Question 3

Intent of Question

The primary goals of this question were to assess a student's ability to (1) perform a probability calculation from a normal distribution; (2) explain an implication of examining the distribution of a sample mean rather than the distribution of a single measurement; and (3) perform a probability calculation involving independent events using the multiplication rule.

Solution

Part (a):

Because the distribution of the daily number of absences is approximately normal with mean 120 students and standard deviation 10.5 students, the z-score for an absence total of 140 students is $z = \frac{140 - 120}{10.5} \approx 1.90$. The table of standard normal probabilities or a calculator reveals that the probability that 140 or fewer students are absent is 0.9713. So the probability that more than 140 students are absent (and that the school will lose some state funding) is $1 - 0.9713 = 0.0287$.

Part (b):

High School A would be *less* likely to lose state funding. With a random sample of 3 days, the distribution of the sample mean number of students absent would have less variability than that of a single day. With less variability, the distribution of the sample mean would concentrate more narrowly around the mean of 120 students, resulting in a smaller probability that the mean number of students absent would exceed 140.

In particular, the standard deviation of the sample mean number of absences, $\bar{x}$, is $\frac{\sigma}{\sqrt{n}} = \frac{10.5}{\sqrt{3}} = 6.062$. So the z-score for a sample mean of 140 is $\frac{140 - 120}{6.062} \approx 3.30$. The probability that High School A loses funding using the suggested plan would be $1 - 0.9995 = 0.0005$, as determined from the table of standard normal probabilities or from a calculator, which is less than a probability of 0.0287 obtained for the plan described in part (a).

Part (c):

For any one typical school week, the probability is $\frac{2}{5} = 0.4$ that the day selected is not Tuesday, not Wednesday, or not Thursday. Therefore, because the days are selected independently across the three weeks, the probability that none of the three days selected would be a Tuesday or Wednesday or Thursday is $(0.4)^3 = 0.064$.

Question 3 (continued)

Scoring

Parts (a), (b), and (c) were scored as essentially correct (E), partially correct (P), or incorrect (I).

Part (a) is scored as follows:

Essentially correct (E) if the response provides the following three components:

1. Indicates use of a normal distribution and clearly identifies the correct parameter values (showing correct components of a z-score calculation is sufficient).
2. Uses the correct boundary value (140, 140.5, or 141 is acceptable).
3. Reports the correct normal probability consistent with components 1 and 2

OR

if the response reports a probability of 0.025 with justification based on the empirical rule for an acceptable boundary value (140, 140.5, or 141 is acceptable).

Partially correct (P) if the response correctly provides only two of the three components listed above.

OR

if the response provides an incorrect probability of 0.05 with justification based on the empirical rule for an acceptable boundary value (140, 140.5, or 141 is acceptable).

Incorrect (I) if the response does not satisfy the criteria for E or P.

Note: An inconsistency in calculations lowers the score for part (a) by one level (that is, from E to P or from P to I).

Part (b) is scored as follows:

Essentially correct (E) if the response provides the correct answer of less likely *AND* the following three components:

1. Clearly references the distribution of the sample <u>mean</u>.
2. Indicates that the variability of the distribution is smaller.
3. Indicates that the distribution is centered at 120.

OR

if the response provides the correct answer of less likely *AND* the following two components:

1. Correctly calculates the probability that the sample mean would exceed 140 (arithmetic errors are not penalized).
2. Correctly compares this probability to the probability in part (a).

Partially correct (P) if the response provides the correct answer of less likely *AND* only two of the following three components:

1. Clearly references the distribution of the sample <u>mean</u>.
2. Indicates that the variability of the distribution is smaller.
3. Indicates that the distribution is centered at 120.

OR

if the response provides the correct answer of less likely *AND* correctly calculates the probability that the sample mean would exceed 140 (arithmetic errors are not penalized) *BUT* does <u>not</u> correctly compare this probability with the probability in part (a).

Question 3 (continued)

Incorrect (I) if the response does not meet the criteria for E or P, including if the response provides the incorrect answer or provides the correct answer of less likely with no explanation or an incorrect explanation.

