Question 4

10 points total

(a) 2 points

For using and substituting values into the equation for the electric field magnitude due to a point charge, for both the \(x\) and \(y\) components

\[ E = \frac{kq}{r^2} \quad \text{or} \quad \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2} \]

For correctly calculating the \(x\) and \(y\) component values with units

\[ |E_x| = \left( \frac{9 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2}{(3 \text{ m})^2} \right) (6 \times 10^{-9} \text{ C}) = 6.0 \text{ N/C} \quad \text{(or V/m)} \]

\[ |E_y| = \left( \frac{9 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2}{(3 \text{ m})^2} \right) (2 \times 10^{-9} \text{ C}) = 2.0 \text{ N/C} \quad \text{(or V/m)} \]

Note: No points were deducted for omitting absolute-value symbols or showing incorrect signs, since direction is assessed in part (b)

(b) 2 points

For showing the direction of the net electric field in quadrant II

For showing a direction of the net electric field consistent with the answers to part (a)

(closer to the \(x\)-axis than to the \(y\)-axis if part (a) correctly has \(|E_x| > |E_y|\)),

or closer to the \(y\)-axis than to the \(x\)-axis if part (a) has \(|E_y| > |E_x|\),

or at 45 degrees if part (a) has \(|E_x| = |E_y|\)

Note: The second point could be earned even if the vector was drawn in the wrong quadrant.
(c) 2 points

For using a correct expression for the potential at the origin as a scalar sum 1 point

\[ V = \frac{kq_1}{r_1} + \frac{kq_2}{r_2} \]

\[ V = \left(9 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2\right) \left[\frac{(-2 \times 10^{-9} \text{ C})}{(3 \text{ m})} + \frac{(6 \times 10^{-9} \text{ C})}{(3 \text{ m})}\right] \]

\[ = \left(9 \text{ N} \cdot \text{m}^2/\text{C}^2\right)(4 \text{ C})/(3 \text{ m}) \]

For correctly calculating the potential, with units 1 point

\[ V = 12 \text{ N} \cdot \text{m/C} \quad \text{(or J/C or V)} \]

(d) 2 points

For selecting “Positive” 1 point

For providing a correct explanation 1 point

Example: Since \( W = q\Delta V \) and both the charge and potential difference are positive, it takes positive work to move charge \( q_3 \) to the origin.

(e) 2 points

For using Coulomb’s law for the forces exerted on \( q_3 \) due to charges \( q_1 \) and \( q_2 \) 1 point

OR

For stating the relation between force and electric field components

\[ F_x = \frac{k|q_2 q_3|}{r^2} \quad \text{and} \quad F_y = \frac{k|q_1 q_3|}{r^2} \quad \text{OR} \quad F_x = q_3 E_x \quad \text{and} \quad F_y = q_3 E_y \]

Calculate the components of \( \mathbf{F} \)

\[ F_x = \left(9 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2\right)\left(6 \times 10^{-9} \text{ C}\right)\left(3 \times 10^{-9} \text{ C}\right)/\left(3^2 \text{ m}^2\right) = 18 \times 10^{-9} \text{ N} = 1.8 \times 10^{-8} \text{ N} \]

\[ F_y = \left(9 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2\right)\left(2 \times 10^{-9} \text{ C}\right)\left(3 \times 10^{-9} \text{ C}\right)/\left(3^2 \text{ m}^2\right) = 6.0 \times 10^{-9} \text{ N} \]

OR

\[ F_x = q_3 E_x = (6.0 \text{ N/C})\left(3 \times 10^{-9} \text{ C}\right) = 1.8 \times 10^{-8} \text{ N} \]

\[ F_y = q_3 E_y = (2.0 \text{ N/C})\left(3 \times 10^{-9} \text{ C}\right) = 6.0 \times 10^{-9} \text{ N} \]

For substituting the calculated components into an expression for the magnitude of \( \mathbf{F} \) 1 point

\[ F = \sqrt{F_x^2 + F_y^2} = \sqrt{(1.8 \times 10^{-8} \text{ N})^2 + (6.0 \times 10^{-9} \text{ N})^2} \]

\[ = 1.9 \times 10^{-8} \text{ N} \]
Alternate Solution

For calculating the net electric field as a vector sum of the components calculated in part (a)

\[ E_{\text{net}} = \sqrt{E_x^2 + E_y^2} = \sqrt{(6 \, \text{N/C})^2 + (2 \, \text{N/C})^2} = 6.2 \, \text{N/C} \]

For substituting into the relationship between force and electric field and using the value of charge \( q_3 \)

\[ F_{\text{net}} = qE_{\text{net}} = q_3E = \left(3 \times 10^{-9} \, \text{C}\right)(6.32 \, \text{N/C}) = 1.9 \times 10^{-8} \, \text{N} \]
4. (10 points)

Two point charges are fixed at the coordinates shown in the diagram above. The charges are \( q_1 = -2.0 \text{ nC} \) and \( q_2 = +6.0 \text{ nC} \).

