Question 3

10 points total	**Distribution of points**

(a)

i. 2 points

For correctly ranking all 4 labeled points — 2 points

<u>1</u> *A* <u>2</u> *B* <u>4</u> *C* <u>3</u> *D*

Note: one point is earned for a ranking that identifies *A* and *C* as having the lowest and highest temperatures, respectively, but incorrectly ranks *B* and *D*. For example:

<u>1</u> *A* <u>3</u> *B* <u>4</u> *C* <u>2</u> *D* or <u>1</u> *A* <u>2</u> *B* <u>3</u> *C* <u>2</u> *D*

A and *C* cannot have the same ranking as other points (e.g., <u>1</u> *A* <u>1</u> *B* <u>2</u> *C* <u>2</u> *D*)

ii. 1 point

For using the ideal gas law with correct substitutions for *P*, *V*, and *n* to get the correct answer — 1 point

$$PV = nRT$$

$$T = \frac{PV}{nR}$$

$$T_D = \frac{P_0(4V_0)}{3R} = \frac{4P_0V_0}{3R} \text{ or } \frac{4P_0V_0}{(3\text{ mol})\bullet R}$$

Note: mole units need not be explicitly stated

(b) 2 points

For selecting “BC” only — 1 point

For providing a correct explanation — 1 point

Examples:

- For the gas to do positive work on its surroundings (or for work done on the gas to be negative), $P\Delta V$ must be positive so ΔV must be positive.
- An expanding gas exerts a force in the direction of motion, doing positive work.

(c) 2 points

“Positive” is the only box checked for which the Justify section will be read

For a correct explanation that mentions internal energy increases with the temperature increase. — 1 point

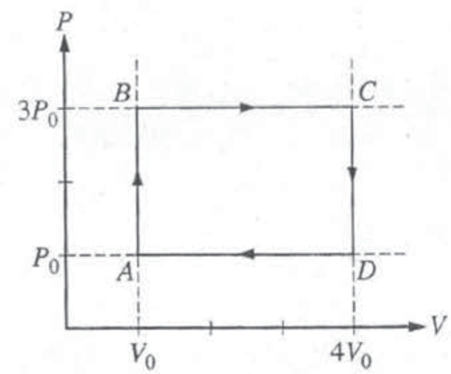
For a correct explanation that mentions the work is zero — 1 point

Example:

The volume does not change in process *AB*, hence the work done is zero. The temperature increase in *AB* means the internal energy increases. Since the work done is zero, the positive change in internal energy must be provided by heating.

Question 3 (continued)

	Distribution of points
(d) 3 points	
For a correct expression of net work done on the gas from segments *BC* and *DA* of the graph	1 point
$W_{net} = -(P\Delta V)_{BC} - (P\Delta V)_{DA}$	
For correctly substituting the values of *P* and ΔV into the above equation	1 point
$W_{net} = -(3P_0)(3V_0) - (P_0)(-3V_0) = -9P_0V_0 + 3P_0V_0 = -6P_0V_0$	
For having a final answer that is negative	1 point
Alternate Solution	*Alternate points*
For a correct expression or statement that the net work done on the gas is given by the area enclosed by the entire path	*1 point*
$W_{net} = -(\Delta P \bullet \Delta V)$	
For correctly substituting the values of P and ΔV *into the above equation*	*1 point*
$W_{net} = -(3P_0 - P_0)\bullet(4V_0 - V_0) = -2P_0 \bullet 3V_0 = -6P_0V_0$	
For having a negative answer	*1 point*



3. (10 points)

A sample containing three moles of an ideal gas is taken through a series of equilibrium states, as represented by the closed path *ABCDA* in the diagram above.

(a)

i. Rank the temperatures at the 4 labeled points from least to greatest, using 1 for the lowest temperature. If two or more points have the same temperature, give them the same ranking.

<u>1</u> *A* <u>2</u> *B* <u>4</u> *C* <u>3</u> *D*

$PV = nRT$ (nR: const.)

ii. Determine the temperature T_D at point *D* in terms of P_0, V_0, and fundamental constants, as appropriate.

