Question 5

15 points total

(a) 2 points

The correct choice is “Q”.

Note: Selecting “Q” earned no points, but was required in order to earn credit for a correct justification.

For recognizing that the current must be to the left in the rod (or clockwise in the circuit) 1 point

For a complete explanation with no incorrect statements 1 point

Example: In order to stretch the springs, the magnetic force must be downward.

According to the right hand rule, this requires a current to the left in the rod.

Therefore, the positive terminal of the battery must be connected to point Q.

(b) 2 points

Note: The spring force may be represented either as a single combined force or as two separate forces due to the two springs.

For the spring force (or forces) drawn upward and properly labeled (with no additional upward forces) 1 point

For the magnetic and gravitational forces drawn downward and properly labeled (with no additional downward forces) 1 point

One earned point is deducted for any of the following:

• Any horizontal forces
• Any vector that is not distinct and/or does not both touch and point away from the dot
• The vectors are not reasonably straight and reasonably vertical

(c) 4 points

For using a correct expression for Newton’s second law 1 point

Using a single spring force: $F_S - mg - F_B = 0$ or $F_S = mg + F_B$

Using two spring forces: $F_{S1} + F_{S2} - mg - F_B = 0$ or $F_{S1} + F_{S2} = mg + F_B$

For any recognition, implicit or explicit, that $mg$ is balanced by the original stretch in the springs and ultimately not part of this calculation 1 point

The spring force is $2k\Delta y$. It is not necessary to explicitly show how the original spring force is canceled by $mg$.

$2k\Delta y = F_B$

For correctly substituting $BIL$ for the magnetic force 1 point

$2k\Delta y = BIL$

For solving for $\Delta y$ to get a correct answer 1 point

$\Delta y = BIL/2k$
(d)  4 points

For correctly labeling both axes with variables and units  
1 point
Note: This point is not earned if axes are labeled with \( I \) as a function of \( \Delta y \).

For using an appropriate linear scale on both axes  
1 point
For correctly plotting the points  
1 point
For drawing a reasonable and straight best fit line for the data  
1 point
Note: The point (0, 0) may be included in the plot without penalty since it is also a data point, though unmeasured and unrecorded.

(e)  3 points

For obtaining a relationship between the slope and the magnetic field \( B \) consistent with part (c).  
1 point

From part (c): \( \Delta y = \frac{BIL}{2k} = \frac{BL}{2k} I \)

slope = \( \frac{BL}{2k} \) so \( B = \frac{2k \cdot \text{slope}}{L} \)

For calculating a slope using two points on the line drawn in part (d), including data points only if they are on that line  
1 point
Example: Using the two points (5.0 A, 14.2 mm) and (1.2 A, 3.0 mm) that are on the line in the graph above.

\[
slope = \left( \frac{14.2 - 3.0}{5.0 - 1.2} \right) \cdot \frac{10^{-3} \text{ m}}{\text{A}} = \frac{11.2}{3.8} \times 10^{-3} \frac{\text{m}}{\text{A}} = 2.95 \times 10^{-3} \frac{\text{m}}{\text{A}}
\]

For a substitution of values into the expression for the magnetic field \( B \) consistent with part (c) and the calculated slope  
1 point
\[
B = \frac{2 \cdot (25 \text{ N/m}) \cdot (2.95 \times 10^{-3} \text{ m/A})}{0.35 \text{ m}} = 0.42 \text{ N/A\cdot m or 0.42 T}
\]
5. (15 points)
A conducting rod of mass $m$ and length $L$ hangs at rest from two identical conducting springs, each with spring constant $k$, as shown in the figure at left above. The upper ends of the springs are fixed at points $P$ and $Q$, and the rod is in a uniform magnetic field $B$ directed into the page. A battery is then connected between points $P$ and $Q$, as shown in the figure at right above, resulting in a current $I$ in the rod. The rod is displaced downward, eventually reaching a new equilibrium position with the springs stretched an additional distance $\Delta y$.

(a) Which point, $P$ or $Q$, is connected to the positive terminal of the battery?

\[ \overline{P} \quad \sqrt{Q} \]

Justify your answer.

The force on the rod is down, so using the right hand rule one can determine that the current must be flowing clockwise.

(b) On the dot below that represents the rod, draw and label the forces (not components) that act on the rod in its new equilibrium position. Each force must be represented by a distinct arrow starting on, and pointing away from, the dot.

