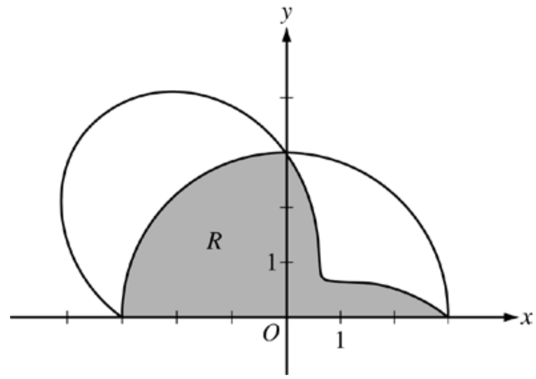
AP® CALCULUS BC
2014 SCORING GUIDELINES

Question 2

The graphs of the polar curves $r = 3$ and $r = 3 - 2\sin(2\theta)$ are shown in the figure above for $0 \le \theta \le \pi$.



(a) Let R be the shaded region that is inside the graph of $r = 3$ and inside the graph of $r = 3 - 2\sin(2\theta)$. Find the area of R.

(b) For the curve $r = 3 - 2\sin(2\theta)$, find the value of $\frac{dx}{d\theta}$ at $\theta = \frac{\pi}{6}$.

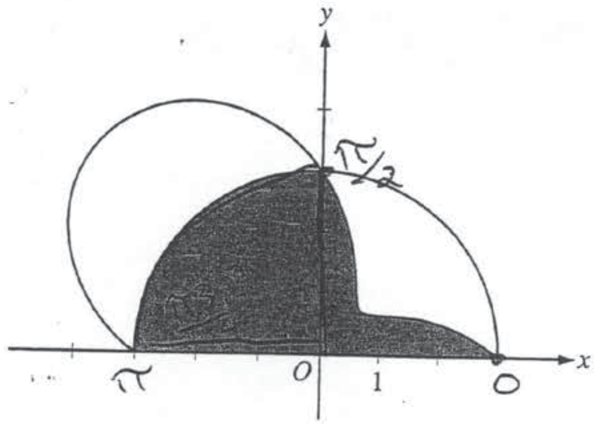
(c) The distance between the two curves changes for $0 < \theta < \frac{\pi}{2}$.

Find the rate at which the distance between the two curves is changing with respect to θ when $\theta = \frac{\pi}{3}$.

(d) A particle is moving along the curve $r = 3 - 2\sin(2\theta)$ so that $\frac{d\theta}{dt} = 3$ for all times $t \ge 0$. Find the value of $\frac{dr}{dt}$ at $\theta = \frac{\pi}{6}$.

(a) Area $= \frac{9\pi}{4} + \frac{1}{2}\int_0^{\pi/2} (3 - 2\sin(2\theta))^2 \, d\theta$ $= 9.708$ (or 9.707)	$3 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{limits} \\ 1 : \text{answer} \end{cases}$
(b) $x = (3 - 2\sin(2\theta))\cos\theta$ $\left.\frac{dx}{d\theta}\right\|_{\theta=\pi/6} = -2.366$	$2 : \begin{cases} 1 : \text{expression for } x \\ 1 : \text{answer} \end{cases}$
(c) The distance between the two curves is $D = 3 - (3 - 2\sin(2\theta)) = 2\sin(2\theta)$. $\left.\frac{dD}{d\theta}\right\|_{\theta=\pi/3} = -2$	$2 : \begin{cases} 1 : \text{expression for distance} \\ 1 : \text{answer} \end{cases}$
(d) $\frac{dr}{dt} = \frac{dr}{d\theta} \cdot \frac{d\theta}{dt} = \frac{dr}{d\theta} \cdot 3$ $\left.\frac{dr}{dt}\right\|_{\theta=\pi/6} = (-2)(3) = -6$	$2 : \begin{cases} 1 : \text{chain rule with respect to } t \\ 1 : \text{answer} \end{cases}$

2A₁



2. The graphs of the polar curves $r = 3$ and $r = 3 - 2\sin(2\theta)$ are shown in the figure above for $0 \le \theta \le \pi$.