$n = 3$

$PV = nRT$

$T = \frac{PV}{nR}$

$T_D = \frac{(P_0)(4V_0)}{3R}$

$$T_D = \frac{4P_0V_0}{3R}$$

(b) Indicate all segments of the path *ABCDA*, if any, for which the work done <u>by</u> the gas is positive. If the work done by the gas is not positive for any of the segments, then check "None".

____ *AB* ✓ *BC* ____ *CD* ____ *DA* ____ None

↓ expansion

Justify your answer.

Segments AB and CD do not do any work on or by the gas because $W_{gas} = P\Delta V$ only works for constant pressure. Segment DA has a negative value for ΔV, resulting in $-W_{gas}$ for that segment. Whereas, segment BC is the only time the gas expands for ΔV to be positive and W_{gas} to be positive.

$W = -P\Delta V$

$W = -W_{gas}$

$+W_{gas} = +P\Delta V$

$W_{gas} = P\Delta V$

GO ON TO THE NEXT PAGE.

(c) In process AB, is the energy transferred to the gas by heating positive, negative, or zero?

__✓__ Positive ____ Negative ____ Zero

Justify your answer.

$\Delta U = Q + W$

$\Delta U = \frac{3}{2} nR\Delta T$

$\Delta U = \frac{3}{2}\Delta(PV)$

$\Delta U = Q + W \quad W = 0$

$\Delta U = Q \quad \Delta U = 3/2\,\Delta(PV) = \frac{3}{2}(3P_0V_0 - P_0V_0) = 3P_0V_0$

$2P_0V_0$

$\Delta U = +3P_0V_0 = Q$

(d) Derive an expression for the net work done on the gas during the entire process $ABCDA$. Express your answer in terms of P_0, V_0, and fundamental constants, as appropriate.

$W = -P\Delta V$

$W_{AB} = W_{CD} = 0$

$W_{BC} = -(3P_0)(4V_0 - V_0) = -9P_0V_0$

$W_{DA} = -(P_0)(V_0 - 4V_0) = (-P_0)(-3V_0) = 3P_0V_0$

$\therefore W_{net} = -9P_0V_0 + 3P_0V_0$

$\boxed{W_{net} = -6P_0V_0}$

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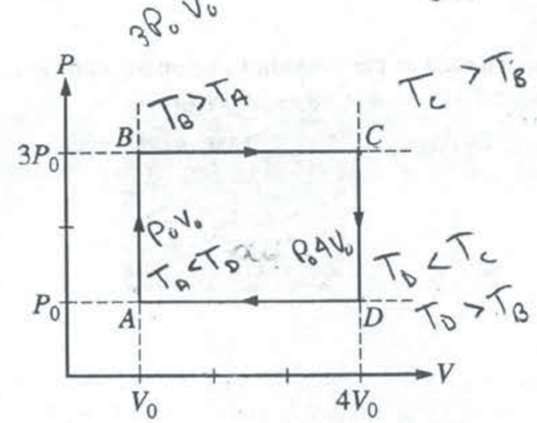
$3P_0\ 4V_0$



$PV = nRT$

$T = \frac{PV}{nR}$

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(a)

i. Rank the temperatures at the 4 labeled points from least to greatest, using 1 for the lowest temperature. If two or more points have the same temperature, give them the same ranking.

<u>1</u> *A* <u>2</u> *B* <u>4</u> *C* <u>3</u> *D*

$T = \frac{PV}{nR}$

ii. Determine the temperature T_D at point *D* in terms of P_0, V_0, and fundamental constants, as appropriate.

$P_D = P_0$

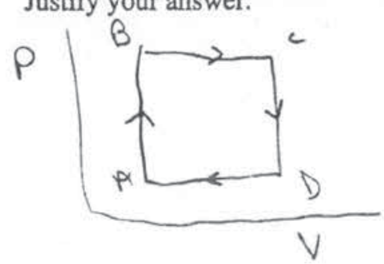
$PV = nRT \quad V_D = 4V_0$

$$\frac{PV}{nR} = T = \frac{P_0 4V_0}{nR} = \boxed{\frac{4P_0V_0}{nR} = T}$$

(b) Indicate all segments of the path *ABCDA*, if any, for which the work done <u>by</u> the gas is positive. If the work done by the gas is not positive for any of the segments, then check "None".

____ *AB*   X *BC*   ____ *CD*   ____ *DA*   ____ None

Justify your answer.