(a) Calculate the magnitudes of the \( x \) and \( y \) components of the net electric field at the origin \((0, 0)\).

\[
E_y = \frac{kq_1}{r^2} = \frac{(9 \times 10^9 \text{ Nm}^2/\text{C}^2)(2 \times 10^{-9} \text{ C})}{(3 \text{ m})^2} \quad E_y = 2 \text{ N/C} \\
E_x = \frac{kq_2}{r^2} = \frac{(9 \times 10^9 \text{ Nm}^2/\text{C}^2)(6 \times 10^{-9} \text{ C})}{(3 \text{ m})^2} \quad E_x = 6 \text{ N/C}
\]

(b) On the diagram below, draw a single vector (not components) originating at the origin \((0,0)\) to represent the direction of the net electric field at that point.
(c) Calculate the electric potential at the origin (0, 0).

\[
V = k \left( \frac{q_1}{r_1} + \frac{q_2}{r_2} \right) = \left(9 \times 10^9 \text{N m}^2/\text{C}^2\right) \left(\frac{-2 \times 10^3}{3 \text{ m}} + \frac{6 \times 10^3}{3 \text{ m}}\right)
\]

\[V = 12 \text{ V}\]

A third charge, \(q_3 = +3.0 \text{ nC}\), is moved by an external force from very far away to the origin. The third charge has the same speed at the start and end of the motion.

(d) Indicate whether the total work done by the external force is positive, negative, or zero.

\[
\begin{array}{ccc}
\sqrt{\text{Positive}} & \_\_\_\_ \text{Negative} & \_\_\_\_ \text{Zero}
\end{array}
\]

Justify your answer.

In order to maintain its speed, it would gain potential energy as it moved against the electric field, therefore the work (\(\Delta U\)) is positive.

(e) Calculate the magnitude of the net force on \(q_3\) due to the other two charges when \(q_3\) is at the origin.

\[
F_1 = k \frac{q_1 q_3}{r^2} = \left(9 \times 10^9 \text{N m}^2/\text{C}^2\right) \left(3 \times 10^3 \text{ C}\right) \left(-2 \times 10^3 \text{ C}\right) = -6 \times 10^{-9} \text{ N}
\]

\[
F_2 = k \frac{q_2 q_3}{r^2} = \left(9 \times 10^9 \text{N m}^2/\text{C}^2\right) \left(3 \times 10^3 \text{ C}\right) \left(6 \times 10^{-9} \text{ C}\right) = 1.8 \times 10^{-8} \text{ N}
\]

\[
\begin{align*}
1.8 \times 10^{-8} \text{ N}^2 + (6 \times 10^{-10} \text{ N})^2 &= F_{\text{net}}^2 \\
F_{\text{net}} &= 1.88 \times 10^{-8} \text{ N}
\end{align*}
\]
4. (10 points)

Two point charges are fixed at the coordinates shown in the diagram above. The charges are \( q_1 = -2.0 \text{ nC} \) and \( q_2 = +6.0 \text{ nC} \).

(a) Calculate the magnitudes of the \( x \) and \( y \) components of the net electric field at the origin (0, 0).

\[
\begin{align*}
E_{x_1} &= \frac{kq_1}{r^2} \\
E_{x_2} &= \frac{kq_2}{r^2} \\
E_{y_1} &= \frac{2 \cdot 10^{-9} \text{ C}}{(3 \text{ m})^2} \\
E_{y_2} &= \frac{6 \cdot 10^{-9} \text{ C}}{(3 \text{ m})^2} \\
E_{x} &= 2 \text{ N/C} \\
E_{y} &= 6 \text{ N/C}
\end{align*}
\]

\[
\begin{align*}
\text{\text{y\ component\ is\ 2N/C}} \\
\text{\text{x\ component\ is\ 6N/C}}
\end{align*}
\]

(b) On the diagram below, draw a single vector (not components) originating at the origin (0,0) to represent the direction of the net electric field at that point.
(c) Calculate the electric potential at the origin (0, 0).

\[
V = K \left( \frac{q_1}{r_1} + \frac{q_2}{r_2} \right) \\
V = K \left( -\frac{2\pi \epsilon_0}{3m} + \frac{6\pi \epsilon_0}{3m} \right) \\
V = K \left( \frac{4\pi \epsilon_0}{3m} \right) \\
V = 12 V
\]

A third charge, \(q_3 = +3.0 \text{ nC}\), is moved by an external force from very far away to the origin. The third charge has the same speed at the start and end of the motion.