(a) Let R be the shaded region that is inside the graph of $r = 3$ and inside the graph of $r = 3 - 2\sin(2\theta)$. Find the area of R.

$\cancel{\frac{9\pi}{4}}$

$3 = 3 - 2\sin(2\theta)$

$0 = 2\sin(2\theta)$

$0 = \sin(2\theta)$

$2\theta = 0, \pi, 2\pi$

$\theta = 0, \frac{\pi}{2}, \pi$

$$\frac{9\pi}{4} + \frac{1}{2}\int_0^{\pi/2} (3 - 2\sin(2\theta))^2 \, d\theta = \boxed{9.707963268}$$

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Continue problem 2 on page 7

2A₂

(b) For the curve $r = 3 - 2\sin(2\theta)$, find the value of $\frac{dx}{d\theta}$ at $\theta = \frac{\pi}{6}$.

$x = r\cos\theta$

$x = (3 - 2\sin(2\theta))\cos\theta$

$\cancel{\frac{d}{d\theta}}\ \frac{dx}{d\theta}\Big|_{\theta=\frac{\pi}{6}} \left[(3-2\sin(2\theta))\cos\theta\right] = \cancel{4.8099998661} \boxed{-2.366025009}$

(c) The distance between the two curves changes for $0 < \theta < \frac{\pi}{2}$. Find the rate at which the distance between the two curves is changing with respect to θ when $\theta = \frac{\pi}{3}$.

$3 - (3 - 2\sin(2\theta))$

$\frac{d}{d\theta} 2\sin(2\theta) = 2 \cdot 2\cos(2\theta)$

$4\cos(2\theta)$

$4\cos(2\frac{\pi}{3}) = \boxed{-2}$

(d) A particle is moving along the curve $r = 3 - 2\sin(2\theta)$ so that $\frac{d\theta}{dt} = 3$ for all times $t \geq 0$. Find the value of $\frac{dr}{dt}$ at $\theta = \frac{\pi}{6}$.

$\frac{dr}{dt} = -2\cos(2\theta) \cdot 2\frac{d\theta}{dt}$

$\frac{dr}{dt} = -2\cos\left(\frac{2\pi}{6}\right) \cdot 2(3)$

$\frac{dr}{dt} = -6$

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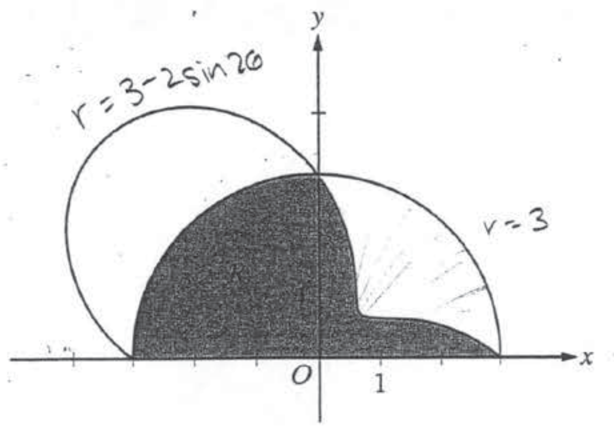
2B.



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(a) Let R be the shaded region that is inside the graph of $r = 3$ and inside the graph of $r = 3 - 2\sin(2\theta)$. Find the area of R.

$$\frac{1}{4}(\pi r^2) = \frac{1}{4}\pi(3)^2 = \frac{9}{4}\pi$$

$$A = \frac{9}{4}\pi + \pi\int_0^{\pi/2}\left((3)^2 - (3-2\sin 2\theta)^2\right)d\theta$$

$$\boxed{A = 34.898}$$

Continue problem 2 on page 7.

