AP® CALCULUS AB
2014 SCORING GUIDELINES

Question 6

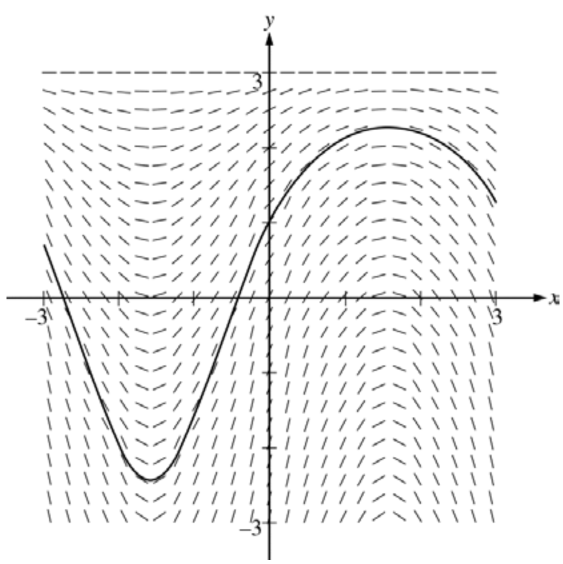
Consider the differential equation $\frac{dy}{dx} = (3 - y)\cos x$. Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(0) = 1$. The function f is defined for all real numbers.

(a) A portion of the slope field of the differential equation is given below. Sketch the solution curve through the point $(0, 1)$.

(b) Write an equation for the line tangent to the solution curve in part (a) at the point $(0, 1)$. Use the equation to approximate $f(0.2)$.

(c) Find $y = f(x)$, the particular solution to the differential equation with the initial condition $f(0) = 1$.

(a)



1 : solution curve

(b) $\left.\frac{dy}{dx}\right|_{(x,\, y)=(0,\, 1)} = 2\cos 0 = 2$

An equation for the tangent line is $y = 2x + 1$.

$f(0.2) \approx 2(0.2) + 1 = 1.4$

$2 : \begin{cases} 1 : \text{tangent line equation} \\ 1 : \text{approximation} \end{cases}$

(c) $\frac{dy}{dx} = (3 - y)\cos x$

$\int \frac{dy}{3 - y} = \int \cos x \, dx$

$-\ln|3 - y| = \sin x + C$

$-\ln 2 = \sin 0 + C \Rightarrow C = -\ln 2$

$-\ln|3 - y| = \sin x - \ln 2$

Because $y(0) = 1$, $y < 3$, so $|3 - y| = 3 - y$

$3 - y = 2e^{-\sin x}$

$y = 3 - 2e^{-\sin x}$

Note: this solution is valid for all real numbers.

$6 : \begin{cases} 1 : \text{separation of variables} \\ 2 : \text{antiderivatives} \\ 1 : \text{constant of integration} \\ 1 : \text{uses initial condition} \\ 1 : \text{solves for } y \end{cases}$

Note: max $3/6$ [1-2-0-0-0] if no constant of integration

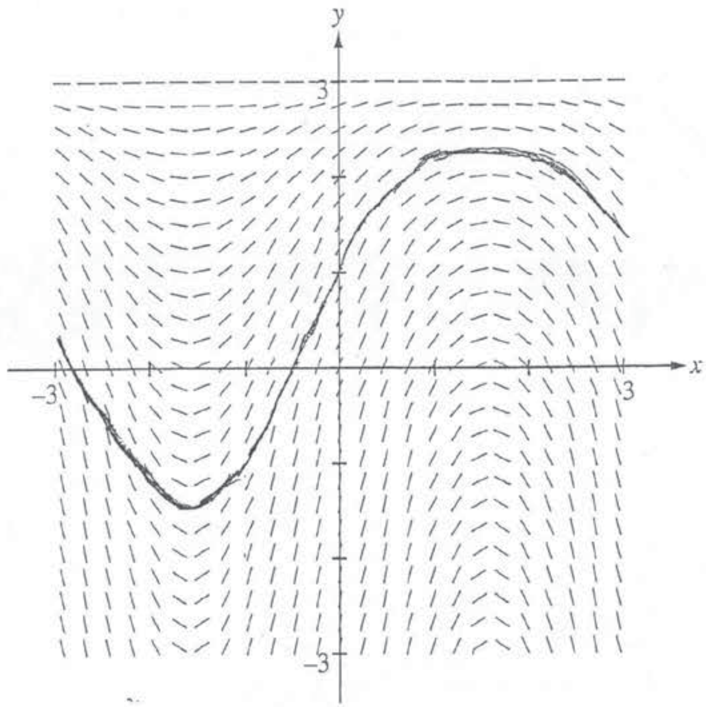
Note: $0/6$ if no separation of variables

NO CALCULATOR ALLOWED

6A₁

6. Consider the differential equation $\frac{dy}{dx} = (3 - y)\cos x$. Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(0) = 1$. The function f is defined for all real numbers.

