# Student Performance Q&A:

## 2014 AP® Calculus AB and Calculus BC Free-Response Questions

The following comments on the 2014 free-response questions for AP® Calculus AB and Calculus BC were written by the Chief Reader, Stephen Kokoska of Bloomsburg University, Bloomsburg, Pa. They give an overview of each free-response question and of how students performed on the question, including typical student errors. General comments regarding the skills and content that students frequently have the most problems with are included. Some suggestions for improving student performance in these areas are also provided. Teachers are encouraged to attend a College Board workshop to learn strategies for improving student performance in specific areas.

### Question AB1/BC1

*What was the intent of this question?*

In this problem students were given $A(t)$, a model for the amount of grass clippings, in pounds, contained in a bin at time $t$ days for $0 \le t \le 30$. In part (a) students were asked to show the calculation of the average rate of change of $A(t)$ over the interval $0 \le t \le 30$ and specify the units of the result – pounds per day. In part (b) students were asked to calculate the derivative of $A(t)$ at $t = 15$, either by using the calculator or by applying basic derivative formulas to $A(t)$ to obtain $A'(t)$ and then evaluating $A'(t)$ at $t = 15$. This answer is negative. Therefore, students needed to interpret the absolute value of this answer as the rate at which the amount of grass clippings in the bin is decreasing, in pounds per day, at time $t = 15$ days. In part (c) students were given two tasks. First, students needed to set up and evaluate the integral expression for the average value of $A(t)$ over the interval $0 \le t \le 30$, namely $\frac{1}{30}\int_0^{30} A(t)\,dt$. Second, students needed to set up and solve the equation $A(t) = \frac{1}{30}\int_0^{30} A(t)\,dt$ for $t$ in the interval $0 \le t \le 30$. In part (d) students needed to compute $A(30)$, $A'(30)$, and write $L(t) = A(30) + A'(30)(t - 30)$. Students were to then solve the equation $L(t) = 0.5$.

*How well did students perform on this question?*

In general, students taking the Calculus AB exam did not perform as well on this problem as they have on the first problem of the exam in recent years. The context of this problem, decomposing grass clippings, may have been an issue for some students. Calculus BC students performed better on this first problem than last year's first problem. The mean score was 2.50 for AB students and 4.26 for BC students out of a possible 9 points. Over 25 percent of AB students did not earn any points on this problem.

Most students found part (a) and the first part of (b) easier than parts (c) and (d). In part (b) many students knew how to find $A'(15)$ and that it was a rate, but students could not describe this value in the context of the problem. In part (c) many students simply did not know how to start this part; they could not write an appropriate equation. Most students who were able to write a correct equation could solve the equation using their calculators. Some students rounded intermediate values, and some students made decimal presentation errors (e.g., not rounding final answers accurately to 3 decimal places).

Many students also had difficulty starting part (d). For those who could find an expression for the tangent line, $L(t)$, there were still some issues involving rounding of intermediate values and decimal presentation errors.

### *What were common student errors or omissions?*

In part (a) some students provided incorrect units or no units, and some computed the average value instead of the average rate of change.

In part (b) many students did not earn the interpretation point. These students often used inappropriate prepositions in their interpretation, for example, “after” or “over,” which suggest a time interval rather than a specific time. Many students also missed the interpretation by writing the amount of grass clippings in the bin is decreasing at $-0.164$. Some students used incorrect mathematical language or terms in their interpretation attempts (e.g., “rate decreasing,” which suggested a second derivative).

In part (c) many students simply did not know how to begin to solve this part. These students could not write an appropriate equation or did not know the definition of the average value of a function over an interval. In addition, many students did not maintain enough accuracy in intermediate values. Therefore, those students were not able to earn the answer point.

For those students who were able to start part (d), many had difficulty determining $A'(30)$ and $A(30)$. Many students did not maintain accuracy of intermediate values here also. Some students did not report the mathematical equation or setup that led to the solution.

### *Based on your experience of student responses at the AP® Reading, what message would you like to send to teachers that might help them to improve the performance of their students on the exam?*

Teachers should continue to present a variety of contextual problems, and this problem is a fine teaching tool. Students should be able to compare and calculate the average rate of change and the average value of a function over an interval. Students should be able to interpret numerical derivatives and definite integrals in various contextual situations.