Note: An equivalent approach is to use the total number of absences for 3 days. The sampling distribution of the total number of absences for the 3 days is approximately normal, with mean $3(120) = 360$ absences and standard deviation $\sqrt{3}(6.026) \approx 18.187$ absences. The z-score for a total of $3(140) = 420$ absences is: $\frac{420 - 360}{18.187} \approx 3.30$. Such a response is scored E if the response provides the correct answer of less likely and references the distribution of the sample total, and includes the correct mean and standard deviation.

Part (c) is scored as follows:

Essentially correct (E) if the response correctly calculates the probability *AND* shows sufficient work.

Partially correct (P) if the response reports the correct probability but shows no work or does not show sufficient work;
OR
if the response uses the multiplication rule involving three events but does so incorrectly and/or with an incorrect probability of not selecting a Tuesday, Wednesday, or Thursday.

Incorrect (I) if the response does not meet the criteria for E or P.

4 Complete Response

All three parts essentially correct

3 Substantial Response

Two parts essentially correct and one part partially correct

2 Developing Response

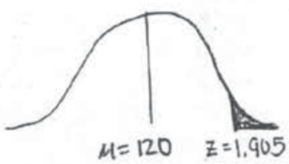
Two parts essentially correct and one part incorrect
OR
One part essentially correct and one or two parts partially correct
OR
Three parts partially correct

1 Minimal Response

One part essentially correct and two parts incorrect
OR
Two parts partially correct and one part incorrect

3. Schools in a certain state receive funding based on the number of students who attend the school. To determine the number of students who attend a school, one school day is selected at random and the number of students in attendance that day is counted and used for funding purposes. The daily number of absences at High School A in the state is approximately normally distributed with mean of 120 students and standard deviation of 10.5 students.

(a) If more than 140 students are absent on the day the attendance count is taken for funding purposes, the school will lose some of its state funding in the subsequent year. Approximately what is the probability that High School A will lose some state funding?



The shaded area represents the probability that more than 140 students are absent on attendance day.

$z = \frac{x_1 - \mu}{6}$

$= \frac{140-120}{10.5}$

$= 1.905$

The probability that more than 140 students will be absent can be calculated by

$P(z > \frac{x_1 - \mu}{6})$

$P(z > \frac{140-120}{10.5})$

$P(z > 1.905) = .028405 \Rightarrow$ There is approximately a 2.84% chance that more than 140 students will be absent on attendence day and High School A will lose funding

(b) The principals' association in the state suggests that instead of choosing one day at random, the state should choose 3 days at random. With the suggested plan, High School A would lose some of its state funding in the subsequent year if the mean number of students absent for the 3 days is greater than 140. Would High School A be more likely, less likely, or equally likely to lose funding using the suggested plan compared to the plan described in part (a)? Justify your choice.

High School A would be less likely to lose funding using this plan than it would in part (a) because taking the mean of multiple samples should result in the same population mean of 120, but with a smaller standard deviation. Decreasing the variability from the mean decreases the chance that the mean will stray as high as 140 and the school losing funding.

(c) A typical school week consists of the days Monday, Tuesday, Wednesday, Thursday, and Friday. The principal at High School A believes that the number of absences tends to be greater on Mondays and Fridays, and there is concern that the school will lose state funding if the attendance count occurs on a Monday or Friday. If one school day is chosen at random from each of 3 typical school weeks, what is the probability that none of the 3 days chosen is a Tuesday, Wednesday, or Thursday?

For each of the 3 weeks, the probability of not choosing a Tuesday, Wednesday, or Thursday (thus choosing either a Monday or Friday) is $\frac{2}{5} = .4$

So, the probability of not choosing a Tues, Wed, or Thurs for 3 typical school weeks is $.4^3 = .064$

There is a 6.4% chance that none of the 3 days chosen is a Tuesday, Wednesday, or a Thursday.

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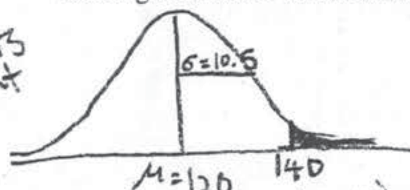
3. Schools in a certain state receive funding based on the number of students who attend the school. To determine the number of students who attend a school, one school day is selected at random and the number of students in attendance that day is counted and used for funding purposes. The daily number of absences at High School A in the state is approximately normally distributed with mean of 120 students and standard deviation of 10.5 students.