(a) A portion of the slope field of the differential equation is given below. Sketch the solution curve through the point $(0, 1)$.



(b) Write an equation for the line tangent to the solution curve in part (a) at the point $(0, 1)$. Use the equation to approximate $f(0.2)$.

$\frac{dy}{dx} = (3-y)\cos x = m$

$\frac{dy}{dx} = (3-1)\cos(0) = m$

$m = 2$

$y - 1 = 2(x - 0)$

$y - 1 = 2x$

$y = 2x + 1$

$y - 1 = 2(0.2 - 0)$

$y - 1 = 0.4$

$y = 1.4$

$\boxed{f(0.2) \approx 1.4}$

Continue problem 6 on page 21.

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6A2

(c) Find $y = f(x)$, the particular solution to the differential equation with the initial condition $f(0) = 1$.

$\int \frac{dy}{3-y} = \int \cos x \, dx$ $\quad 3 - y = u$

$du = -dx$

$-\int \frac{du}{u} = \sin x + C$

$-\ln|u| = \sin x + C$

$-\ln|3-y| = \sin x + C$

$-\ln|3-1| = \sin(0) + C$

$-\ln|2| = 0 + C$

$C = -\ln 2$

$-\ln|3-y| = \sin x - \ln 2$

$\ln|3-y| = \ln 2 - \sin x$

$|3-y| = e^{\ln 2 - \sin x}$

$|3-y| = \frac{e^{\ln 2}}{e^{\sin x}}$

$|3-y| = 2e^{-\sin x}$

$3-y = 2e^{-\sin x}$

$-y = 2e^{-\sin x} 3$

$\boxed{y = -2e^{-\sin x} + 3}$

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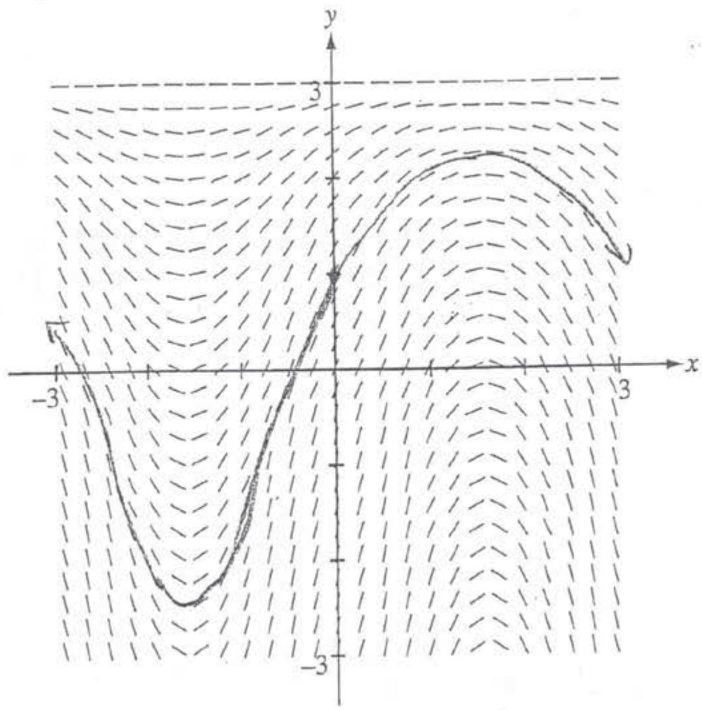
6B1

6. Consider the differential equation $\frac{dy}{dx} = (3 - y)\cos x$. Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(0) = 1$. The function f is defined for all real numbers.

(a) A portion of the slope field of the differential equation is given below. Sketch the solution curve through the point $(0, 1)$.



(b) Write an equation for the line tangent to the solution curve in part (a) at the point $(0, 1)$. Use the equation to approximate $f(0.2)$.

$y - 1 = 2x$ - tang line

$y - y_1 = 2(x - x_1)$

$y - 1 = 2(.2 - 0)$

$y - 1 = 2(.2)$

$y - 1 = .4$

$\boxed{y = 1.4}$

Continue problem 6 on page 21.