Despite the continued emphasis on the use of technology, students still need to understand how to store and use intermediate values in their graphing calculators in order to ensure accuracy of the final answer. Grading experience on this problem suggests that students need more practice using their graphing calculators to find the numerical derivative of a function at a point.

Mathematical notation, communication, and presentation of results are very important. There were some students who seemed to know important calculus concepts, but could not communicate precisely and thoroughly. It is important for students to remember that if they use their calculators for one of the four required capabilities, they must indicate the mathematical setup using proper mathematical notation.

## Question AB2

*What was the intent of this question?*

In this problem a sketch of the boundary curves of a planar region $R$ in the first quadrant was given. One boundary is the graph of $f(x) = x^4 - 2.3x^3 + 4,$ and the other boundary is the line $y = 4.$ In part (a) students were expected to compute the volume of the solid generated when $R$ is rotated about the horizontal line $y = -2,$ using the method of washers. Both the integral setup and evaluation were required. Students needed to find the limits of integration and the integrand. The limits of integration are the solutions of $f(x) = 4.$ The solutions can be found by algebra or using the calculator. By the method of washers, the integrand is $\pi\left(6^2 - (f(x) + 2)^2\right)$ because the outer radius of the washer centered at $(x, 0)$ is $4 + 2 = 6$ and the inner radius of that washer is $f(x) + 2.$ Students were expected to evaluate the resulting integral by using the calculator. In part (b) students were expected to find the volume of the solid by integrating $A(x),$ the area of the cross section of the solid at $(x, 0),$ from $x = 0$ to $x = 2.3.$ By geometry, $A(x) = \left(\frac{1}{2}\right)(4 - f(x))^2.$ In part (c) students were expected to realize that the area inside $R$ to the left of $x = k$ can be written as $\int_0^k (4 - f(x))\, dx$ and the area inside $R$ to the right of $x = k$ can be written as $\int_k^{2.3} (4 - f(x))\, dx.$ Thus, if the vertical line $x = k$ divides $R$ into two regions with equal areas, then $\int_0^k (4 - f(x))\, dx = \int_k^{2.3} (4 - f(x))\, dx.$

*How well did students perform on this question?*

This problem was a typical area/volume problem. The mean score was 3.39 out of a possible 9 points. Nearly 23 percent of students did not earn any points on this problem.

In part (a) some students had presentation errors, most often by omitting a pair of parentheses. This occurred most often if the student used the polynomial expression rather than the function name, $f(x).$ Some students had difficulty expressing the lower bound of the definite integral, which was probably related to calculator use. Calculator syntax rather than mathematical notation was presented by some students. Some students were not able to compute the value of the definite integral using the graphing calculator.

In part (b) students had difficulty finding the area of an arbitrary cross section in terms of $x$. There were some presentation errors in part (b) involving parentheses, and some students could not find the value of the definite integral.

In part (c) many students received either 0 or 2 points. Students either knew exactly how to treat the parameter $k$, or students did not know how to begin this part. Some students computed the area of the region $R$ without any supporting mathematical setup, using their graphing calculators without explaining their numerical result.

*What were common student errors or omissions?*

In part (a) many students reported the $x$-coordinate of the left-hand point of intersection using calculator syntax for scientific notation. Many did not recognize this value as 0. Similarly, for the right-hand point of intersection, many students without thinking used the calculator rather than noticing the $x$-coordinate was 2.3. Some students set up a definite integral to find the volume of a solid rotated about the $x$-axis, and many students had presentation errors involving parentheses and distribution of terms. Some could not find the final answer using technology.

In part (b) many students had difficulty finding the area of a cross section of the solid described. It appeared many students did not know the area of an isosceles triangle. Some students included additional constants times the definite integral. The most common incorrect constants included were $\pi$ and $\sqrt{3}$. Some students had trouble evaluating the definite integral using the calculator.

