### Question 3

<table>
<thead>
<tr>
<th>10 points total</th>
<th>Distribution of points</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td></td>
</tr>
<tr>
<td>i. 2 points</td>
<td>2 points</td>
</tr>
<tr>
<td>For correctly ranking all 4 labeled points</td>
<td></td>
</tr>
<tr>
<td>(1 \quad A \quad 2 \quad B \quad 4 \quad C \quad 3 \quad D)</td>
<td></td>
</tr>
<tr>
<td>Note: one point is earned for a ranking that identifies (A) and (C) as having the lowest and highest temperatures, respectively, but incorrectly ranks (B) and (D). For example:</td>
<td></td>
</tr>
<tr>
<td>(1 \quad A \quad 3 \quad B \quad 4 \quad C \quad 2 \quad D) or (1 \quad A \quad 2 \quad B \quad 3 \quad C \quad 2 \quad D)</td>
<td></td>
</tr>
<tr>
<td>(A) and (C) cannot have the same ranking as other points (e.g., (1 \quad A \quad 1 \quad B \quad 2 \quad C \quad 2 \quad D))</td>
<td></td>
</tr>
<tr>
<td>ii. 1 point</td>
<td></td>
</tr>
<tr>
<td>For using the ideal gas law with correct substitutions for (P), (V), and (n) to get the correct answer</td>
<td></td>
</tr>
<tr>
<td>(PV = nRT)</td>
<td></td>
</tr>
<tr>
<td>(T = \frac{PV}{nR})</td>
<td></td>
</tr>
<tr>
<td>(T_D = \frac{P_0(4V_0)}{3R} = \frac{4P_0V_0}{3R}) or (\frac{4P_0V_0}{(3 \text{ mol})\cdot R})</td>
<td></td>
</tr>
<tr>
<td>Note: mole units need not be explicitly stated</td>
<td></td>
</tr>
<tr>
<td>(b) 2 points</td>
<td>1 point</td>
</tr>
<tr>
<td>For selecting “BC” only</td>
<td></td>
</tr>
<tr>
<td>For providing a correct explanation</td>
<td></td>
</tr>
<tr>
<td>Examples:</td>
<td></td>
</tr>
<tr>
<td>• For the gas to do positive work on its surroundings (or for work done on the gas to be negative), (P\Delta V) must be positive so (\Delta V) must be positive.</td>
<td></td>
</tr>
<tr>
<td>• An expanding gas exerts a force in the direction of motion, doing positive work.</td>
<td></td>
</tr>
<tr>
<td>(c) 2 points</td>
<td></td>
</tr>
<tr>
<td>“Positive” is the only box checked for which the Justify section will be read</td>
<td></td>
</tr>
<tr>
<td>For a correct explanation that mentions internal energy increases with the temperature increase.</td>
<td></td>
</tr>
<tr>
<td>For a correct explanation that mentions the work is zero</td>
<td></td>
</tr>
<tr>
<td>Example:</td>
<td></td>
</tr>
<tr>
<td>The volume does not change in process (AB), hence the work done is zero. The temperature increase in (AB) means the internal energy increases. Since the work done is zero, the positive change in internal energy must be provided by heating.</td>
<td></td>
</tr>
</tbody>
</table>
Question 3 (continued)

(d) 3 points

For a correct expression of net work done on the gas from segments $BC$ and $DA$ of the graph

$$W_{\text{net}} = -(P \Delta V)_{BC} - (P \Delta V)_{DA}$$

For correctly substituting the values of $P$ and $\Delta V$ into the above equation 1 point

$$W_{\text{net}} = -(3P_0)(3V_0) - (P_0)(-3V_0) = -9P_0V_0 + 3P_0V_0 = -6P_0V_0$$

For having a final answer that is negative 1 point

Alternate Solution

For a correct expression or statement that the net work done on the gas is given by the area enclosed by the entire path 1 point

$$W_{\text{net}} = -(\Delta P \cdot \Delta V)$$

For correctly substituting the values of $P$ and $\Delta V$ into the above equation 1 point

$$W_{\text{net}} = -(3P_0 - P_0)(4V_0 - V_0) = -2P_0 \cdot 3V_0 = -6P_0V_0$$

For having a negative answer 1 point
3. (10 points)

A sample containing three moles of an ideal gas is taken through a series of equilibrium states, as represented by the closed path ABCDA in the diagram above.

