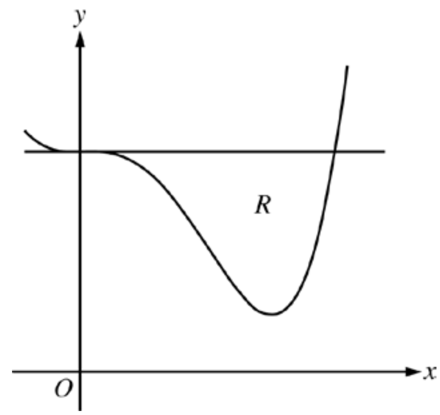
Question 2

Let R be the region enclosed by the graph of $f(x) = x^4 - 2.3x^3 + 4$ and the horizontal line $y = 4$, as shown in the figure above.

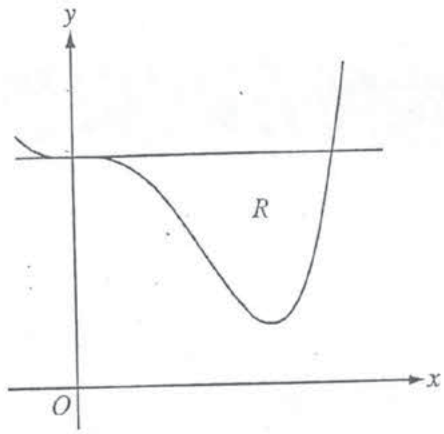


(a) Find the volume of the solid generated when R is rotated about the horizontal line $y = -2$.

(b) Region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is an isosceles right triangle with a leg in R. Find the volume of the solid.

(c) The vertical line $x = k$ divides R into two regions with equal areas. Write, but do not solve, an equation involving integral expressions whose solution gives the value k.

(a) $f(x) = 4 \Rightarrow x = 0, 2.3$ $\text{Volume} = \pi\int_0^{2.3}\left[(4+2)^2 - (f(x)+2)^2\right]dx$ $= 98.868$ (or 98.867)	$4 : \begin{cases} 2 : \text{integrand} \\ 1 : \text{limits} \\ 1 : \text{answer} \end{cases}$
(b) $\text{Volume} = \int_0^{2.3} \frac{1}{2}(4 - f(x))^2\,dx$ $= 3.574$ (or 3.573)	$3 : \begin{cases} 2 : \text{integrand} \\ 1 : \text{answer} \end{cases}$
(c) $\int_0^k (4 - f(x))\,dx = \int_k^{2.3} (4 - f(x))\,dx$	$2 : \begin{cases} 1 : \text{area of one region} \\ 1 : \text{equation} \end{cases}$



2. Let R be the region enclosed by the graph of $f(x) = x^4 - 2.3x^3 + 4$ and the horizontal line $y = 4$, as shown in the figure above.

(a) Find the volume of the solid generated when R is rotated about the horizontal line $y = -2$.

$f(x)=4 \Big|_{x=0 \text{ or } 2.3}$ $\pi\int_0^{2.3} \left((2+4)^2 - (2+f(x))^2\right) dx = 98.868 \text{ units}^3$

$V = \pi(R^2 - r^2)h$

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Continue problem 2 on page 7.

(b) Region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is an isosceles right triangle with a leg in R. Find the volume of the solid.

$A = \frac{1}{2} b h$

$A = \frac{1}{2} b^2$

$b = h$

$b = 4 - f(x)$

$\frac{1}{2} \int_0^{2.3} (4 - f(x))^2 \, dx = 3.574 \text{ units}^3$

(c) The vertical line $x = k$ divides R into two regions with equal areas. Write, but do not solve, an equation involving integral expressions whose solution gives the value k.

$\int_0^k (4 - f(x)) \, dx = \frac{1}{2} \int_0^{2.3} (4 - f(x)) \, dx$

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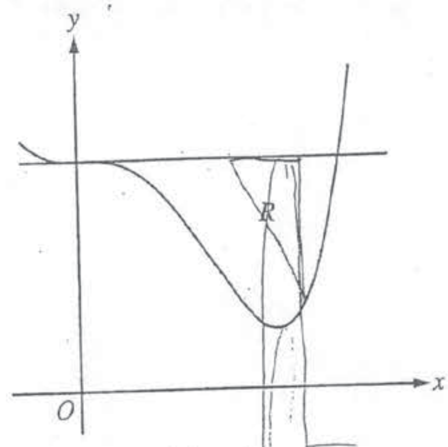
2B1



2. Let R be the region enclosed by the graph of $f(x) = x^4 - 2.3x^3 + 4$ and the horizontal line $y = 4$, as shown in the figure above.

(a) Find the volume of the solid generated when R is rotated about the horizontal line $y = -2$.

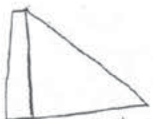
$O.R. = 4 \cancel{-}(-2))$

$I.R. = x^4 - 2.3x^3 + 4$

$$\pi \int_0^{2.3} [6^2 - (x^4 - 2.3x^3 + 4)^2]\,dx = 202.940$$

Continue problem 2 on page 7.

(b) Region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is an isosceles right triangle with a leg in R. Find the volume of the solid.