In part (c), some students had difficulty working with an unknown parameter $k$. Several students started part (c) by reporting the total area of the region $R$ without any supporting work. Some students presented an equation that expressed the total area of $R$ as the sum of two arbitrary parts.

*Based on your experience of student responses at the AP® Reading, what message would you like to send to teachers that might help them to improve the performance of their students on the exam?*

Teachers should continue to stress presentation skills, especially the proper use of parentheses and distributing a negative sign. Students should be encouraged to use the names of functions if given, for example, $y = f(x)$ rather than the polynomial expression. Students need more experience in finding cross-sectional areas of solids and in problems involving unknown parameters.

## Question AB3/BC3

*What was the intent of this question?*

In this problem students were given the graph of a piecewise continuous function $f$ defined on the closed interval $[-5, 4]$. The graph of $f$ consists of line segments whose slopes can be determined precisely. A second function $g$ is defined by $g(x) = \int_{-3}^{x} f(t)\, dt$. In part (a) students must calculate $g(3) = \int_{-3}^{3} f(t)\, dt$ by using a decomposition of $\int_{-3}^{3} f(t)\, dt$, such as $\int_{-3}^{3} f(t)\, dt = \int_{-3}^{2} f(t)\, dt + \int_{2}^{3} f(t)\, dt$, and by applying the relationship between the definite integral of a continuous function and the area of the region between the graph of that function and the $x$-axis. In part (b) students were expected to apply the Fundamental Theorem of Calculus to conclude that $g'(x) = f(x)$ on the interval $[-5, 4]$. Students were to then conclude that $g''(x) = f'(x)$ wherever $f'(x)$ is defined on $[-5, 4]$. Students needed to explain that the intervals $(-5, -3)$ and $(0, 2)$ are the only open intervals where both $g'(x) = f(x)$ is positive and decreasing. In part (c) students were expected to apply the quotient rule to find $h'(3)$ using the result from part (a) and the value $g'(3) = f(3)$ from the graph of $f$. In part (d) students were expected to apply the chain rule to find $p'(-1)$. This required finding $f'(2)$ from the graph of $f$.

*How well did students perform on this question?*

In general, students performed well on this problem. Problems of this type, in which a function is defined by a definite integral, have appeared on recent AP Calculus Exams, and teachers seem to be preparing students for this type of question. The mean score was 3.26 for AB students and 5.24 for BC students out of a possible 9 points. Over 22 percent of AB students did not earn any points on this problem.

Most students were able to answer part (a). They correctly calculated the areas of triangles and used these areas to compute the final answer. In part (b) many students found at least one of the intervals. However, some students did not explain the connection between the functions $g'$ and $f$.

In part (c) many students used the quotient rule properly. However, there were some presentation errors, primarily missing parentheses. In part (d) many students recognized the need for the chain rule. However, many students made presentation errors or did not properly differentiate $x^2 - x$.

*What were common student errors or omissions?*

In part (a) the most common student error involved the area of the region below the $x$-axis. Some students treated the area of this region as negative. In part (b) many students had difficulty stating a correct reason for their answers. Some gave a "recipe" for the reason, e.g., $g'(x) > 0$ and $g''(x) < 0.$ These students failed to connect $g'$ to $f$. Other students added extra, incorrect information to their reasons. These students did not earn the reason point as a result of writing incorrect mathematical statements in the context of the problem.

In part (c) the most common student error was a presentation mistake of omitting the parentheses in the denominator. These students wrote $5x^2$ instead of $(5x)^2$. Many of these students then evaluated the denominator as 45 rather than 225 when $x = 3.$ In part (d) many students simply did not know how to apply the chain rule. Of those students who attempted to find the derivative using the chain rule, some forgot the parentheses around $2x - 1.$ Some students went on to find an equation of the line tangent to the graph of $y = p(x)$ and made an error in their work.

*Based on your experience of student responses at the AP® Reading, what message would you like to send to teachers that might help them to improve the performance of their students on the exam?*

Use of parentheses correctly, proper mathematical notation, and presentation of work are important. Teachers should continue to stress succinct, appropriate communication. Students need to be specific and use names when referring to a function or a derivative. Students cannot simply say "the derivative," "the function," or "the slope." These phrases are usually too vague. Many students are writing too much in their reasons or justifications. Succinct responses that directly address the question asked are recommended. Teachers should remind students to be very careful when simplifying a numerical or algebraic result. Final answers do not need to be simplified. However, if a student tries to simplify and makes a mistake, the student will typically not earn the answer point.

