Question 1

15 points total

(a) 2 points

For using a correct expression for the potential energy of a spring expressed as the integral of the force
\[ U_S = \int F(x)dx \]
\[ U_S = \int_{0}^{s} \left( A x^2 + B x \right) dx \]
Evaluate the definite integral to get an answer with the correct magnitude.
\[ U_S = \left[ \frac{1}{3} A x^3 + \frac{1}{2} B x^2 \right]_{0}^{s} \]
For any correct answer with a local minimum at \( x = 0 \) 1 point
\[ U_S = \frac{1}{3} A s^3 + \frac{1}{2} B s^2 \]
Note: Full credit is given for any consistent use of sign, since the guidelines are concerned with the magnitude. Full credit is also given if there is a constant term added to the correct expression.

(b) 3 points

i. 3 points

For any statement of conservation of mechanical energy 1 point
\[ U_S = K \]
For using the expression for potential energy from part (a) and a correct expression for kinetic energy 1 point
\[ \frac{1}{3} A s^3 + \frac{1}{2} B s^2 = \frac{1}{2} m v^2 \]
Solve for \( v^2 \)
\[ v^2 = 2 \left( \frac{1}{3} A s^3 + \frac{1}{2} B s^2 \right) / m \]
For correct substitution 1 point
\[ v^2 = 2 \left( \frac{1}{3} (200 \text{ N/m}^2)(0.040 \text{ m})^3 + \left( \frac{1}{2} \right) (150 \text{ N/m})(0.040 \text{ m})^2 \right) / (0.30 \text{ kg}) \]
\[ v^2 = 0.828 \text{ m}^2/\text{s}^2 \]
\[ v = 0.91 \text{ m/s} \]
(b) continued

ii. 3 points

For using a correct expression for impulse in terms of change in velocity
\[ J = \Delta p = m(v_2 - v_1) \]
1 point

For substitution of values consistent with the answer in part (b)i
\[ J = (0.30 \text{ kg})(0.91 \text{ m/s} - 0) \]
\[ J = 0.27 \text{ kg} \cdot \text{m/s} \text{ or Ns} \]
1 point

Units 1 point

For correct units in both answers in part (b)
1 point

(c) 3 points

For correctly labeling both axes with variables and units
1 point

For correctly scaling both axes with scales that are linear and such that the curvature in the data is apparent
1 point

For correctly plotting the data points
1 point
(d) i. 2 points

For stating that the measured initial speed of the cart is greater than the predicted value 1 point
For correctly identifying a source of error regarding the initial speed of the cart 1 point
Examples:
The student compressed the spring more than was determined. This would lead to more potential energy in the spring and greater kinetic energy for the cart. The cart would therefore move faster than predicted.
The table is not level, sloping downward would result in a greater measured speed.
The constants \( A \) and \( B \) for the spring are not accurate. The true values are larger than what is given. This would lead to smaller predicted potential energy of the spring and a smaller predicted value for the kinetic energy of the cart. Therefore, the cart would move faster than predicted.

ii. 2 points

For correctly identifying the trend 1 point
For a correct physical explanation for the cart slowing down 1 point
Examples:
Friction in the axles and air resistance against the cart are slowing it down.
The track is not perfectly level and the cart is going uphill. This is slowing down the cart.
PHYSICS C: MECHANICS
SECTION II
Time—45 minutes
3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.

Mech. 1.

In an experiment, a student wishes to use a spring to accelerate a cart along a horizontal, level track. The spring is attached to the left end of the track, as shown in the figure above, and produces a nonlinear restoring force of magnitude \( F_s = As^2 + Bs \), where \( s \) is the distance the spring is compressed, in meters. A measuring tape, marked in centimeters, is attached to the side of the track. The student places five photogates on the track at the locations shown.

(a) Derive an expression for the potential energy \( U \) as a function of the compression \( s \). Express your answer in terms of \( A, B, s \), and fundamental constants, as appropriate.

\[
U = -\frac{W_s}{s}
\]

\[
U = -\int_0^s (As^2 + Bs) \, ds
\]

\[
U = -\left[\frac{As^3}{3} + \frac{Bs^2}{2}\right]
\]
In a preliminary experiment, the student pushes the cart of mass 0.30 kg into the spring, compressing the spring 0.040 m. For this spring, \( A = 200 \text{ N/m}^2 \) and \( B = 150 \text{ N/m} \). The cart is released from rest. Assume friction and air resistance are negligible only during the short time interval when the spring is accelerating the cart.