(c) Derive an expression for $\Delta y$ in terms of $k$, $m$, $L$, $I$, the magnetic field strength $B$, and fundamental constants, as appropriate.

\[ F = -kx \]
\[ F = k\Delta y \]
\[ \Delta y = \frac{mg + BIL}{k} \]
\[ \Delta y = \frac{BIL}{k} \]

\[ F_B = BIL \]

GO ON TO THE NEXT PAGE.
An experiment is conducted with batteries of different emf connected between points P and Q. The current I in the rod and the stretch of the springs Δy are measured and recorded in the table below.

<table>
<thead>
<tr>
<th>I (amperes)</th>
<th>Δy (meters)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>0.0028</td>
</tr>
<tr>
<td>2.0</td>
<td>0.0050</td>
</tr>
<tr>
<td>3.0</td>
<td>0.0084</td>
</tr>
<tr>
<td>4.0</td>
<td>0.0119</td>
</tr>
<tr>
<td>5.0</td>
<td>0.0140</td>
</tr>
</tbody>
</table>

(d) On the grid below, plot the data points for Δy as a function of I. Be sure to label your axes with variables, units, and scale. Draw a straight line that best represents the data.

(e) Using the straight line you drew in part (d), calculate the value B for the magnetic field if \( m = 0.019 \text{ kg} \), \( L = 0.35 \text{ m} \), and \( k = 25 \text{ N/m} \).

\[
\frac{\Delta y}{I} = \frac{B I}{L}
\]

\[
\Delta y = \frac{k}{L} \cdot \Delta y = \frac{B I \cdot 3.5}{25 \text{ N/m}}
\]

\[
\frac{0.0028}{I} = \frac{B \cdot 3.5}{25 \text{ N/m}}
\]

\[
0.12 = B
\]
5. (15 points)

A conducting rod of mass \( m \) and length \( L \) hangs at rest from two identical conductors, each with spring constant \( k \), as shown in the figure at left above. The upper ends of the springs are fixed at points \( P \) and \( Q \), and the rod is in a uniform magnetic field \( B \) directed into the page. A battery is then connected between points \( P \) and \( Q \), as shown in the figure at right above, resulting in a current \( I \) in the rod. The rod is displaced downward, eventually reaching a new equilibrium position with the springs stretched an additional distance \( \Delta y \).

(a) Which point, \( P \) or \( Q \), is connected to the positive terminal of the battery?

\[ X \quad P \quad O \]

Justify your answer.

If the contraction moves down in this field, the right hand rule implies a current from \( P \) to \( Q \). Current flows from positive to negative, implying \( P \) is positive.

(b) On the dot below that represents the rod, draw and label the forces (not components) that act on the rod in its new equilibrium position. Each force must be represented by a distinct arrow starting on, and pointing away from, the dot.

\[ F_{\text{spring}_1} \quad \uparrow \quad F_{\text{spring}_2} \quad \downarrow F_B \]

(c) Derive an expression for \( \Delta y \) in terms of \( k \), \( m \), \( L \), \( I \), the magnetic field strength \( B \), and fundamental constants, as appropriate.

\[ \sum F_y = 2F_3 - F_B = 0 \]
\[ F_B = BIL \]
\[ F_3 = -kx = -k \Delta y \]
\[ -2k \Delta y - BIL = 0 \]
\[ \Delta y = \frac{BIL}{2k} \]

Unauthorized copying or reuse of any part of this page is illegal.
An experiment is conducted with batteries of different emf connected between points P and Q. The current $I$ in the rod and the stretch of the springs $\Delta y$ are measured and recorded in the table below.

<table>
<thead>
<tr>
<th>$I$ (amperes)</th>
<th>$\Delta y$ (meters)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>0.0028</td>
</tr>
<tr>
<td>2.0</td>
<td>0.0050</td>
</tr>
<tr>
<td>3.0</td>
<td>0.0084</td>
</tr>
<tr>
<td>4.0</td>
<td>0.0119</td>
</tr>
<tr>
<td>5.0</td>
<td>0.0140</td>
</tr>
</tbody>
</table>

(d) On the grid below, plot the data points for $\Delta y$ as a function of $I$. Be sure to label your axes with variables, units, and scale. Draw a straight line that best represents the data.

