Question 2

10 points total

(a) 2 points

For showing the three force vectors for buoyancy, weight (gravity), and tension 1 point
For showing all the forces labeled and in the correct direction 1 point
One point is deducted if either or both of the following occur:
• Any other forces are indicated
• Any vector does not both touch and point away from the dot

(b) 3 points

For using Newton’s second law to sum the forces in the vertical direction 1 point
\[ F_B - F_T - F_g = 0 \]
\[ F_B = F_g + F_T \]
For equating the buoyant force with the weight of displaced water, and expressing this weight in terms of density, volume, and \( g \) 1 point
\[ F_B = F_g + 0.25F_g = 1.25F_g \]
\[ \rho_W V_{\text{cube}} g = 1.25mg \]
\[ \left(1000 \text{ kg/m}^3\right)V_{\text{cube}} = 1.25m \]
For solving the equation to get a correct answer with units 1 point
\[ \left(1000 \text{ kg/m}^3\right)/1.25 = m/V_{\text{cube}} \]
\[ 800 \text{ kg/m}^3 = \rho_{\text{cube}} \]
Alternate solution

Alternate Points

Answer based on direct comparison of densities of water and the cube: since the buoyant force on the cube only depends on the density of water, and the net downward force only depends on the density of the cube, the ratio of densities can be determined.

For correctly substituting density and mass and determining the ratio of densities 2 points
\[ \rho_W V g = mg + 0.25mg \]
\[ \rho_W V = 1.25m \]
\[ \rho_W = 1.25\rho_{\text{cube}} \]
Note: 2 points will be awarded for correctly writing down the ratio of the densities even without showing any work
For the correct answer 1 point
\[ \rho_{\text{cube}} = \rho_W/1.25 = \left(1000 \text{ kg/m}^3\right)/1.25 = 800 \text{ kg/m}^3 \]
(c) 3 points

For any use of Newton’s second law to sum the forces in the vertical direction 1 point

\( F_{net} = ma = F_B - F_g \)

For correct substitution 1 point
\begin{align*}
ma = 1.25F_g - F_g = 0.25mg \\
a = 0.25g \\
\end{align*}

OR

\( F_B = \rho_w gV = \rho_w g\left(\frac{m}{\rho_{cube}}\right) \)

\( F_B = \left(1000 \text{ kg/m}^3\right) \left(g \left(\frac{m}{800 \text{ kg/m}^3}\right)\right) \), or \( \rho_{cube} \) consistent with answer from (b)

\( a = \frac{F_B - F_g}{m} = \left(1000 \text{ kg/m}^3\right) \left(g \left(\frac{m}{800 \text{ kg/m}^3}\right) - mg \right) = \left(\frac{1000}{800}\right)g - g \)

For calculating a correct answer, with units 1 point
\( a = 2.5 \text{ m/s}^2 \)

(d) 2 points

For selecting “Remains the same” 1 point

For providing a correct explanation 1 point

Example:
\( F_B \) relates to density, volume, and \( g \), none of which change.

Note: If the wrong choice is selected, the explanation is not considered and no points are awarded.
2. (10 points)

A cube of mass $m$ and side length $L$ is completely submerged in a tank of water and is attached to the bottom of the tank by a string, as shown in the figure above. The tension in the string is 0.25 times the weight of the cube. The density of water is $1000 \text{ kg/m}^3$.

(a) On the dot below that represents the cube, draw and label the forces (not components) that act on the cube while it is attached to the string. Each force must be represented by a distinct arrow starting on, and pointing away from, the dot.

(b) Calculate the density of the cube.

(c) The string is now cut. Calculate the magnitude of the acceleration of the cube immediately after the string is cut. If you need to draw anything other than what you have shown in part (a) to assist in your solution, use the space below. Do NOT add anything to the figure in part (a).
(d) Indicate whether the magnitude of the buoyant force on the cube increases, decreases, or remains the same while the cube is rising, but before it reaches the surface.

___ Increases  ___ Decreases  ___ Remains the same

Justify your answer.

The buoyant force will remain the same because density of an object is constant, the volume under water won’t change, and g is a constant.
2. (10 points)

A cube of mass \( m \) and side length \( L \) is completely submerged in a tank of water and is attached to the bottom of the tank by a string, as shown in the figure above. The tension in the string is 0.25 times the weight of the cube. The density of water is 1000 kg/m\(^3\).

(a) On the dot below that represents the cube, draw and label the forces (not components) that act on the cube while it is attached to the string. Each force must be represented by a distinct arrow starting on, and pointing away from, the dot.