(a) 

i. Rank the temperatures at the 4 labeled points from least to greatest, using 1 for the lowest temperature. If two or more points have the same temperature, give them the same ranking.

\[ pV = nRT \]

\[ n = 3 \]

\[ T = \frac{pV}{nR} \]

\[ T_D = \frac{4P_0V_0}{3R} \]

ii. Determine the temperature \( T_D \) at point D in terms of \( P_0, V_0 \), and fundamental constants, as appropriate.

(b) Indicate all segments of the path ABCDA, if any, for which the work done by the gas is positive. If the work done by the gas is not positive for any of the segments, then check “None”.

\[ \text{AB} \quad \checkmark \text{BC} \quad \text{CD} \quad \text{DA} \quad \text{None} \]

Justify your answer.

Segments AB and CD do not do any work on or by the gas because \( W_{\text{gas}} = P\Delta V \) only works for constant pressure.

Segment DA has a negative value for \( \Delta V \), resulting in \( -W_{\text{gas}} \) for that segment. Whereas, segment BC is the only time the gas expands for \( \Delta V \) to be positive and \( W_{\text{gas}} \) to be positive.
(c) In process $AB$, is the energy transferred to the gas by heating positive, negative, or zero?

[ ] Positive  [ ] Negative  [ ] Zero

Justify your answer.

\[ \Delta U = Q + W \]
\[ \Delta U = Q + 0 \]
\[ \Delta U = Q \]

\[ \Delta U = \frac{3}{2} n R \Delta T \]
\[ \Delta U = \frac{3}{2} \Delta (PV) \]
\[ \Delta U = \frac{3}{2} (3P_0V_0 - P_0V_0) = 3P_0V_0 \]

\[ \Delta U = +3P_0V_0 = Q \]

(d) Derive an expression for the net work done on the gas during the entire process $ABCDA$. Express your answer in terms of $P_0$, $V_0$, and fundamental constants, as appropriate.

\[ W = -P_0 \Delta V \]
\[ W_{AB} = W_{CD} = 0 \]
\[ W_{BC} = - (3P_0)(4V_0 - V_0) = -9P_0V_0 \]
\[ W_{DA} = -(P_0)(V_0 - 4V_0) = -(P_0)(-3V_0) = 3P_0V_0 \]

\[ W_{net} = -9P_0V_0 + 3P_0V_0 \]
\[ W_{net} = -6P_0V_0 \]
3. (10 points)

A sample containing three moles of an ideal gas is taken through a series of equilibrium states, as represented by the closed path ABCDA in the diagram above.

(a) 

i. Rank the temperatures at the 4 labeled points from least to greatest, using 1 for the lowest temperature. If two or more points have the same temperature, give them the same ranking.

\[1\ A \ 2\ B \ 4\ C \ 3\ D\]

\[T = \frac{PV}{nR}\]

ii. Determine the temperature \(T_D\) at point D in terms of \(P_0, V_0\), and fundamental constants, as appropriate.

\[P_D = P_0\]

\[\frac{PV}{nR} = T = \frac{P_0 V_0}{nR}\]

(b) Indicate all segments of the path ABCDA, if any, for which the work done by the gas is positive. If the work done by the gas is not positive for any of the segments, then check “None”.

\[\_\_\_\_\ AB \ \_\_\_\ BC \ \_\_\_\ CD \ \_\_\_\ DA \ \_\_\_\ None\]

Justify your answer.

\[W = 0\ for\ AB\ and\ CD\ because\ it\ is\ Isochoric\ so\ W = \cdot P \Delta V = 0,\]

Work done is positive for BC because \(V\) increases but is negative for DA because \(V\) decreases.