$W = 0$ for AB and CD because it is Isochloric so $W = -P\Delta V = 0$.

Work done is positive for BC because V increases but is negative for DA because V is decreasing

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(c) In process *AB*, is the energy transferred to the gas by heating positive, negative, or zero?

_K_ Positive   ____ Negative   ____ Zero

Justify your answer.

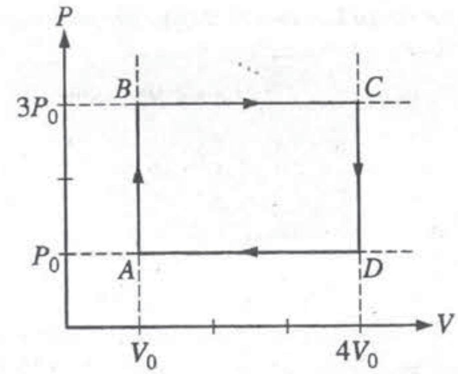
P increases so work is done on the gas. Process AB is isochlorric so W = 0 therefore $\Delta U = Q + W = \Delta U = Q$ Temperature Increase from A to B so ΔU increases so Q increases

(d) Derive an expression for the net work done on the gas during the entire process *ABCDA*. Express your answer in terms of P_0, V_0, and fundamental constants, as appropriate.

Net work by the cycle/process equals the area enclosed by the process.

$A = bh = (2P_0)(3V_0) = 6P_0V_0$

$\Sigma W = 6P_0V_0$



$PV = nRT$

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A sample containing three moles of an ideal gas is taken through a series of equilibrium states, as represented by the closed path *ABCDA* in the diagram above.

(a)

i. Rank the temperatures at the 4 labeled points from least to greatest, using 1 for the lowest temperature. If two or more points have the same temperature, give them the same ranking.

1 *A* 2 *B* 4 *C* 3 *D*

ii. Determine the temperature T_D at point *D* in terms of P_0, V_0, and fundamental constants, as appropriate.

$PV = nRT$

$T = \frac{PV}{nR}$

$T_D = \frac{P_0(4V_0)}{nR}$

$T_D = \frac{4P_0V_0}{3R}$

(b) Indicate all segments of the path *ABCDA*, if any, for which the work done by the gas is positive. If the work done by the gas is not positive for any of the segments, then check "None".

✓ *AB* ____ *BC* ____ *CD* ✓ *DA* ____ None

Justify your answer.

GO ON TO THE NEXT PAGE.

(c) In process AB, is the energy transferred to the gas by heating positive, negative, or zero?

✓ Positive ____ Negative ____ Zero

Justify your answer.

Energy is being transferred to the gas, the gas is gaining energy, therefore, it is positive

(d) Derive an expression for the net work done <u>on</u> the gas during the entire process $ABCDA$. Express your answer in terms of P_0, V_0, and fundamental constants, as appropriate.

$\Delta U = Q + W$

Question 3

Overview

The intent of this question was to assess the students' understanding of thermodynamic cycles. Part (a) tested their interpretation of a PV diagram and the ideal gas law. Part (b) tested their ability to determine whether work was done on or by a gas. Part (c) tested their understanding of the first law of thermodynamics, heat, and internal energy. Finally, part (d) tested the students' ability to find the work done on a gas using a PV diagram.

Sample: B3 A
Score: 10

This was an excellent response earning full credit on all parts. In part (b) the reasoning explains all four segments in the cycle. It is actually only necessary to explain the positive work in the correctly checked segment BC. Part (c) has excellent reasoning in symbolic form that was easy for the graders to follow. Part (d) also has a well presented step-by-step reasoning.

Sample: B3 B
Score: 7

Part (a)(ii) lost 1 point because the given value of the number of moles was not used. Part (b) earned full credit in another straightforward argument. In part (c) the first statement contradicts an otherwise fine answer, and only 1 point was earned. Part (d) did not earn the final point for an indication that the net work was negative.

Sample: B3 C
Score: 3

In part (a), the temperatures were ranked correctly and T_D was correctly determined, so 3 points were earned. The correct choice was not selected in part (b). In part (c) the correct choice was made, but points are only awarded for the explanation, which was not correct. No credit was earned for the equation in part (d).