2B₂

(b) For the curve $r = 3 - 2\sin(2\theta)$, find the value of $\frac{dx}{d\theta}$ at $\theta = \frac{\pi}{6}$.

$x = r\cos\theta$

$\frac{dx}{d\theta} = -r\sin\theta + r'\cos\theta$

$\frac{dx}{d\theta} = -(3-2\sin 2\theta)\sin\theta + -4\cos 2\theta \cdot \cos\theta$

$\frac{dx}{d\theta}$ at $\pi/6 = \boxed{-2.366}$

$-.63397$

(c) The distance between the two curves changes for $0 < \theta < \frac{\pi}{2}$. Find the rate at which the distance between the two curves is changing with respect to θ when $\theta = \frac{\pi}{3}$.

$y = r\sin\theta$

$dy/dx = \frac{r\cos\theta + r'\sin\theta}{-r\sin\theta + r'\cos\theta}$

-1.5

$dy/dx = \frac{-2\sin 2\theta\cos\theta - 4\cos 2\theta\sin\theta}{2\sin 2\theta\sin\theta - 4\cos 2\theta\cos\theta}$; $\theta = \pi/3$

$\boxed{dy/dx = 2.8868}$

(d) A particle is moving along the curve $r = 3 - 2\sin(2\theta)$ so that $\frac{d\theta}{dt} = 3$ for all times $t \geq 0$. Find the value of $\frac{dr}{dt}$ at $\theta = \frac{\pi}{6}$.

$\frac{dr}{d\theta} = -4\cos 2\theta$ $\frac{dr}{dt} = \frac{dr/d\theta}{dt/d\theta} = \frac{-4\cos 2\theta}{1/3}$

$\frac{d\theta}{dt} = t$

$(-4\cos(\pi/3))3 = \boxed{-6}$

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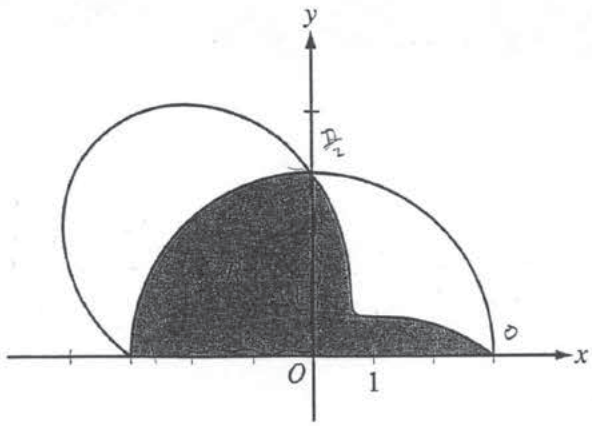
2C1



2. The graphs of the polar curves $r = 3$ and $r = 3 - 2\sin(2\theta)$ are shown in the figure above for $0 \le \theta \le \pi$.

(a) Let R be the shaded region that is inside the graph of $r = 3$ and inside the graph of $r = 3 - 2\sin(2\theta)$. Find the area of R.

$\frac{\pi (3)^2}{4}$

$\frac{9\pi}{4} + \int_0^{\frac{\pi}{2}} 3 - 2\sin(2\theta)$

$\boxed{\approx 9.781}$

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Continue problem 2 on page 7.

$2C_2$

(b) For the curve $r = 3 - 2\sin(2\theta)$, find the value of $\frac{dx}{d\theta}$ at $\theta = \frac{\pi}{6}$.

$x = r\cos\theta$

$x = (3 - 2\sin(2\theta))\cos\theta$

$\frac{Dx}{D\theta} = -4\cos(2\theta)(-\sin\theta)$

$4\cos(2\theta)\sin(\theta)$

$4\cos\left(\frac{\pi}{3}\right)\sin\left(\frac{\pi}{6}\right)$

$4\left(\frac{1}{2}\right)\frac{1}{2} = \boxed{1}$

(c) The distance between the two curves changes for $0 < \theta < \frac{\pi}{2}$. Find the rate at which the distance between the two curves is changing with respect to θ when $\theta = \frac{\pi}{3}$.