6B2

(c) Find $y = f(x)$, the particular solution to the differential equation with the initial condition $f(0) = 1$.

$$\frac{dy}{dx} = (3-y)\cos x$$

$$\int \frac{1}{3-y}\, dy = \int \cos x \, dx$$

$$\ln|3-y| = \sin x + C$$

$$\ln|3-1| = \sin(0) + C$$

$$\ln|2| = C$$

$$\ln|3-y| = \sin x + \ln|2|$$

$$3-y = \pm e^{\sin x + \ln(2)}$$

$$\boxed{y = e^{\sin x + \ln(2)} - 3}$$

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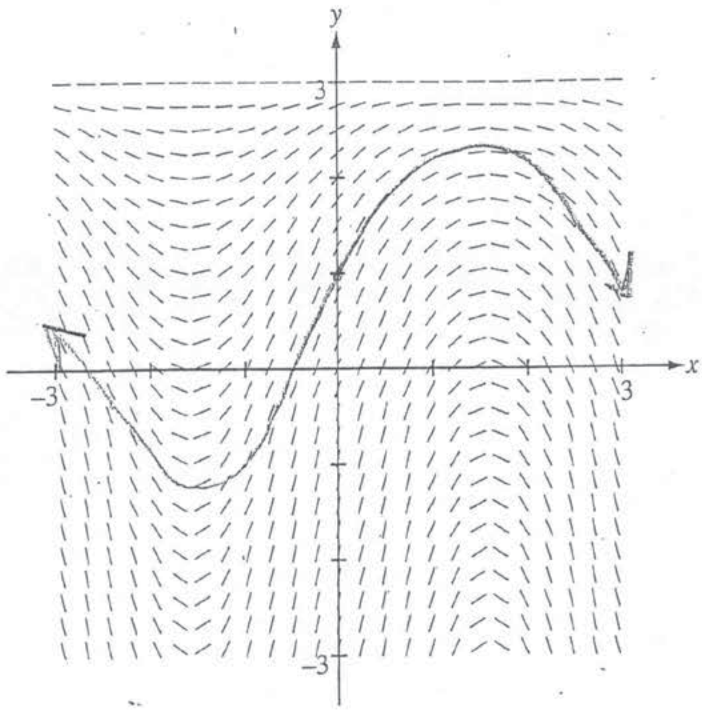
NO CALCULATOR ALLOWED

6C1

6. Consider the differential equation $\frac{dy}{dx} = (3 - y)\cos x$. Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(0) = 1$. The function f is defined for all real numbers.

(a) A portion of the slope field of the differential equation is given below. Sketch the solution curve through the point $(0, 1)$.



(b) Write an equation for the line tangent to the solution curve in part (a) at the point $(0, 1)$. Use the equation to approximate $f(0.2)$.

$\frac{dy}{dx} = (3-1)\cos(0)$

$\boxed{\frac{dy}{dx} = 2}$

$y - 1 = 2(x - 0)$

$f(x) = 2x + 1$

$f(0.2) = 0.4 + 1$

$\boxed{f(0.2) = 1.4}$

$f(0.2)$ is 1.4

6C2

(c) Find $y = f(x)$, the particular solution to the differential equation with the initial condition $f(0) = 1$.

$$(3-y)\,dy = \cos x\,dx$$

$$3y - \frac{y^2}{2} = -\sin(x) + C$$

$$6y - y^2 = -2\sin(x) + C$$

$$y^2 - 6y = 2\sin(x) + C$$

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AP® CALCULUS AB
2014 SCORING COMMENTARY

Question 6

Overview

This problem presented students with a differential equation and defined $y = f(x)$ to be the particular solution to the differential equation passing through a given point. Part (a) presented students with a portion of the slope field of the differential equation and asked students to draw a solution curve through the point $(0, 1)$. Part (b) asked students to write an equation for the line tangent to the solution curve from part (a) at a given point, and then to use this tangent line to approximate $f(x)$ at a nearby value of x. Students needed to recognize that the slope of the tangent line is the value of the derivative given in the differential equation at the given point. Part (c) asked for the particular solution to the differential equation that passes through the given point. Students were expected to use the method of separation of variables to solve the differential equation.

Sample: 6A
Score: 9

The student earned all 9 points.

Sample: 6B
Score: 6

The student earned 6 points: 1 point in part (a), 1 point in part (b), and 4 points in part (c). In part (a) the student's solution curve is correct. In part (b) the student does not show use of $\frac{dy}{dx}$ to find an equation of the tangent line, so the first point was not earned. The student uses a correct tangent line to approximate $f(0.2)$. In part (c) the student earned the separation of variables point, 1 antiderivative point, the constant of integration point, and the initial condition point. The student has an incorrect antiderivative on the left side of the equation, so the student is not eligible for the answer point.

Sample: 6C
Score: 3

The student earned 3 points: 1 point in part (a), 2 points in part (b), and no points in part (c). In part (a) the student's solution curve is correct. In part (b) the student's work is correct. In part (c) the student's work did not earn any points.