## Question AB4/BC4

*What was the intent of this question?*

In this problem students were given a table of values of a differentiable function $v_A(t)$, the velocity of Train $A$, in meters per minute, for selected values of $t$ in the interval $0 \le t \le 12,$ where $t$ is measured in minutes. In part (a) students were expected to know that the average acceleration of Train $A$ over the interval $2 \le t \le 8$ is the average rate of change of $v_A(t)$ over that interval. The unit of the average acceleration is meters per minute per minute. In part (b) students were expected to state clearly that $v_A$ is continuous because it is differentiable, and thus the Intermediate Value Theorem implies the existence of a time $t$ between $t = 5$ and $t = 8$ at which $v_A(t) = -100.$ In part (c) students were expected to show that the change in position over a time interval is given by the definite integral of the velocity over that time interval. If $s_A(t)$ is the position of Train $A$, in meters, at time $t$ minutes, then $s_A(12) - s_A(2) = \int_2^{12} v_A(t)\,dt$, which implies that $s_A(12) = 300 + \int_2^{12} v_A(t)\,dt$ is the position at $t = 12.$ Students approximated $\int_2^{12} v_A(t)\,dt$ using a trapezoidal approximation. In part (d) students had to determine the relationship between train $A$'s position, train $B$'s position, and the distance between the two trains. Students needed to put together several pieces of information from different parts of the problem and use implicit differentiation to determine the rate at which the distance between the two trains is changing at time $t = 2.$

### *How well did students perform on this question?*

Both AB and BC students did not perform as well as expected on this problem. The mean score was 2.64 for AB students and 3.97 for BC students out of a possible 9 points. Over 20 percent of AB students did not earn any points on this problem.

In part (a) the majority of students were able to find the average acceleration. Because the answer was not dependent on units, it was only necessary to present evidence of a difference quotient and the answer. Students did not need to simplify. Most errors were the result of arithmetic mistakes in simplifying the answer. In part (b) many students seemed to know to use the Intermediate Value Theorem. However, many had trouble applying the theorem in this context.

In part (c) many students presented a correct position expression. Some students omitted the initial position of 300. Many students presented correct trapezoidal sums. Some students extended the interval to $[0, 12]$. In part (d) many students were unable to present a distance relationship. Therefore, students were unable to find the derivative using implicit differentiation.

### *What were common student errors or omissions?*

The most common student errors in part (a) involved arithmetic errors and dropping a sign in the final answer. In part (b) many students did not mention the specific values that bracketed $-100$. Some students cited an incorrect or incomplete reason. Some students cited differentiability, the Mean Value Theorem, or the Extreme Value Theorem as a reason.

In part (c) the most common student error was a mistake in the expression for position. Some students omitted the 300. Some students were not able to write the appropriate trapezoidal sum. This error was often due to extending the interval or using incorrect coefficients. Students who correctly set up the position often presented a correct answer of $-150$ meters; those who omitted the initial condition most often reported $-450$ meters. Few students went on to explain the meaning of the answer in terms of the position of the train with respect to Origin Station. In part (d) many students were unable to begin the problem; these students could not present an expression for the distance between trains $A$ and $B$. For those students who were able to find an expression for distance, there were several errors in implicit differentiation.

### *Based on your experience of student responses at the AP® Reading, what message would you like to send to teachers that might help them to improve the performance of their students on the exam?*

Teachers should remind students to provide a complete answer to each question, providing appropriate reasons. Vague and ambiguous answers often do not earn points. Teachers should continue to stress presentation of work and the use of proper mathematical notation. Students who presented a trapezoidal sum in part (c) often made presentation errors involving fractions and grouping.

Teachers should also make sure that students understand and can apply principal named theorems: the Intermediate Value Theorem, the Mean Value Theorem, and the Extreme Value Theorem. In addition, students should know that differentiability implies continuity.

