# Question 5

<table>
<thead>
<tr>
<th>Distribution of points</th>
<th>7 points total</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>(a)</strong> 3 points</td>
<td></td>
</tr>
<tr>
<td>For reasoning that since the strings all have the same length, and since the wavelength of the fundamental depends on the length, all four waves have the same wavelength ($\lambda_1 = 2L$)</td>
<td>1 point</td>
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<tr>
<td>For reasoning that since the wavelengths are all the same, different frequencies correspond to different velocities of the waves on the strings</td>
<td>1 point</td>
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<tr>
<td>For reasoning that all the string tensions are the same due to the same mass $M$ of each block, and therefore the linear mass densities must be different for different velocities since $v = \sqrt{F_T/(m/L)}$ (or since the vertical component of the tension will result in different vertical accelerations for strings with different masses)</td>
<td>1 point</td>
</tr>
<tr>
<td>Note: Responses may refer to the physical differences between the strings in a variety of ways, e.g., different linear mass density, different total mass, different thicknesses of the same material</td>
<td></td>
</tr>
</tbody>
</table>

| **(b)** 2 points       |               |
| For combining $v = f\lambda$ with $v = \sqrt{F_T/(m/L)}$ (or referring to such an equation written in part (a)) | 1 point |
| For indicating how the equation leads to the conclusion that frequency would not be proportional to the inverse of the linear mass density | 1 point |

| **(c)** 2 points       |               |
| For any indication of the second harmonic on the string, or a wave drawn such that $\lambda_2 = L$. | 1 point |
| For points which are at the antinodes of the second harmonic, or at the antinodes of any standing wave drawn on the string | 1 point |

Notes:
- Full credit is earned for having two points that are located one fourth the length of the string and three fourths the length of the string from the oscillator.
- One earned point is deducted for each incorrect point marked on the figure.
5. (7 points, suggested time 13 minutes)

The figure above shows a string with one end attached to an oscillator and the other end attached to a block. The string passes over a massless pulley that turns with negligible friction. Four such strings, A, B, C, and D, are set up side by side, as shown in the diagram below. Each oscillator is adjusted to vibrate the string at its fundamental frequency $f$. The distance between each oscillator and pulley $L$ is the same, and the mass $M$ of each block is the same. However, the fundamental frequency of each string is different.

![Diagram of string system with labeled frequencies]

The equation for the velocity $v$ of a wave on a string is $v = \sqrt{\frac{F_T}{m/L}}$, where $F_T$ is the tension of the string and $m/L$ is the mass per unit length (linear mass density) of the string.

(a) What is different about the four strings shown above that would result in their having different fundamental frequencies? Explain how you arrived at your answer.

By manipulating the equation given and $f \lambda = v$, it can be found that $f = \sqrt{\frac{F_T}{m/L}} \cdot \frac{1}{2L}$.

Since $F_T$ and $L$ are the same for all four strings, this means that the frequency changes based on the linear mass density of each string. So, all four strings have different linear mass densities.
(b) A student graphs frequency as a function of the inverse of the linear mass density. Will the graph be linear? Explain how you arrived at your answer.

As was found in part a), \( f \alpha \frac{1}{\sqrt{mL}} \), so \( f^2 \alpha \frac{1}{mL} \) since the tension in the string and the length of the string are constant. Therefore, to graph \( f \) as a function of \( \frac{1}{(mL)} \) (as the student has done) would not yield a linear graph. Since \( f \alpha \sqrt{\frac{1}{(mL)}} \) the graph would appear to be a sideways parabola, concave down for all values of \( (mL) \) — so definitely not linear.

(c) The frequency of the oscillator connected to string \( D \) is changed so that the string vibrates in its second harmonic. On the side view of string \( D \) below, mark and label the points on the string that have the greatest average vertical speed.
5. (7 points, suggested time 13 minutes)

The figure above shows a string with one end attached to an oscillator and the other end attached to a block. The string passes over a massless pulley that turns with negligible friction. Four such strings, A, B, C, and D, are set up side by side, as shown in the diagram below. Each oscillator is adjusted to vibrate the string at its fundamental frequency $f$. The distance between each oscillator and pulley $L$ is the same, and the mass $M$ of each block is the same. However, the fundamental frequency of each string is different.

\[
f = \frac{\nu}{2L}\]

The equation for the velocity $\nu$ of a wave on a string is $\nu = \sqrt{\frac{F_T}{m/L}}$, where $F_T$ is the tension of the string and $m/L$ is the mass per unit length (linear mass density) of the string.

(a) What is different about the four strings shown above that would result in their having different fundamental frequencies? Explain how you arrived at your answer.

