$\dfrac{\text{change in } y}{\text{change in } x}$:   slope:

MATH TERMS

Slope is the ratio of vertical change to horizontal change, or $\dfrac{\textit{change in y}}{\textit{change in x}}$.
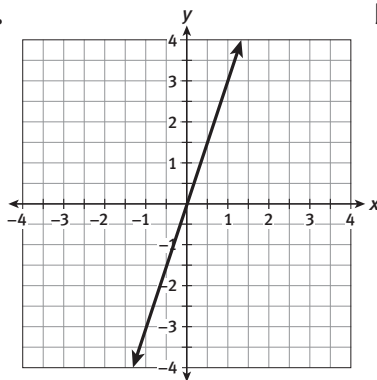
READING MATH

The slope of a line, $\dfrac{\textit{change in y}}{\textit{change in x}}$, is also expressed symbolically as $\dfrac{\Delta y}{\Delta x}$.

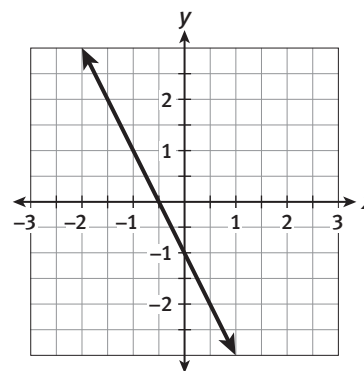Δ is the Greek letter delta, and in mathematics it means "change in."

Check Your Understanding

15. Find the slope and the *y*-intercept for each of the following. Remember to use the ratio $\dfrac{change\ in\ y}{change\ in\ x}$.

a.

b.

c.

x	y
0	0
1	2.5
2	5
4	10

d.

x	y
−1	4
0	2
1	0
3	−4

e. Look back at the figure for Item 12. Would a point *P* that is 9 units up from point *W* and 15 units to the right be on the line that contains points *W*, *V*, and *Z*? Use similar triangles to explain your answer.

16. John is longboarding at a constant rate down the road. If 2 minutes after he leaves his house he is 1,000 feet away and at 5 minutes he is 2,500 feet from his house, what would his average rate of change be?

LESSON 11-1 PRACTICE

The Tran family is driving across the country. They drive 400 miles each day. Use the table below to answer Items 17–20.

Day	Total Miles Driven
1	400
2	800
3	
4	
5	

17. Complete the table.

18. Draw a graph for the data in the table. Be sure to title the graph and label the axes. Draw a line through the points.

19. Write an equation that can be used to determine the total miles, M, driven over d days.

20. Find the slope and the y-intercept of the line you created, using the graph you drew or the equation you wrote. Explain what each represents for the Tran family's situation.

The graph below shows the money a student earns as she tutors. Use the graph to answer Items 21–24.

Money Earned Tutoring
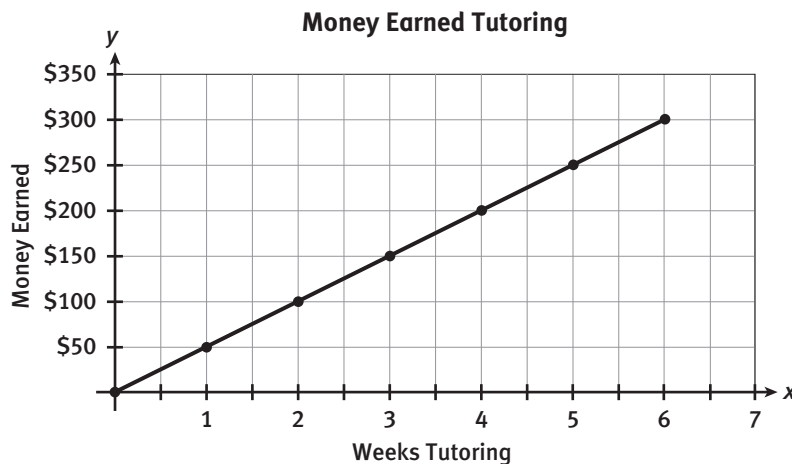
21. What is the slope of the line?

22. What is the y-intercept of the line?

23. Write an equation that can be used to determine how much money, D, the student has earned after w weeks.

24. **Attend to precision.** Calculate how much money the student will have earned after 52 weeks.