Sample Syllabus 3 Contents

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CR4 Students and teachers have access to a college-level calculus textbook.
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AP Calculus BC Syllabus

Course Overview

In our BC Calculus program, we believe in taking the time to have students work together during class to develop and explore ideas, discuss approaches to problems, and develop skills. We frequently use guided worksheets in class to introduce new concepts. Tests are given about every two weeks, with some quizzes in between, and we also have several problem sets each semester. These problem sets contain more difficult problems, which require more extensive writing than a regular, timed test.

All students have graphing calculators and access to laptops. If a family cannot afford a graphing calculator, then one is provided for the student. **[CR3a]**

[CR3a] — Students have access to graphing calculators.

Textbook

Hughes-Hallett, Gleason, McCallum, et al. *Calculus*. 5th ed. Hoboken, NJ: John Wiley & Sons, Inc., 2009. **[CR4]**

[CR4] — Students and teachers have access to a college-level calculus textbook.

Big Idea 1: Limits

Important Concepts and Definitions:

i. Introductory ideas: Rates of change; connections between graphs of position and velocity; and simple Euler's method (with $y' = f(x)$ only).

ii. Limits: Intuitive sense of continuity and “nice” limits; algebraic limits to deal with holes in graphs; one-sided limits and piecewise-defined function; limits as $x \to \infty$ and $y \to \infty$ and asymptotes; formal definition of continuity; introduction to $\delta - \varepsilon$ definition; and Intermediate Value Theorem. **[CR1a]**

[CR1a] — The course is structured around the enduring understandings within Big Idea 1: Limits.

Examples:

1. One early worksheet for limits explores the function $f(x) = \frac{x^2+x-6}{2x-4}$.
2. Students use the table (with Ask) on their calculator to explore the function near $x = 2$ and interpret the results in terms of the definition of “limit.” **[CR3c]** Students graph the function with windows that both miss and show the hole. Then students find the limit analytically. Following that work, students write to explain their understanding of a limit, especially when the function is not defined at the point in question. **[CR2c]** **[CR2d: connection among graphical, numerical, and analytical]**

[CR3c] — Students have opportunities to use a graphing calculator to explore and interpret calculus concepts.

[CR2c] — The course provides opportunities for students to implement algebraic/computational processes.

[CR2d] — The course provides opportunities for students to engage with graphical, numerical, analytical, and verbal representations and demonstrate connections among them.

Big Idea 2: Derivatives

Important Concepts and Definitions:

i. Introductory ideas: The definition of the derivative; average rate of change vs. instantaneous rate of change; local linearity and tangent lines; meaning of the derivative in context; derivative function; analysis of graphs, including concavity and graphs of f, f', and f''; and how differentiability implies continuity. **[CR1b: derivatives]**

ii. Computing Derivatives: Use the definition of the derivative and approximations to develop the rules for derivatives of power functions and exponential functions; work on limits involving trigonometric functions; definition of the derivative for sine and cosine; product, quotient, and chain rules, with development of rules for all trigonometric functions; and derivatives of inverse functions and implicit differentiation.

[CR1b] — The course is structured around the enduring understandings within Big Idea 2: Derivatives.

Accompanying work:

i. Mean Value Theorem and more theorems proved with the MVT. **[CR1b: Mean Value Theorem]**

ii. Optimization, related rates, families of functions, and L'Hospital's Rule (Big Idea 1).

[CR1b] — The course is structured around the enduring understandings within Big Idea 2: Derivatives.

Examples:

1. When studying the meaning of derivatives in context, such as position/velocity/acceleration problems described in words, students are asked to interpret the concept using the specific numbers both in writing on homework, using complete sentences, and orally for new questions in class. **[CR2f] [CR2d]**

2. In a worksheet on the Mean Value Theorem, students draw secants and parallel tangent lines on a variety of graphs, considering properties such as continuity and differentiability. More problems are given that ask students about hypotheses and conclusions. In particular, students are asked about the conclusion even when the hypotheses do not hold. Then students move to finding (if possible) the appropriate value of c described by the MVT, both by hand and with their graphing calculators. **[CR3b]** Finally, students are asked to explain the conclusions of the MVT in writing for multiple scenarios in context. **[CR2a] [CR2b]**

3. Students use analytic techniques to find derivatives. They use derivative functions to write equations of tangent lines, find extrema, and solve other problems requiring derivatives.

[CR2f] — The course provides opportunities for students to communicate mathematical ideas in words, both orally and in writing.