(b) Calculate the density of the cube.

\[
F_T = F_B = F_g + F_{\text{up}}
\]

\[
F_B = F_g (L) = mg (L)
\]

\[
(1000)(L^3) = m(9.8)(L^3)
\]

\[
\frac{1000}{9.8} = \frac{mL^3}{L^3}
\]

\[
= 101.6 \text{ kg/m}^3
\]

(c) The string is now cut. Calculate the magnitude of the acceleration of the cube immediately after the string is cut. If you need to draw anything other than what you have shown in part (a) to assist in your solution, use the space below. Do NOT add anything to the figure in part (a).

\[
f_{\text{net}} = F_B - F_g
\]

\[
w_a = (1000)(L^3) - (101.6)(L^3)(9.8)
\]

\[
w_a = \frac{2000}{L^3}
\]

\[
w = \frac{2000}{81.6} = 2.45 \text{ m/s}^2
\]
(d) Indicate whether the magnitude of the buoyant force on the cube increases, decreases, or remains the same while the cube is rising, but before it reaches the surface.

___ Increases    ___ Decreases    ___ Remains the same

Justify your answer.

since it has not yet broken the surface, it is displacing the same amount of water. Since $F_B = \rho V g$, $F_B$ is the same.

$F_{buoy} = \rho V g$
2. (10 points)

A cube of mass $m$ and side length $L$ is completely submerged in a tank of water and is attached to the bottom of the tank by a string, as shown in the figure above. The tension in the string is 0.25 times the weight of the cube. The density of water is 1000 kg/m$^3$.

(a) On the dot below that represents the cube, draw and label the forces (not components) that act on the cube while it is attached to the string. Each force must be represented by a distinct arrow starting on, and pointing away from, the dot.

\[ \text{b) Calculate the density of the cube.} \]
\[ \rho = \frac{m}{V} \]
\[ \text{f_bouy} = \rho_0 + \rho \text{g} \]
\[ \rho_{\text{water}} = 1000 \text{ kg/m}^3 \]
\[ f_T = 0.25 \text{ weight} \]
\[ p = \frac{m}{V} \]
\[ p_{\text{cube}} = m_{\text{cube}} \cdot V + \text{atm} \]

(b) The string is now cut. Calculate the magnitude of the acceleration of the cube immediately after the string is cut. If you need to draw anything other than what you have shown in part (a) to assist in your solution, use the space below. Do NOT add anything to the figure in part (a).
(d) Indicate whether the magnitude of the buoyant force on the cube increases, decreases, or remains the same while the cube is rising, but before it reaches the surface.

[ ] Increases  [ ] Decreases  [ ] Remains the same

Justify your answer.

The buoyant force will remain the same b/c the initial pressure, density, and volume and gravity will all remain the same.
Question 2

Overview

This problem explored the topic of buoyant force and the application of Newton’s second law to solve problems involving the buoyant force. The concepts stressed are: objects in a fluid experience an upward force, the ratio of densities (of object and fluid) determines the behavior of an object in a fluid, and the buoyant force is equal in magnitude to the weight of the fluid displaced by the object.

Part (a) required the students to draw a free body diagram to show the three forces acting: the buoyant force, tension, and the gravitational force. This demonstrated an understanding of the directions of these forces and provided preparation for the next section.

Part (b) required the students to use Newton’s second law to determine the density of the cube. The necessary connection here was that the sum of forces in equilibrium is zero. It also required an understanding of the equation for the buoyant force. This could also be solved by recognizing that the ratio of densities of the water and the cube is easily determined by considering the relationship between weight and tension.

Part (c) required the students to use Newton’s second law again, this time with the tension force no longer acting, to find the acceleration. Conceptually, the students needed to demonstrate an understanding that the forces are now unbalanced, with the buoyant force greater in magnitude than the weight, resulting in the cube accelerating upwards.

Part (d) required understanding that the two forces remaining, weight and buoyant force, are constants and therefore the cube has a constant acceleration. Primarily, it was the concept that the buoyant force depends only on the weight of the displaced fluid that was the goal.

Sample: B2 A
Score: 9

Both points were earned in part (a) for showing all three force vectors, each labeled and in the correct direction. Only the first 2 of 3 points were earned in part (b): 1 point for using Newton’s law to sum the forces and 1 point for equating the buoyant force with the weight of displaced water. However, the third point was not earned because the final density of cube was not calculated. Full credit was earned in parts (c) and (d).

Sample: B2 B
Score: 7

Only the second of 2 points was earned in part (a) for labeling all three forces in the correct direction. The first point was not awarded because each force was not represented by a distinct arrow. Only the first of 3 points were earned in part (b) for a clear use of Newton’s law to sum forces. The second point was not earned because buoyant force was expressed incorrectly, and the third point was not earned because the final density of the cube was not calculated correctly. Full credit was earned in parts (c) and (d).
Sample: B2 C
Score: 3

The second of 2 points was awarded in part (a) for labeling all three forces in the correct direction. The first point was not earned because each force was not represented by a distinct arrow. No points were awarded for parts (b) or (c). Full 2 points of credit were awarded in part (d): 1 point for correctly selecting “Remains the same” and the additional 1 point for correctly explaining why the buoyant force would remain constant due to its dependence on volume, density, and acceleration of gravity, all of which do not change.