## Question AB5

*What was the intent of this question?*

In this problem students were provided with a table giving values of two twice-differentiable functions $f$ and $g$ at various values of $x$. Part (a) asked students to find the $x$-coordinate of each relative minimum of $f$ on the given interval. Students should have determined that $x = 1$ is a critical point and that $f'$ changes sign from negative to positive at that point. In part (b) students had to explain why there is a value $c$, for $-1 < c < 1$, such that $f''(c) = 0$. Because the function is twice differentiable, $f'$ is continuous on the interval $-1 \le x \le 1$, and because $f'(1) = f'(-1) = 0$, the Mean Value Theorem guarantees that there is at least one value $c$, $-1 < c < 1$, such that $f''(c) = 0$. In part (c) students needed to differentiate $h(x)$ using the chain rule to get $h'(x) = \frac{1}{f(x)} \cdot f'(x)$. Using values from the table, $h'(3) = \frac{1}{14}$. Part (d) required students to find the antiderivative of the integrand to get $f(g(3)) - f(g(-2))$. Using values from the table, the result is $-6$.

*How well did students perform on this question?*

Many students performed well on this question, and most students showed significant work in all four parts. Students had a difficult time justifying in part (a) and explaining in part (b). It was common to see a student earn 6 points on this problem with all points earned in parts (c) and (d). Difficulty understanding and using notation caused problems for some students in parts (c) and (d). Many students did not show knowledge of the derivative of the natural logarithm function in part (c), and some had trouble handling fractional arithmetic, also in part (c). The mean score was 3.62 out of a possible 9 points. Nearly 28 percent of students did not earn any points on this problem.

Most students were able to use the information presented in tabular form and focus on the specific entries needed. Some students misread the table, and some students did not show sufficient work, especially in part (d). Most students communicated the computations that led to their answers.

*What were common student errors or omissions?*

In part (a) some students did not earn this point due to vague language in their explanations. There were some unclear references to the derivative or the slope. Some students even referred to “the graph,” even though no graph was presented in the problem. Some students wrote fragments of explanations, e.g., “because negative to positive,” without any reference to what was changing sign. Another common error involved a reference to the decreasing-then-increasing behavior of the function $f$ rather than justifying the answer using the function $f'$.

In part (b) many students attempted to make an argument based only on the existence of a point of inflection or a change in concavity. These students were not able to provide evidence for this argument. Some students correctly identified the relevance of the Mean Value Theorem or Rolle’s Theorem but failed to apply the theorem to the problem. Often, these students made statements about $f'$ and $f$ rather than $f''$ and $f'$. Many students did not refer to the Mean Value Theorem or Rolle’s theorem by name, and instead attempted to construct their own complete argument. Many of these students failed to explicitly state that $f'$ is continuous, a necessary condition for the argument.

In part (c) some students did not use the chain rule. Some students presented work that included an incorrect derivative of the natural logarithm function. The most common presentation error was $h'(x) = \frac{1}{\ln(f(x))} \cdot f'(x)$.

Some students did not simplify correctly, so they did not earn the answer point.

In part (d) many students incorrectly assumed the antiderivative of a product is the product of the antiderivatives. Some students did not antidifferentiate and substituted the limits of integration directly into the integrand. There were also simplification errors in part (d).

*Based on your experience of student responses at the AP® Reading, what message would you like to send to teachers that might help them to improve the performance of their students on the exam?*

Most students managed this tabular presentation well. Teachers should continue to include problems with functions presented analytically, graphically, and in tabular form. Students need more opportunities to solve problems in which they must justify their conclusions. Students must be specific when referring to a function and use the proper notation. Students must learn the importance of differentiability and/or continuity in making claims about function behavior, and they must be able to apply certain theorems to specific information.

Students need more practice with problems that involve notational fluency. Students must use correct notation in a variety of contexts, including working with derivatives and antiderivatives of composite functions. Teachers should provide students with more routine applications of the chain rule and antidifferentiation.

