AP® PHYSICS C - MECHANICS
2014 SCORING GUIDELINES

Question 2

15 points total **Distribution of points**

(a) 3 points

For some correct application of conservation of energy — 1 point

$U_i = K_f$

For substituting correct expressions for potential and kinetic energy — 1 point

$mgh_i = \frac{1}{2}mv_f^2$

$gh = \frac{1}{2}v_0^2$

For a correct answer — 1 point

$h = \frac{v_0^2}{2g}$

Alternate solution — *Alternate points*

For using correct kinematics and dynamics equations — *1 point*

$v_f^2 = v_i^2 + 2a(s_f - s_i)$

$F = ma$

For substituting correct variables — *1 point*

$mg\sin\theta = ma$, $\sin\theta = h/L$ *where L is the length of the ramp, so* $a = gh/L$

$v_0^2 = 0 + 2\left(\frac{gh}{L}\right)L$

For a correct answer — *1 point*

$h = \frac{v_0^2}{2g}$

(b)

i. 2 points

For selecting "Zero" — 1 point

For a correct explanation of why the vertical component of the net force is zero. The explanation must be linked to the fact that there is no acceleration. — 1 point

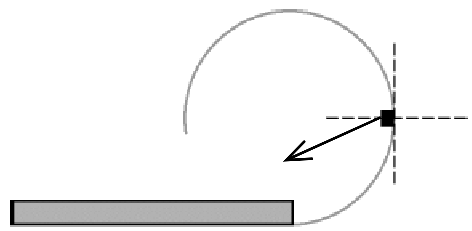
Example:

The block does not accelerate vertically, therefore the component of the net force in the vertical direction must be zero.

Note: No credit is earned if an incorrect choice is selected.

Question 2 (continued)

	Distribution of points
(b) (continued)	
ii. 3 points	



For drawing a single arrow pointing into the third quadrant	2 points
For correct justification of both components	1 point
Example:	
The horizontal component of the net force must provide a centripetal force and also slow down the block. So it must point inward and against the direction of motion.	
Note: If components are drawn, partial credit can be earned for the following.	
For drawing an arrow pointing left (toward the center of the circle) with correct justification	*1 point*
For drawing an arrow pointing toward the bottom of the page with correct justification	*1 point*
A point is deducted for each incorrect vector drawn, with a maximum 2 point deduction	

(c) 1 point

The normal force exerted by the wall is the centripetal force.

$F_N = F_C$

For a correct answer — 1 point

$F_N = mv^2/R$

Note: Since the statement of part (c) says to "determine" an expression, credit is given for just stating the correct answer.

(d) 3 points

For correctly using the frictional force in an expression of Newton's second law — 1 point

$-F_f = ma_t$

$-\mu F_N = ma_t$

For substituting the normal force from part (c) into a correct expression — 1 point

$-\mu mv^2/R = ma_t$

For an answer consistent with part (c) — 1 point

$a_t = -\mu v^2/R$

Note: Since the question asks for the magnitude of the acceleration, the negative sign is not needed but students are not penalized for including it.

Question 2 (continued)

Distribution of points

(e) 3 points

For substituting dv/dt for a into the answer from part (d), <u>or</u> substituting dv/dt for a and the friction force for F_{net} into Newton's second law — 1 point

$$\frac{dv}{dt} = -\frac{\mu v^2}{R} \quad \text{or} \quad m\frac{dv}{dt} = -F_f$$

For including the negative sign — 1 point

Substituting for F_f produces the same relationship as the first equation above.

For separation of variables and using correct limits — 1 point

$$\frac{1}{v^2}dv = -\frac{\mu}{R}dt$$

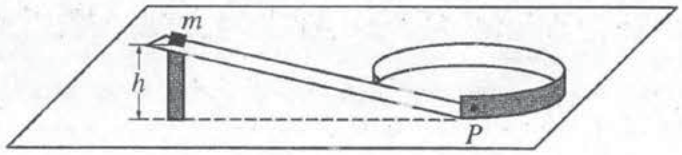
$$\int_{v_0}^{v} \frac{1}{v^2}dv = \int_0^t -\frac{\mu}{R}dt$$

Integrate the equation to solve for v.