(b) Calculate the following:

i. The speed of the cart immediately after it loses contact with the spring

\[
\frac{(200)(0.040)^3}{3} + \frac{(150)(0.040)^2}{2} = \frac{1}{2} (0.3) v^2
\]

\[ v = 0.91 \text{ m/s} \]

ii. The impulse given to the cart by the spring

\[ J = m 4v \]

\[ = 0.30(0.91) \]

\[ J = 0.273 \]
In a second experiment, the student collects data using the photogates. Each photogate measures the speed of the cart as it passes through the gate. The student calculates a spring compression that should give the cart a speed of \(0.320\ \text{m/s}\) after the cart loses contact with the spring. The student runs the experiment by pushing the cart into the spring, compressing the spring the calculated distance, and releasing the cart. The speeds are measured with a precision of \(\pm 0.002\ \text{m/s}\). The positions are measured with a precision of \(\pm 0.005\ \text{m}\).

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<thead>
<tr>
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(c) On the axes below, plot the data points for the speed \(v\) of the cart as a function of position \(x\). Clearly scale and label all axes, as appropriate.
(d) i. Compare the speed of the cart measured by photogate 1 to the predicted value of the speed of the cart just after it loses contact with the spring. List a physical source of error that could account for the difference.

\[3.20 \text{ m/s} \quad 4.12 \text{ m/s}\]

The student pushed the cart back too far causing the resulting impulse from the spring to be greater.

ii. From the measured speed values of the cart as it rolls down the track, give a physical explanation for any trend you observe.

A frictional force causes an acceleration that opposes the cart's vel, causing it to slow down.
Mech. 1.

In an experiment, a student wishes to use a spring to accelerate a cart along a horizontal, level track. The spring is attached to the left end of the track, as shown in the figure above, and produces a nonlinear restoring force of magnitude \( F_s = As^2 + Bs \), where \( s \) is the distance the spring is compressed, in meters. A measuring tape, marked in centimeters, is attached to the side of the track. The student places five photogates on the track at the locations shown.

(a) Derive an expression for the potential energy \( U \) as a function of the compression \( s \). Express your answer in terms of \( A, B, s \), and fundamental constants, as appropriate.

\[
\begin{align*}
F_s &= kx \\
U &= \frac{1}{2}kx^2 = \frac{1}{2}F_s \cdot x \\
U &= \frac{1}{2} (As^2 + Bs) \cdot s \\
U &= \frac{1}{2} As^3 + \frac{1}{2}Bs^2
\end{align*}
\]
In a preliminary experiment, the student pushes the cart of mass 0.30 kg into the spring, compressing the spring 0.040 m. For this spring, \( A = 200 \text{ N/m}^2 \) and \( B = 150 \text{ N/m} \). The cart is released from rest. Assume friction and air resistance are negligible only during the short time interval when the spring is accelerating the cart.

(b) Calculate the following:

i. The speed of the cart immediately after it loses contact with the spring

\[
U = \frac{1}{2} m v^2
\]

\[
\frac{A s^2 + B s}{2} = \frac{1}{2} (0.3) v^2
\]

\[
v = 4.477 \text{ m/s}
\]

ii. The impulse given to the cart by the spring

\[
J = \Delta p = m v = (0.3)(4.477) = 1.343 \text{ kg \cdot m/s}
\]
In a second experiment, the student collects data using the photogates. Each photogate measures the speed of the cart as it passes through the gate. The student calculates a spring compression that should give the cart a speed of 0.320 m/s after the cart loses contact with the spring. The student runs the experiment by pushing the cart into the spring, compressing the spring the calculated distance, and releasing the cart. The speeds are measured with a precision of ±0.002 m/s. The positions are measured with a precision of ±0.005 m.

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(c) On the axes below, plot the data points for the speed \( v \) of the cart as a function of position \( x \). Clearly scale and label all axes, as appropriate.
(d)

i. Compare the speed of the cart measured by photogate 1 to the predicted value of the speed of the cart just after it loses contact with the spring. List a physical source of error that could account for the difference.

\[ 0.412 - 0.320 = 0.092 \text{ m/s} \]

- Human measurement errors
- Air resistance
- Friction

ii. From the measured speed values of the cart as it rolls down the track, give a physical explanation for any trend you observe.