GO ON TO THE NEXT PAGE.
(c) In process AB, is the energy transferred to the gas by heating positive, negative, or zero?

\[ \begin{array}{ccc}
\text{Positive} & \text{Negative} & \text{Zero} \\
\hline
\end{array} \]

Justify your answer.

P increases so work is done on the gas. Process AB is isochoric, so \( W = 0 \)

therefore \( \Delta U = Q + W = \Delta U = Q \)

Temperature Increase from A to B so \( \Delta U \) increases so \( Q \) increases.

(d) Derive an expression for the net work done on the gas during the entire process ABCDA. Express your answer in terms of \( P_0, V_0 \), and fundamental constants, as appropriate.

Net work by the cycle/process equals the area enclosed by the process.

\[ A = bh = (2P_0)(3V_0) = 6P_0V_0 \]

\[ \Sigma W = 6P_0V_0 \]
3. (10 points)

A sample containing three moles of an ideal gas is taken through a series of equilibrium states, as represented by the closed path $ABCDA$ in the diagram above.

(a)

i. Rank the temperatures at the 4 labeled points from least to greatest, using 1 for the lowest temperature. If two or more points have the same temperature, give them the same ranking.

$\begin{align*}
1 & \quad A \\
2 & \quad B \\
3 & \quad C \\
4 & \quad D
\end{align*}$

ii. Determine the temperature $T_D$ at point $D$ in terms of $P_0$, $V_0$, and fundamental constants, as appropriate.

$\begin{align*}
P_V &= nRT \\
T &= \frac{P_V}{nR} \\
T_D &= \frac{P_0(V_0)}{3R}
\end{align*}$

(b) Indicate all segments of the path $ABCDA$, if any, for which the work done by the gas is positive. If the work done by the gas is not positive for any of the segments, then check “None”.

$\begin{align*}
\underline{AB} & \quad BC & \quad CD & \quad DA & \quad \text{None}
\end{align*}$

Justify your answer.
(c) In process AB, is the energy transferred to the gas by heating positive, negative, or zero?

☑ Positive  ___ Negative  ___ Zero

Justify your answer.

Energy is being transferred to the gas, the gas is gaining energy, therefore it is positive.

(d) Derive an expression for the net work done on the gas during the entire process ABCDA. Express your answer in terms of $P_0$, $V_0$, and fundamental constants, as appropriate.

$$\Delta u = Q + W$$
Overview

The intent of this question was to assess the students' understanding of thermodynamic cycles. Part (a) tested their interpretation of a $PV$ diagram and the ideal gas law. Part (b) tested their ability to determine whether work was done on or by a gas. Part (c) tested their understanding of the first law of thermodynamics, heat, and internal energy. Finally, part (d) tested the students' ability to find the work done on a gas using a $PV$ diagram.

Sample: B3 A
Score: 10

This was an excellent response earning full credit on all parts. In part (b) the reasoning explains all four segments in the cycle. It is actually only necessary to explain the positive work in the correctly checked segment $BC$. Part (c) has excellent reasoning in symbolic form that was easy for the graders to follow. Part (d) also has a well presented step-by-step reasoning.

Sample: B3 B
Score: 7

Part (a)(ii) lost 1 point because the given value of the number of moles was not used. Part (b) earned full credit in another straightforward argument. In part (c) the first statement contradicts an otherwise fine answer, and only 1 point was earned. Part (d) did not earn the final point for an indication that the net work was negative.

Sample: B3 C
Score: 3

In part (a), the temperatures were ranked correctly and $T_D$ was correctly determined, so 3 points were earned. The correct choice was not selected in part (b). In part (c) the correct choice was made, but points are only awarded for the explanation, which was not correct. No credit was earned for the equation in part (d).