The masses of the strings would have to be different. Knowing that the equation for fundamental frequency is $f = \frac{\nu}{2L}$ and that $L$ is the same in all circumstances, $\nu$ must be the changing variable. Then in the equation $\nu = \sqrt{\frac{F_T}{m/L}}$, $F_T$ is constant (the masses on the end of the strings are equal) and so is $L$, so therefore the mass (mass) of the spring is the changing factor causing the strings to have different fundamental frequencies.
(b) A student graphs frequency as a function of the inverse of the linear mass density. Will the graph be linear? Explain how you arrived at your answer.

The graph will not be linear because there is a square root in the equation \( v = \sqrt{\frac{f}{m/L}} \), therefore causing the graph to be curved rather than linear.

(c) The frequency of the oscillator connected to string \( D \) is changed so that the string vibrates in its second harmonic. On the side view of string \( D \) below, mark and label the points on the string that have the greatest average vertical speed.

The places marked with stars (\( \star \)) are the antinodes of the wave and have the greatest average vertical speed.
5. (7 points, suggested time 13 minutes)

The figure above shows a string with one end attached to an oscillator and the other end attached to a block. The string passes over a massless pulley that turns with negligible friction. Four such strings, A, B, C, and D, are set up side by side, as shown in the diagram below. Each oscillator is adjusted to vibrate the string at its fundamental frequency \( f \). The distance between each oscillator and pulley \( L \) is the same, and the mass \( M \) of each block is the same. However, the fundamental frequency of each string is different.

\[
\begin{array}{c}
\text{Top View} \\
\hline
f_A = 400 \text{ Hz} \\
f_B = 382 \text{ Hz} \\
f_C = 364 \text{ Hz} \\
f_D = 350 \text{ Hz}
\end{array}
\]

The equation for the velocity \( v \) of a wave on a string is \( v = \sqrt{\frac{F_T}{m/L}} \), where \( F_T \) is the tension of the string and \( m/L \) is the mass per unit length (linear mass density) of the string.

(a) What is different about the four strings shown above that would result in their having different fundamental frequencies? Explain how you arrived at your answer.

The difference about the four strings shown above that would result in their having different fundamental frequencies is the string's medium. The string's medium can relate to the thickness of the string and also the type of material the string is made of and can cause frequencies to change based off of that medium.
(b) A student graphs frequency as a function of the inverse of the linear mass density. Will the graph be linear? Explain how you arrived at your answer.

Yes the graph would be linear because the frequency and the linear mass density would correspond with each other. If the linear mass density is increased, the medium of the string becomes thicker and causes the frequency to travel faster through the string causing a higher frequency. Since a more dense string will produce a higher frequency, the graph will be linear.

(c) The frequency of the oscillator connected to string D is changed so that the string vibrates in its second harmonic. On the side view of string D below, mark and label the points on the string that have the greatest average vertical speed.

- Points A and B are the same point but in different positions as well as C and D.
- Points A/B and C/D have the greatest average vertical speed because the string is vibrating in its second harmonic.
Overview

The intent of this question was to examine the properties of standing waves and harmonics and use the appropriate relationships between wavelength, frequency, and wave speed.

Sample: P1Q5 A
Score: 7

In this full-credit response, the student mathematically combines the given equation with the relationship between wave speed and frequency to derive an equation for frequency in terms of the given quantities. The student then reasons about which quantities are the same for all strings and accurately explains why each string must have a different linear mass density. The student does a nice job of referring to the equation in part (a) as part of the explanation in part (b). The student accurately draws the first and second harmonics in part (c) and chooses the antinodes of the second harmonic as the correct points of greatest average vertical speed.

Sample: P1Q5 B
Score: 5

Part (a) received 3 points for full credit. The student indicates in part (a) that the fundamental frequency is \( f = \frac{v}{2L} \) and therefore relates the wavelength to the length of the string accurately. The student also indicates that the tension is constant because the masses at the end of the strings are constant. The student then accurately concludes that the mass of the strings all have to be different. Writing “spring” when the student clearly meant “string” was a common mistake that was overlooked as there is no spring in the problem. Part (b) earned 1 point. The student only relates the speed to the linear density and thus did not earn the point for combining the two wave speed equations. The student’s answer to (c) seems correct at first glance, but on further examination, the stars are on the wrong lines since the second harmonic is not drawn accurately and the antinodes are not centered appropriately. As a result, the student earned 1 point for part (c).

Sample: P1Q5 C
Score: 2

Part (a) earned no credit. The student uses several sentences simply to state that the linear mass density must be different, with none of the reasoning for which credit could be earned. Similarly, there is no correct reasoning in part (b), and the conclusion is incorrect, so there is no credit in part (b). However, in part (c), the student correctly shows the second harmonic and correctly identifies the antinodes as the points of greatest average vertical speed, earning 2 points.