(e) Using the straight line you drew in part (d), calculate the value $B$ for the magnetic field if $m$ is 0.019 kg, $L$ is 0.35 m, and $k$ is 25 N/m.

\[
\Delta y = \frac{BIL}{2k}
\]

$L$, $I$, and $k$ are constants, $I$ is the independent variable, so $B$ is the slope of $\Delta y(I)$.

\[
B = \frac{(\Delta y_{5} - \Delta y_{1})}{(I_{5} - I_{1})} = \frac{0.014 - 0.0028}{5 - 1}
\]

$B = 0.0028 \, T$
5. (15 points)

A conducting rod of mass $m$ and length $L$ hangs at rest from two identical conducting springs, each with spring constant $k$, as shown in the figure at left above. The upper ends of the springs are fixed at points $P$ and $Q$, and the rod is in a uniform magnetic field $B$ directed into the page. A battery is then connected between points $P$ and $Q$, as shown in the figure at right above, resulting in a current $I$ in the rod. The rod is displaced downward, eventually reaching a new equilibrium position with the springs stretched an additional distance $\Delta y$.

(a) Which point, $P$ or $Q$, is connected to the positive terminal of the battery?

\[ \sqrt{\text{(b) \quad \text{Since the field is going into the page, the current is moving clockwise. Since batteries go from high potential to low potential, the only the system would work is if the current was going from positive to negative.}} \}

(b) On the dot below that represents the rod, draw and label the forces (not components) that act on the rod in its new equilibrium position. Each force must be represented by a distinct arrow starting on, and pointing away from, the dot.

(c) Derive an expression for $\Delta y$ in terms of $k$, $m$, $L$, $I$, the magnetic field strength $B$, and fundamental constants, as appropriate.

\[ F_B = I l B \cos \Theta \]
\[ \Delta \Phi = B \Delta \Phi \cos \Theta \]
\[ \frac{B}{\Delta \Phi} = \frac{\Delta y}{\cos \Theta} \]
An experiment is conducted with batteries of different emf connected between points P and Q. The current I in the rod and the stretch of the springs Δy are measured and recorded in the table below.

<table>
<thead>
<tr>
<th>I (amperes)</th>
<th>Δy (meters)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>0.0028</td>
</tr>
<tr>
<td>2.0</td>
<td>0.0050</td>
</tr>
<tr>
<td>3.0</td>
<td>0.0084</td>
</tr>
<tr>
<td>4.0</td>
<td>0.0119</td>
</tr>
<tr>
<td>5.0</td>
<td>0.0140</td>
</tr>
</tbody>
</table>

(d) On the grid below, plot the data points for Δy as a function of I. Be sure to label your axes with variables, units, and scale. Draw a straight line that best represents the data.

(c) Using the straight line you drew in part (d), calculate the value B for the magnetic field if m is 0.019 kg, L is 0.35 m, and k is 25 N/m.

\[
\frac{1}{\Delta y} \quad m = 0.019 \text{ kg} \quad L = 0.35 \quad k = 25 \text{ N/m}
\]

\[
\text{Slope} = \frac{2.8 - 1.4}{0.008 - 0.004} = 350 \frac{I}{\Delta y}
\]
Overview

The intent of this question was to assess student knowledge of electromagnetism, their understanding of forces, equilibrium and dynamics, and their graphing and data analysis skills.

Sample: B5 A
Score: 13

This response earned full credit in parts (a) and (b). In part (c) the answer point was not awarded because the factor of 2 from the two springs was not included in the final expression. Part (d) earned full credit. Only two points were awarded for part (e) because it was not clear how the slope was calculated, but the relationship between slope and magnetic field was consistent with part (b).

Sample: B5 B
Score: 9

This response earned no credit in part (a) because “P” was checked. One point was earned in part (b) for the upward spring forces. The “downward” force point was not awarded because weight was missing (and the magnetic force vector does not touch the dot). This response earned full credit in part (c). Note that all of the points are addressed, even though there was a sign error in the problem that does not drop out until the last step. Only 3 points were earned in part (d); the best-fit line point was not awarded because this appears to be more of a “connect-the-dots” graph. One point was earned in part (e) for calculating the slope, but it was not used or interpreted correctly.

Sample: B5 C
Score: 4

This response earned no points in part (a) because “P” was checked. There also was no credit earned in part (b) because, though \( F_s \) was drawn and labeled correctly for 1 point, a point was then deducted for the horizontal force. No credit was earned in part (c). The magnetic force was written as \( ILB \cos \theta \) rather than \( ILB \sin \theta \), and the \( \cos \theta \) term was not appropriately dealt with. There was no clear demonstration of use of Newton’s second law. The response earned full credit in part (d). No credit was earned in part (e) because the slope was incorrectly determined and then not used.