(d) Indicate whether the total work done by the external force is positive, negative, or zero.

___ Positive ___ Negative ___ Zero

Justify your answer.

\[\text{work is change in energy, particle does not change height, and kinetic energy is constant} \]

(e) Calculate the magnitude of the net force on \(q_3\) due to the other two charges when \(q_3\) is at the origin.

\[
F_{q_1} = \frac{Kq_1 q_3}{r^2} \\
F_{q_2} = K \frac{(2\pi \epsilon_0)(3\pi \epsilon_0)}{(3m)^2} \\
F_{q_3} = 6 \cdot 10^{-9} \text{ N} \\
F_{net} = \sqrt{F_{q_1}^2 + F_{q_2}^2} \\
F_{net} = 1.9 \cdot 10^{-8} \text{ N}
\]
4. (10 points)

Two point charges are fixed at the coordinates shown in the diagram above. The charges are \( q_1 = -2.0 \text{ nC} \) and \( q_2 = +6.0 \text{ nC} \).

(a) Calculate the magnitudes of the \( x \) and \( y \) components of the net electric field at the origin \((0, 0)\).

\[
F = \frac{kq_1q_2}{r^2}
\]

\[
k = q \times 10^9
\]

\[
F_x = \frac{(q \times 10^9) \times (60 \text{ C})}{3^2} = 6 \text{ N}
\]

\[
F_y = \frac{(1 \times 10^9) \times (-2 \text{ nC})}{3^2} = -2 \text{ N}
\]

(b) On the diagram below, draw a single vector (not components) originating at the origin \((0,0)\) to represent the direction of the net electric field at that point.
(c) Calculate the electric potential at the origin (0, 0).

\[ V = k \left( \frac{q_1}{r_1} + \frac{q_2}{r_2} \right) \]

\[ q_1 \times 10^8 \left( \frac{6nC}{3m} + \frac{-2nC}{3} \right) = V \]

\[ V = 12 \]

A third charge, \( q_3 = +3.0 \text{ nC} \), is moved by an external force from very far away to the origin. The third charge has the same speed at the start and end of the motion.

(d) Indicate whether the total work done by the external force is positive, negative, or zero.

___ Positive ___ Negative ___ Zero

Justify your answer.

b/c the forces that pushed away/attached it from the origin will then push towards/attract away from the origin

(e) Calculate the magnitude of the net force on \( q_3 \) due to the other two charges when \( q_3 \) is at the origin.

\[ F_x = \frac{kq_1q_2}{r^2} \]

\[ F_x = 6N \]

\[ F_N = 2N \]

\[ F_x = \frac{9 \times 10^9 \times 6nC \times 3nC}{3m^2} = 1.8 \times 10^{-8} \text{ N} \]

\[ F_y = \frac{9 \times 10^9 \times (-2nC) \times 3nC}{3m^2} = -6 \times 10^{-9} \text{ N} \]
Question 4

Overview

The intent of this question was to assess student understanding of electrostatics, specifically the concepts of Coulomb’s law, electric fields, and voltage.

Sample: B4 A
Score: 10

This response earned full credit and showed calculations that treated units and orders of magnitude clearly and carefully. While the ratio of vector components in part (b) does not match the answer from part (a), the rubric only requires that the vector be closer to the $x$-axis than the $y$-axis.

Sample: B4 B
Score: 7

Part (a) earned full credit. The vector in part (b) has the correct ratio of components, but it was drawn in the incorrect quadrant, so only 1 point was earned. Part (c) earned full credit, but part (d) earned nothing because the incorrect choice was selected. Part (e) also earned full credit.

Sample: B4 C
Score: 3

In part (a), the equation for the force was written instead of the electric field. While substitutions are made that are equivalent to the field, the equations clearly note them as forces and have units of newtons, so no credit was earned. In part (b) the ratio of components was consistent with part (a), but the vector is in the wrong quadrant, so only 1 point was earned. In part (c) the correct expression was used but there are no units on the answer, so only 1 point was earned. Part (d) earned nothing because the incorrect choice was selected. Part (e) earned 1 point for using Coulomb’s law to calculate the force components.