$x = r\cos\theta \quad y = r\sin\theta$

$x = 3 - 2\sin(2\theta)\cos\theta \quad y = 3 - 2\sin(2\theta)\sin\theta$

$x' = \approx 2.500 \quad y' \approx .866$

$\sqrt{(x')^2 + (y')^2}$

$\sqrt{6.250 + 0.750}$

$\boxed{\approx 2.646}$ units

(d) A particle is moving along the curve $r = 3 - 2\sin(2\theta)$ so that $\frac{d\theta}{dt} = 3$ for all times $t \geq 0$. Find the value of $\frac{dr}{dt}$ at $\theta = \frac{\pi}{6}$.

?

$\frac{DR}{Dt} = -4\cos(2\theta)\frac{D\theta}{Dt}$

$\frac{DR}{Dt} = -4\cos\left(\frac{\pi}{3}\right)3$

$-4\left(\frac{1}{2}\right)3$

$\boxed{\frac{DR}{Dt} = -6}$

GO ON TO THE NEXT PAGE.

Question 2

Overview

In this problem students were given the graphs of the polar curves $r = 3 - 2\sin(2\theta)$ and $r = 3$ for $0 \le \theta \le \pi$. In part (a) students had to find the area of the shaded region R that is the common area inside both graphs. Students needed to find the area bounded by the polar curve $r = 3 - 2\sin(2\theta)$ in the first quadrant and add it to the area of the quarter circle in the second quadrant resulting in $\frac{1}{2}\int_0^{\pi/2}(3 - 2\sin(2\theta))^2\,d\theta + \frac{9\pi}{4}$. In part (b) students needed to find $\frac{dx}{d\theta}$ at $\theta = \frac{\pi}{6}$. Students had to realize that $x = r\cos(\theta)$ and then differentiate to find $\frac{dx}{d\theta}$ at $\theta = \frac{\pi}{6}$. In part (c) students were asked to find the rate at which the distance between the curves in the first quadrant was changing at $\theta = \frac{\pi}{3}$. Students needed to set up an equation for the distance between the curves in the first quadrant, $D = 3 - (3 - 2\sin(2\theta))$, and then evaluate the derivative of D at the required value. Finally, in part (d) students were given that $\frac{d\theta}{dt} = 3$ and were asked to find the value of $\frac{dr}{dt}$ at $\theta = \frac{\pi}{6}$. Students had to invoke the chain rule to get $\frac{dr}{dt} = \frac{dr}{d\theta} \cdot \frac{d\theta}{dt}$ and evaluate the result at $\theta = \frac{\pi}{6}$.

Sample: 2A
Score: 9

The student earned all 9 points.

Sample: 2B
Score: 6

The student earned 6 points: 2 points in part (a), 2 points in part (b), no points in part (c), and 2 points in part (d). In part (a) the student writes a correct integrand for the unshaded portion in the first quadrant. The limits of 0 and $\frac{\pi}{2}$ are correct for that integrand. The student earned the integrand and limits points. The student uses an incorrect constant on the integral and appears to be combining the integral with the shaded quarter circle in the second quadrant. The student did not earn the answer point. In part (b) the student's work is correct. In part (c) the student's work does not present a valid approach to the question. In part (d) the student's work is correct.

Sample: 2C
Score: 3

The student earned 3 points: no points in part (a), 1 point in part (b), no points in part (c), and 2 points in part (d). In part (a) the student does not square the integrand, so the integrand point was not earned. Because this is not a valid integrand for polar area, the student is not eligible for the limits and answer points. In part (b) the student earned the first point with a correct expression for x on the second line of work. The student has an incorrect value for $\frac{dx}{d\theta}$. In part (c) the student's work does not present a valid approach to the question. In part (d) the student's work is correct.