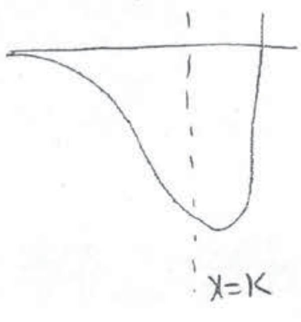


$dx \quad 4-(x^4-2.3x^3+4)$

$$A = \frac{\left(4-(x^4-2.3x^3+4)\right)^2}{2}$$

$$\int_0^{2.3} \left[\frac{\left(4-(x^4-2.3x^3+4)\right)^2}{2}\right] dx = 7.147$$

(c) The vertical line $x = k$ divides R into two regions with equal areas. Write, but do not solve, an equation involving integral expressions whose solution gives the value k.



$$\int_0^{k} \left[4-(x^4-2.3x^3+4)\right] dx = \int_k^{2.3} \left[4-(x^4-2.3x^3+4)\right] dx$$

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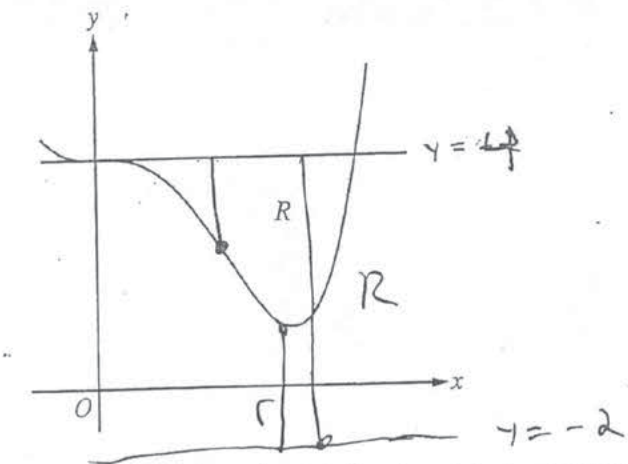
2C1



2. Let R be the region enclosed by the graph of $f(x) = x^4 - 2.3x^3 + 4$ and the horizontal line $y = 4$, as shown in the figure above.

(a) Find the volume of the solid generated when R is rotated about the horizontal line $y = -2$.

$$\pi \int_0^{2.3} \left((4) - (x^4 - 2.3x^3 + 4) + 2\right)^2 dx$$

$4 = x^4 - 2.3x^3 + 4$

$x = 2.3$

24.220127π

91.79771 units^3

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Continue problem 2 on page 7.

2C2

(b) Region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is an isosceles right triangle with a leg in R. Find the volume of the solid.

$A = \frac{1}{2}bh$

$$\pi \int_0^{2.3} \frac{1}{2} \cdot \left(4-(x^4-2.3x^3+4)\right) \left(\frac{(4-(x^4-2.3x^3+4))}{\sqrt{2}}\right) d$$

$$\frac{1}{2}(3.2181715)\left(\frac{3.2181715}{\sqrt{2}}\right)$$

11.50332 units^3

Isosceles: $\sqrt{3}$? ----

(c) The vertical line $x = k$ divides R into two regions with equal areas. Write, but do not solve, an equation involving integral expressions whose solution gives the value k.

$$\int_0^{k} \left(4-(x^4-2.3x^3+4)\right) dx + \int_k^{2.3} \left(4-(x^4-2.3x^3+4)\right) dx$$

$$=$$

$$\int_0^{2.3} \left(4-(x^4-2.3x^3+4)\right) dx$$

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AP® CALCULUS AB
2014 SCORING COMMENTARY

Question 2

Overview

In this problem a sketch of the boundary curves of a planar region R in the first quadrant was given. One boundary is the graph of $f(x) = x^4 - 2.3x^3 + 4,$ and the other boundary is the line $y = 4.$ In part (a) students were expected to compute the volume of the solid generated when R is rotated about the horizontal line $y = -2,$ using the method of washers. Both the integral setup and evaluation were required. Students needed to find the limits of integration and the integrand. The limits of integration are the solutions of $f(x) = 4.$ The solutions can be found by algebra or using the calculator. By the method of washers, the integrand is $\pi\left(6^2 - (f(x) + 2)^2\right)$ because the outer radius of the washer centered at $(x, 0)$ is $4 + 2 = 6$ and the inner radius of that washer is $f(x) + 2.$ Students were expected to evaluate the resulting integral by using the calculator. In part (b) students were expected to find the volume of the solid by integrating $A(x),$ the area of the cross section of the solid at $(x, 0),$ from $x = 0$ to $x = 2.3.$ By geometry, $A(x) = \left(\frac{1}{2}\right)(4 - f(x))^2.$ In part (c) students were expected to realize that the area inside R to the left of $x = k$ can be written as $\int_0^k (4 - f(x))\, dx$ and the area inside R to the right of $x = k$ can be written as $\int_k^{2.3} (4 - f(x))\, dx.$ Thus, if the vertical line $x = k$ divides R into two regions with equal areas, then $\int_0^k (4 - f(x))\, dx = \int_k^{2.3} (4 - f(x))\, dx.$

Sample: 2A
Score: 9

The student earned all 9 points.

Sample: 2B
Score: 6

The student earned 6 points: 2 points in part (a), 2 points in part (b), and 2 points in part (c). In part (a) the student presents the correct square of the outer radius, but the student does not present the correct square of the inner radius. The student earned 1 of the 2 integrand points and the limits point. The student is not eligible for the answer point. In part (b) the student has a correct integrand for the cross-sectional area. The volume is not calculated correctly. In part (c) the student's work is correct.

Question 2 (continued)

Sample: 2C
Score: 3

The student earned 3 points: 1 point in part (a), 1 point in part (b), and 1 point in part (c). In part (a) the student has neither the square of the outer radius nor the square of the inner radius. The student did not earn any integrand points, but the limits point was earned. The student is not eligible for the answer point. In part (b) the student attempts to work with the area of a cross section involving an isosceles right triangle. The student presents a correct expression for the length of one of the sides of the triangle, but presents an incorrect expression for the length of the other side. The student earned 1 of the 2 integrand points and is not eligible for the answer point. In part (c) the student has the parameter k as the upper limit in an integral expression for the area of a portion of the region R. The student earned the point for the area of one region. Although the student writes an equation, the equation is true for any value of k. The student did not earn the equation point.