## Question AB6

*What was the intent of this question?*

This problem presented students with a differential equation and defined $y = f(x)$ to be the particular solution to the differential equation passing through a given point. Part (a) presented students with a portion of the slope field of the differential equation and asked students to draw a solution curve through the point $(0, 1)$. Part (b) asked students to write an equation for the line tangent to the solution curve from part (a) at a given point, and then to use this tangent line to approximate $f(x)$ at a nearby value of $x$. Students needed to recognize that the slope of the tangent line is the value of the derivative given in the differential equation at the given point. Part (c) asked for the particular solution to the differential equation that passes through the given point. Students were expected to use the method of separation of variables to solve the differential equation.

*How well did students perform on this question?*

The mean score was 3.92 out of a possible 9 points. Most students were able to sketch a solution curve in part (a). Most students were able to separate the variables in part (c) and earn at least half of the points. Nearly 19 percent of students did not earn any points on this problem.

*What were common student errors or omissions?*

In part (a) the most common student error was failure to use the initial condition $f(0) = 1$. These students provided a sketch of the solution curve that did not pass through the point $(0, 1)$. Some students extended the solution curve above $y = 3$.

In part (b) many students produced a slope of the tangent line without an algebraic connection to $\frac{dy}{dx} = (3 - y)\cos x$. Most students earned at least one point in part (b).

Very few students failed to separate the variables in part (c). However, some students incorrectly separated the variables. In the antidifferentiation step, some students missed the factor of $(-1)$ in the expression $-\ln|3 - y|$. Some students made algebraic errors when manipulating the exponential function.

*Based on your experience of student responses at the AP® Reading, what message would you like to send to teachers that might help them to improve the performance of their students on the exam?*

Teachers should provide more practice for students in drawing solutions curves for differential equations with an initial condition. Students should understand and use $\frac{dy}{dx}$ to compute the slope of a function at a specific point. Students still need practice in finding antiderivatives of basic functions. Teachers should review the properties of exponential functions; the ability to perform algebraic manipulation is still important.

## Question BC2

*What was the intent of this question?*

In this problem students were given the graphs of the polar curves $r = 3 - 2\sin(2\theta)$ and $r = 3$ for $0 \le \theta \le \pi$. In part (a) students had to find the area of the shaded region $R$ that is the common area inside both graphs. Students needed to find the area bounded by the polar curve $r = 3 - 2\sin(2\theta)$ in the first quadrant and add it to the area of the quarter circle in the second quadrant resulting in $\frac{1}{2}\int_0^{\pi/2}(3 - 2\sin(2\theta))^2\,d\theta + \frac{9\pi}{4}$. In part (b) students needed to find $\frac{dx}{d\theta}$ at $\theta = \frac{\pi}{6}$. Students had to realize that $x = r\cos(\theta)$ and then differentiate to find $\frac{dx}{d\theta}$ at $\theta = \frac{\pi}{6}$. In part (c) students were asked to find the rate at which the distance between the curves in the first quadrant was changing at $\theta = \frac{\pi}{3}$. Students needed to set up an equation for the distance between the curves in the first quadrant, $D = 3 - (3 - 2\sin(2\theta))$, and then evaluate the derivative of $D$ at the required value. Finally, in part (d) students were given that $\frac{d\theta}{dt} = 3$ and were asked to find the value of $\frac{dr}{dt}$ at $\theta = \frac{\pi}{6}$. Students had to invoke the chain rule to get $\frac{dr}{dt} = \frac{dr}{d\theta} \cdot \frac{d\theta}{dt}$ and evaluate the result at $\theta = \frac{\pi}{6}$.

*How well did students perform on this question?*

Most students were able to set up an integral representing polar area. In part (b) most students were successful in writing an expression for $x$. However, there were some presentation errors involving parentheses. In part (c) many students had difficulty determining an expression for the distance between the two curves. In part (d) most students recognized the need to use the chain rule.

The mean score was 3.96 out of a possible 9 points. Over 16 percent of students did not earn any points on this problem.