(a) If more than 140 students are absent on the day the attendance count is taken for funding purposes, the school will lose some of its state funding in the subsequent year. Approximately what is the probability that High School A will lose some state funding?

= # of students absent

μ = mean daily # of absences at High School A

σ = std. dev of daily # of absences at High School A



$P(x>140) = \text{ncdf}(140, \infty, 120, 10.5) = .032$

(b) The principals' association in the state suggests that instead of choosing one day at random, the state should choose 3 days at random. With the suggested plan, High School A would lose some of its state funding in the subsequent year if the mean number of students absent for the 3 days is greater than 140. Would High School A be more likely, less likely, or equally likely to lose funding using the suggested plan compared to the plan described in part (a)? Justify your choice.

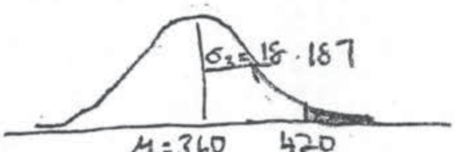
μ_3 = mean # of absences at High School A for 3 days

σ_3 = std. dev of # of absences at High School A for 3 days

$\mu_3 = 120 \times 3 = 360$

$\sigma_3 = \sqrt{10.5^2 \times 3} = 18.187$

$140 \times 3 = 420$



$P(x>420) = \text{ncdf}(420, \infty, 360, 18.187) = .000485$

x = # students gone

less likely b/c of lower probability that the mean # of students gon for 3 days will be over 140 as compared to the prob. that 140 will be gone for 1 day

(c) A typical school week consists of the days Monday, Tuesday, Wednesday, Thursday, and Friday. The principal at High School A believes that the number of absences tends to be greater on Mondays and Fridays, and there is concern that the school will lose state funding if the attendance count occurs on a Monday or Friday. If one school day is chosen at random from each of 3 typical school weeks, what is the probability that none of the 3 days chosen is a Tuesday, Wednesday, or Thursday?

$P(x=3) = \binom{3}{3} .4^3 (1-.4)^{3-3}$

$= .064$

x = # of Mondays or Fridays picked

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3C

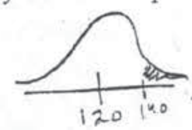
3. Schools in a certain state receive funding based on the number of students who attend the school. To determine the number of students who attend a school, one school day is selected at random and the number of students in attendance that day is counted and used for funding purposes. The daily number of absences at High School A in the state is approximately normally distributed with mean of 120 students and standard deviation of 10.5 students.

(a) If more than 140 students are absent on the day the attendance count is taken for funding purposes, the school will lose some of its state funding in the subsequent year. Approximately what is the probability that High School A will lose some state funding?

1) SRS: given ✓

2) Normal: approx Normal ✓

$$z = \frac{x-\mu}{\sigma} = \frac{140-120}{10.5} = 1.90$$



$P(z > 1.90) =$ **0.0287**

(b) The principals' association in the state suggests that instead of choosing one day at random, the state should choose 3 days at random. With the suggested plan, High School A would lose some of its state funding in the subsequent year if the mean number of students absent for the 3 days is greater than 140. Would High School A be more likely, less likely, or equally likely to lose funding using the suggested plan compared to the plan described in part (a)? Justify your choice.

approx Normal

The school would be less likely.

$$z = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}} = \frac{140-120}{10.5/\sqrt{3}} = 3.299 \qquad P(z > 3.299) \approx 0.0005$$

Since the probability of an average being this extreme for 3 days is only 0.0005, which is less than the probability of one day, 0.0287, the school is less likely to lose money.

(c) A typical school week consists of the days Monday, Tuesday, Wednesday, Thursday, and Friday. The principal at High School A believes that the number of absences tends to be greater on Mondays and Fridays, and there is concern that the school will lose state funding if the attendance count occurs on a Monday or Friday. If one school day is chosen at random from each of 3 typical school weeks, what is the probability that none of the 3 days chosen is a Tuesday, Wednesday, or Thursday?