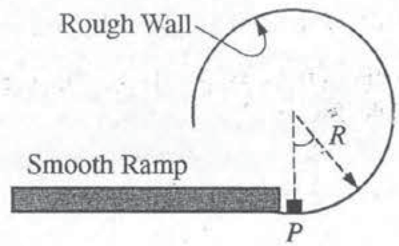
$$\left[-\frac{1}{v}\right]_{v_0}^{v} = \left[-\frac{\mu t}{R}\right]_0^t$$

$$\frac{1}{v} - \frac{1}{v_0} = \frac{\mu t}{R}$$

$$v = \frac{Rv_0}{R + \mu v_0 t} \quad \text{or} \quad \frac{v_0}{1 + \mu v_0 t/R}$$



Side View



Top View

Mech. 2.

A small block of mass m starts from rest at the top of a frictionless ramp, which is at a height h above a horizontal tabletop, as shown in the side view above. The block slides down the smooth ramp and reaches point P with a speed v_0. After the block reaches point P at the bottom of the ramp, it slides on the tabletop guided by a circular vertical wall with radius R, as shown in the top view. The tabletop has negligible friction, and the coefficient of kinetic friction between the block and the circular wall is μ.

(a) Derive an expression for the height of the ramp h. Express your answer in terms of v_0, m, and fundamental constants, as appropriate.

$W_{nc} = \Delta K + \Delta U$

$0 = \frac{1}{2}mv^2 - mgh$

$\frac{1}{2}mv_0^2 = mgh \rightarrow \therefore \boxed{h = \frac{v_0^2}{2g}}$

A short time after passing point P, the block is in contact with the wall and moves with a speed of v.

(b)

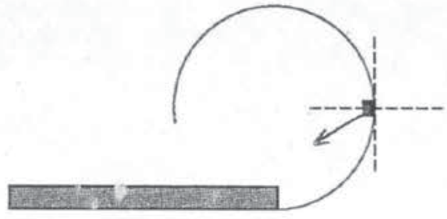
i. Is the vertical component of the net force on the block upward, downward, or zero?

____ Upward ____ Downward _✓_ Zero

Justify your answer.

The block shows centripetal acceleration, but $\Sigma F_{\hat{y}} = 0$ because it does not accelerate up or down. Thus, $mg - N = 0$.

ii. On the figure below, draw an arrow starting on the block to indicate the direction of the horizontal component of the net force on the moving block when it is at the position shown.



Top View

Justify your answer.

The block is in circular motion about the center of the loop. Thus, it must accelerate toward the center of the loop. Furthermore, friction with the loop's wall provides a retarding force (tangential acceleration). These 2 forces together provide the vector shown above.

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Express your answers to the following in terms of v_0, v, m, R, μ, and fundamental constants, as appropriate.

(c) Determine an expression for the magnitude of the normal force N exerted on the block by the circular wall as a function of v.

$$F_c = \frac{mv^2}{r} \longrightarrow \therefore \boxed{N = \frac{mv^2}{R}}$$

(d) Derive an expression for the magnitude of the tangential acceleration of the block at the instant the block has attained a speed of v.

$$\Sigma \vec{F} = ma$$

$$-\mu N = ma$$

$$-\mu\left(\frac{mv^2}{R}\right) = ma$$

$$\therefore \boxed{a = -\frac{\mu v^2}{R}}$$

(e) Derive an expression for $v(t)$, the speed of the block as a function of time t after passing point P on the track.