When the cart is more closer to lose the contact with the spring, the speed of the cart decreases.
PHYSICS C: MECHANICS
SECTION II
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Mech. 1.

In an experiment, a student wishes to use a spring to accelerate a cart along a horizontal, level track. The spring is attached to the left end of the track, as shown in the figure above, and produces a nonlinear restoring force of magnitude \( F_s = As^2 + Bs \), where \( s \) is the distance the spring is compressed, in meters. A measuring tape, marked in centimeters, is attached to the side of the track. The student places five photogates on the track at the locations shown.

(a) Derive an expression for the potential energy \( U \) as a function of the compression \( s \). Express your answer in terms of \( A, B, s \), and fundamental constants, as appropriate.

\[
\frac{1}{2} K s^2
\]

\[
\frac{1}{2} K s^2
\]

\[
\frac{1}{2} (A+B) s^2 = U
\]
In a preliminary experiment, the student pushes the cart of mass 0.30 kg into the spring, compressing the spring 0.040 m. For this spring, \( A = 200 \text{ N/m}^2 \) and \( B = 150 \text{ N/m} \). The cart is released from rest. Assume friction and air resistance are negligible only during the short time interval when the spring is accelerating the cart.

(b) Calculate the following:

i. The speed of the cart immediately after it loses contact with the spring

\[ \frac{1}{2} m v_f^2 = \frac{1}{2} m (3v)^2 = 2.8 \]

\[ v = 1.3663 \text{ m/s} \]

ii. The impulse given to the cart by the spring

\[ \Delta E = \frac{1}{8} (3)(1.3663)^2 = 0.279 \text{ J} \]
In a second experiment, the student collects data using the photogates. Each photogate measures the speed of the cart as it passes through the gate. The student calculates a spring compression that should give the cart a speed of 0.320 m/s after the cart loses contact with the spring. The student runs the experiment by pushing the cart into the spring, compressing the spring the calculated distance, and releasing the cart. The speeds are measured with a precision of ±0.002 m/s. The positions are measured with a precision of ±0.005 m.

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(c) On the axes below, plot the data points for the speed $v$ of the cart as a function of position $x$. Clearly scale and label all axes, as appropriate.
i. Compare the speed of the cart measured by photogate 1 to the predicted value of the speed of the cart just after it loses contact with the spring. List a physical source of error that could account for the difference. 

\[ \text{friction} \]

ii. From the measured speed values of the cart as it rolls down the track, give a physical explanation for any trend you observe.

\[ \text{there is a nonlinear negative acceleration meaning friction is not constant} \]
Question 1

Overview

This question assessed understanding of the relationships between force, energy, and impulse, as well as application of conservation of energy. In addition, the analysis of data was used to evaluate the students' understanding of the system and ability to provide physical explanations for the results.

Sample: M1 A
Score: 14

The only point lost by this response was the units point in part (b), since there are no units on the answer for impulse. Writing the two values of speed in part (d)(i) was minimal but does acknowledge their difference.

Sample: M1 B
Score: 9

The work in part (a) has no attempt to evaluate an integral and earned no credit. Part (b) earned full credit, including the units point, since the incorrect answer from (a) was used correctly. One point was lost in part (c) for having the variables on the wrong axes. Part (d)(i) earned 1 point for the numerical comparison, but the explanation was not correct for an actual speed that is greater than the predicted one. Part (d)(ii) earned no credit, since the speed of the cart decreases as it moves farther from the point of losing contact.

Sample: M1 C
Score: 5

The work in part (a) has no attempt to evaluate an integral and earned no credit. Part (b)(i) earned full credit. The expression from (a) was correctly used to calculate a potential energy, and then set equal to a correct expression for kinetic energy. While the final answer does not follow from the work, all the substitutions are correct. Part (b)(ii) has nothing related to impulse and earned no credit. The units for impulse are wrong, so the units point was not earned. Parts (c) and (d)(i) also earned no credit. In part (c), "v" does not designate a unit, the horizontal scale is not linear, and the last three points are clearly not plotted at the correct speeds. Part (d)(ii) earned full credit.