[CR2d] — The course provides opportunities for students to engage with graphical, numerical, analytical, and verbal representations and demonstrate connections among them.

[CR3b] — Students have opportunities to use calculators to solve problems.

[CR2a] — The course provides opportunities for students to reason with definitions and theorems.

[CR2b] — The course provides opportunities for students to connect concepts and processes.

Big Idea 3: Integrals

Important Concepts and Definitions:

i. Integrals (notation, signed area): Riemann and trapezoidal sums; definition of an integral with limits; and properties of integrals, including linearity and average value of a function. **[CR1c: integrals]**

ii. Euler's method: Develop accumulation and work with $F(x) = \int_a^x f(t)dt$.

iii. Development: Fundamental Theorem (both parts) with f continuous, $\int_a^b f(x)dx = F(b) - F(a)$, where F is an antiderivative of f; then development of the second part of the Fundamental Theorem stating $\frac{d}{dx}\left(\int_a^x f(t)dt\right) = f(x)$ (with appropriate hypotheses) and examples requiring the chain rule; and functions defined by the integral of a function f, given the graph of f. **[CR1c: Fundamental Theorem of Calculus parts 1 and 2]**

[CR1c] — The course is structured around the enduring understandings within Big Idea 3: Integrals and the Fundamental Theorem of Calculus.

Additional ideas:

i. Basic antiderivatives, indefinite integrals, and evaluation of more definite integrals.

ii. Techniques of integration: Substitution, integration by parts, partial fractions, powers of trigonometric functions, and trigonometric substitution.

iii. More on area; volume with washers, shells, and general cross sections; and arc length.

iv. Improper integrals, including evaluation with limits.

Examples:

1. Worksheets on the definition of an integral and the FTCs require students to use and connect notation involving summations, limits, integrals, and derivatives. Students learn how the limit of a Riemann sum connects to the definite integral starting with simple area calculations. **[CR2e]**

2. Students use calculators to evaluate integrals to obtain areas, volumes, arc lengths, and other quantities. Technology is used particularly for functions that do not have elementary antiderivative forms, including many of those on Free Response Questions from released AP exams.

[CR2e] — The course provides opportunities for students to build notational fluency.

Big Idea 4: Series

Important Concepts and Definitions:

i. Sequences and convergence: Series and computations of the related sequence of partial sums; proof of divergence of the harmonic series; tests of convergence of series – integral test, comparison and limit comparison tests, and ratio test; alternating series with conditional and absolute convergence; and alternating series error bound.

ii. Power series with ratio test and checking endpoints. **[CR1d: power series]**

iii. Series: Develop Taylor polynomials and extend to infinite series; more work on intervals of convergence; manipulation of series to create new series; and approximations and error bounds with alternating series and with Lagrange error bound. **[CR1d: series and Taylor polynomials]**

[CR1d] — The course is structured around the enduring understandings within Big Idea 4: Series.

Examples:

1. Multiple worksheets and textbook problems require students to work with hypotheses for tests of convergence of series.

2. Students must deal with notation and analytic processes to compute Taylor polynomials, the interval of convergence for series, and error bounds.

Big Idea 2: Derivatives and Big Idea 3: Integrals

Important Concepts and Definitions:

i. Vector-valued functions: Derivatives, tangent lines to curves, magnitude, speed, acceleration, and modeling projectiles.
ii. Polar functions: Basic graphs, derivatives, tangent lines, and area.
iii. Differential Equations:
  a. Definitions, examples, and verifying solutions.
  b. Slope fields and drawing solution curves, analytic solutions by separation of variables, and Euler's method for numerical approximations.
  c. Modeling: Newton's law of cooling, air drag, exponential and logistic population growth, and others; and combine slope fields, Euler's method, and analytic solutions for each model, as possible.

Example:

The worksheet that explores Newton's law of cooling begins by stating the verbal "rule" and asks students to write the differential equation and find the general solution. **[CR2d: connection among graphical, analytical, and verbal]** Information is given to find a specific solution for a cooling cup of coffee. Students examine the slope field on their calculator. They use Euler's method and compare the result to the actual value of the solution where they note how concavity will tell us why the real value of the temperature is higher. Students are then asked what might affect the proportionality constant k and what would change if they took a soda from the refrigerator.

> **[CR2d]** — The course provides opportunities for students to engage with graphical, numerical, analytical, and verbal representations and demonstrate connections among them.