$$\frac{dv}{dt} = -\frac{\mu v^2}{R}$$

$$\int \frac{dv}{v^2} = \int -\frac{\mu}{R}\,dt$$

$$-\frac{1}{v} = -\frac{\mu t}{R} + C_1$$

When $t = 0$, $v = v_0$

$$\therefore -\frac{1}{v_0} = 0 + C_1$$

$$\therefore C_1 = -\frac{1}{v_0}$$

$$\therefore \frac{1}{v} = \frac{\mu t}{R} + \frac{1}{v_0}$$

$$\frac{1}{v} = \frac{\mu t v_0 + R}{R v_0}$$

$$\therefore \boxed{v(t) = \frac{R v_0}{\mu v_0 t + R}}$$

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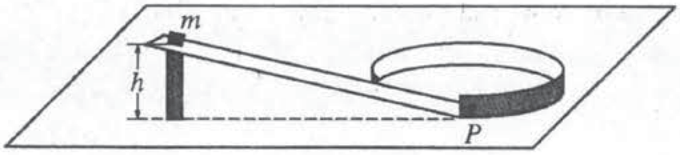
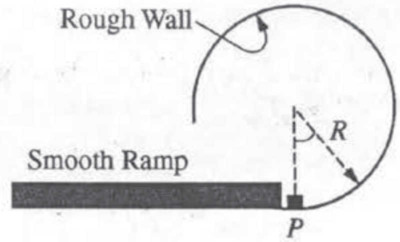


Side View



Top View

Mech. 2.

A small block of mass m starts from rest at the top of a frictionless ramp, which is at a height h above a horizontal tabletop, as shown in the side view above. The block slides down the smooth ramp and reaches point P with a speed v_0. After the block reaches point P at the bottom of the ramp, it slides on the tabletop guided by a circular vertical wall with radius R, as shown in the top view. The tabletop has negligible friction, and the coefficient of kinetic friction between the block and the circular wall is μ.

(a) Derive an expression for the height of the ramp h. Express your answer in terms of v_0, m, and fundamental constants, as appropriate.

$$mgh = \frac{1}{2}mv_0^2$$
$$h = \frac{v_0^2}{2g}$$

A short time after passing point P, the block is in contact with the wall and moves with a speed of v.

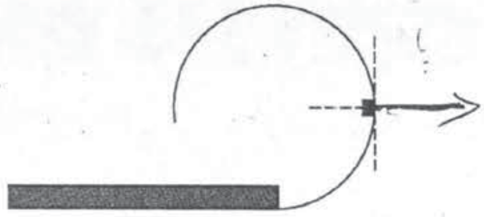
(b)

i. Is the vertical component of the net force on the block upward, downward, or zero?

____ Upward   ____ Downward   X Zero

Justify your answer.
The block is not moving in the vertical direction and the only two vertical forces are F_N and mg which cancel eachother out

ii. On the figure below, draw an arrow starting on the block to indicate the direction of the horizontal component of the net force on the moving block when it is at the position shown.



Top View

Justify your answer.
Since it is going in a circle its net force will be equal to centripetal force which will go in this direction

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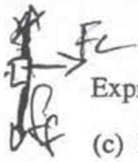

Express your answers to the following in terms of v_0, v, m, R, μ, and fundamental constants, as appropriate.

(c) Determine an expression for the magnitude of the normal force N exerted on the block by the circular wall as a function of v.

$$\Sigma F_x = F_N - F_C = 0 \qquad \Sigma F_y = -F_f = ma$$

$$F_N = F_C$$

$$\boxed{N = \frac{mv^2}{R}}$$

(d) Derive an expression for the magnitude of the tangential acceleration of the block at the instant the block has attained a speed of v.

$$a_c = \frac{v^2}{R}$$

(e) Derive an expression for $v(t)$, the speed of the block as a function of time t after passing point P on the track.

$$-F_f = ma$$

$$-\mu F_N = ma$$

$$-\mu\left(\frac{\cancel{m}v^2}{R}\right) = \cancel{m}a$$

$$\frac{-\mu v^2}{R} = \frac{dv}{dt}$$

$$\int_0^t dt = \int \frac{dv}{\frac{-\mu v^2}{R}} = -\frac{R}{\mu}\int_{v_0}^{v} \frac{dv}{v^2}$$

$$t = -\frac{R}{\mu}\left(-\frac{1}{3v^3}\Big|_{v_0}^{v}\right)$$

$$t = \frac{-R}{\mu}\left(-\frac{1}{3v^3} - \frac{1}{3v_0}\right)$$

$$+\frac{\mu}{R}t = \frac{1}{3}\left(\frac{1}{v^3} + \frac{1}{v_0}\right)$$

$$\frac{3\mu}{R}t = \frac{1}{v} + \frac{1}{v_0}$$

$$\boxed{v(t) = \frac{R}{3\mu t} - v_0}$$

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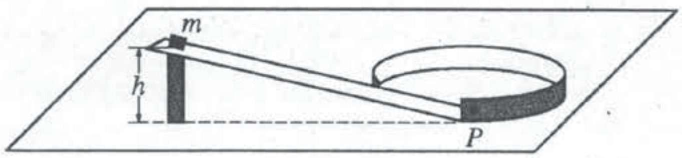