Mon	Tues	Wed	Thurs	Fri
.2	.2	.2	.2	.2

$[P(M,M,M) + P(MMF) + P(MFF) + P(FFF)]$

= .032

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Question 3

Overview

The primary goals of this question were to assess a student's ability to (1) perform a probability calculation from a normal distribution; (2) explain an implication of examining the distribution of a sample mean rather than the distribution of a single measurement; and (3) perform a probability calculation involving independent events using the multiplication rule.

Sample: 3A
Score: 4

In part (a) the response indicates the use of the normal distribution with a sketch as well as with a correct z-score equation and z-score inequality. The values of the parameters are identified with correct substitution into the z-score equation and z-score inequality. Additionally, the mean is again identified in the sketch. With the normal distribution indicated and the parameter values identified the first component is satisfied. The boundary value is also identified in the z-score equation and z-score inequality and the correct normal probability is computed. Thus, the second and third components are satisfied. With all three components satisfied, part (a) was scored as essentially correct. In part (b) the response begins by stating the correct answer, "High School A would be less likely to lose funding using this plan than it would in part (a)." The next statement refers to the distribution of the sample mean, indicates that this distribution remains centered at 120, and indicates that its standard deviation is smaller, so each of the three components is satisfied. With the correct answer and each of the three components satisfied, part (b) was scored as essentially correct. In part (c) the response correctly uses the multiplication rule with the correct probability and reports the correct answer. Since the multiplication rule is used correctly with work shown, part (c) was scored as essentially correct. Because all three parts were scored as essentially correct, the response earned a score of 4.

Sample: 3B
Score: 3

In part (a) the response indicates the use of the normal distribution and identifies the parameter values with a well-labeled sketch. Thus, the first component is satisfied. The inequality indicates that the probability that X exceeds 140 is being computed, satisfying the second component. However, an incorrect probability is reported, so the third component is not satisfied. With two of the three components satisfied, part (a) was scored as partially correct. In part (b) the response correctly calculates the mean and standard deviation for the distribution of the total number of absences for 3 randomly selected days and correctly computes the probability that the total number of absences for 3 randomly selected days exceeds $420 = 3(140)$. This is mathematically equivalent to computing the probability that the mean number of absences for 3 randomly selected days exceeds 140. The response gives the correct answer, "less likely" and gives a correct comparison to the probability computed in part (a). Since the correct answer, the correct probability, and a correct comparison to the probability in part (a) are given, part (b) was scored as essentially correct. In part (c) the response correctly uses the binomial probability formula with $n = 3$ and the correct value of $p = \frac{2}{5} = 0.4$ to compute the probability that only Mondays or Fridays are selected. Since the correct probability is computed with work shown, part (c) was scored as essentially correct. Because two parts were scored as essentially correct and one part was scored as partially correct, the response earned a score of 3.

Sample: 3C
Score: 2

In part (a) the response indicates the use of the normal distribution with a sketch as well as with a correct z-score equation and a note in the left margin. The values of the parameters are identified with correct substitution into the z-score equation and with labels under the appropriate values in the stem of the problem. Additionally, the value of the mean is identified for a third time in the sketch. With the normal distribution indicated and the parameter values identified, the first component is satisfied. The boundary value is also identified in the z-score equation and on the sketch and the correct normal probability is computed. Thus, the second and third components are satisfied. Another note in the left margin, "1) SRS: given √" indicates that day was randomly selected. Since this is a correct observation, but was not required, it was considered to be extraneous. With all three components satisfied, part (a) was scored as essentially correct. In part (b) the response correctly calculates the probability that the mean number of absences would exceed 140, gives a correct comparison to the probability computed in part (a), and gives the correct answer of "less likely." Since the correct answer, the correct probability, and a correct comparison to the probability in part (a) are given, part (b) was scored as essentially correct. In part (c) the response gives an incorrect probability without justification that the multiplication rule is used, so part (c) was scored as incorrect. Because two parts were scored as essentially correct and one part was scored as incorrect, the response earned a score of 2.