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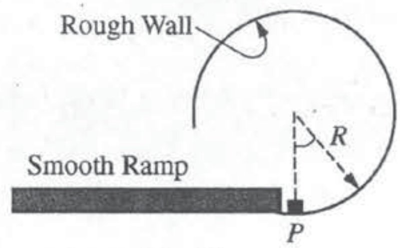

Top View

Mech. 2.

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(a) Derive an expression for the height of the ramp h. Express your answer in terms of v_0, m, and fundamental constants, as appropriate.

$U_1 + K_1 = U_2 + K_2$ for just the ramp

$mgh + 0 = 0 + \frac{1}{2}mv^2$

$\cancel{m}gh = \frac{1}{2}\cancel{m}v^2$

$gh = \frac{1}{2}v^2$

$\boxed{h = \frac{v_0^2}{2g} \text{ meters}}$

A short time after passing point P, the block is in contact with the wall and moves with a speed of v.

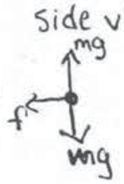
(b)

i. Is the vertical component of the net force on the block upward, downward, or zero?

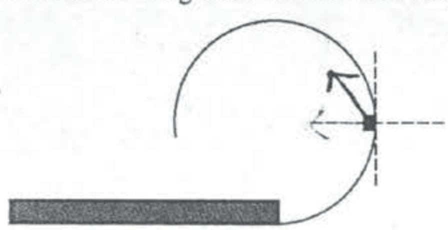
____ Upward ____ Downward _X_ Zero

Justify your answer.

The table is flat, so the normal force equals the weight ($n=mg$).



ii. On the figure below, draw an arrow starting on the block to indicate the direction of the horizontal component of the net force on the moving block when it is at the position shown.


Top View

Justify your answer.

The centripetal force pulls block in & makes it go around the curve while the velocity wants to move it forward at a 90° angle from the F_c. This results in the vector being between the two.

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Express your answers to the following in terms of v_0, v, m, R, μ, and fundamental constants, as appropriate.

(c) Determine an expression for the magnitude of the normal force N exerted on the block by the circular wall as a function of v.

$F = ma$

$-f = m\frac{dv}{dt}$

$-\frac{f}{m} = \frac{dv}{dt}$

$\int_0^t -\frac{f}{m}\,dt = \int_{v_0}^{v} dv$

$-\frac{f}{m}t = v - v_0$

$\frac{\mu mg}{m}$

(d) Derive an expression for the magnitude of the tangential acceleration of the block at the instant the block has attained a speed of v.

$a = \frac{dv}{dt}$

$\int_0^t a\,dt = \int_{v_0}^{v} dv$

$at = v - v_0$

$\boxed{a = t(v - v_0)}$ m/s²

(e) Derive an expression for $v(t)$, the speed of the block as a function of time t after passing point P on the track.

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Question 2

Overview

The primary intent of this question was to assess the students' understanding of Newton's laws as they apply to bodies moving with linear or circular motion, and with constant or nonconstant acceleration. Students' ability to visualize and process physical situations from multiple perspectives was also examined.

Sample: M2 A
Score: 15

This response was clear and straightforward and earned full credit. Note how the work in part (e) does not put limits on the integrals but uses a constant of integration and a boundary condition.

Sample: M2 B
Score: 8

Part (a) earned full credit. Part (b)(i) earned just 1 point, because there was no explicit mention of zero acceleration. Part (b)(ii) earned no credit. Part (c) earned full credit, but (d) earned no credit because there was no Newton's second law expression. However, the acceleration that part (d) asked for was correctly produced and used in (e), so full credit was earned there.

Sample: M2 C
Score: 4

Part (a) earned full credit, and part (b) earned 1 point for "zero" being selected. None of the remaining work was correct, and no further credit was earned.