*What were common student errors or omissions?*

In part (a) many students found the area of the shaded portion in the first quadrant and correctly added it to the quarter circle area in the second quadrant. Others found the area of the unshaded region in the first quadrant and correctly subtracted that from the area of a semicircle. Several students wrote a correct integral expression that could be used to find the area of the region $R$, but combined it with an incorrect integral (e.g., the sum of the unshaded regions) or combined it with an incorrect circular area. Some students struggled to determine which area to find and how to combine areas. While many students had the correct answer after the correct setup, several had difficulty producing the correct answer using their calculators. Common incorrect values included the value obtained by failing to use a constant of $\frac{1}{2}$ or the value obtained by failing to square the integrand. Some students earned the integrand point but failed to include correct limits.

In part (b), of those students who were able to enter the problem, very few failed to substitute the expression for $r$ into $x = r\cos\theta$. Many students used the calculator to compute the value of the derivative. Some students used symbolic differentiation and tried to evaluate the trigonometric functions at $\theta = \frac{\pi}{6}$, which often resulted in incorrect answers. Parentheses were an issue for many students when presenting their expression for $x$.

In part (c) many students presented an expression for distance involving the square root function with arguments that were perhaps from the arc length formula. Some students tried to calculate $\frac{dy}{dx}$. Of the students who correctly determined an expression for the distance between the curves, many had parentheses issues or reversal issues resulting in the opposite of the correct answer.

In part (d) most students understood that to find $\frac{dr}{dt}$, they needed to use the chain rule. Some students incorrectly differentiated $\frac{dr}{d\theta}$ and some divided by $\frac{d\theta}{dt}$.

*Based on your experience of student responses at the AP® Reading, what message would you like to send to teachers that might help them to improve the performance of their students on the exam?*

Teachers should emphasize the need to present mathematical notation correctly and clearly communicate work, especially the use of parentheses and labeling of expressions. Students still need practice using their calculators to find the value of a definite integral that involves a complicated integrand and to find the derivative of a function at a specific value.

## Question BC5

*What was the intent of this question?*

In this problem students were given the graph of a region $R$ bounded by the graph of $y = xe^{x^2}$, the line $y = -2x$, and the vertical line $x = 1$. In part (a) students were asked to find the area of $R$, requiring an appropriate integral setup and evaluation. Students needed to correctly evaluate $\int_0^1 \left(xe^{x^2} - (-2x)\right) dx$. Part (b) required students to find the volume of the solid generated when $R$ is rotated about the horizontal line $y = -2$. Students needed to set up an integral where the integrand represents a cross-sectional area of a circular disc with inner radius $(-2x + 2)$ and outer radius $\left(xe^{x^2} + 2\right)$. This yielded the integral $\pi \int_0^1 \left(\left(xe^{x^2} + 2\right)^2 - (-2x + 2)^2\right) dx$. In part (c) students needed to write an expression involving one or more integrals that gives the perimeter of $R$. Students should have recognized that part of the perimeter involves finding the length of the curve $y = xe^{x^2}$ from $x = 0$ to $x = 1$ as well as the length of the two line segments. The resulting expression is $\int_0^1 \sqrt{1 + \left(e^{x^2}\left(1 + 2x^2\right)\right)^2}\, dx + \sqrt{5} + (2 + e)$.

*How well did students perform on this question?*

Students performed well on this question. The mean score was 4.45 out of a possible 9 points. Most students were able to enter the problem, and there were very few blank papers.

*What were common student errors or omissions?*

In part (a) most students attempted to set up an integral to find the area of the region $R$. Some students used a geometric approach. Many students successfully found the antiderivative of $xe^{x^2}$. The most common error involved finding this antiderivative. Some students tried using integration by parts and others treated $x$ as a constant. Some students added the functions $xe^{x^2}$ and $-2x,$ rather than subtracting them. Some students failed to express the area of the triangular region below the $x$-axis as a positive number in a sum to represent the total area of the region $R$. Some students made algebra and/or arithmetic errors. Some students simplified $e^0$ incorrectly, and others presented poorly organized work that was difficult to follow.

In part (b) most students attempted to set up an integral using the washer method. There were several errors involving squaring: squaring the difference of the radii, only squaring one radius, and not squaring either radii. Some students incorrectly combined the radii, taking the sum of the squares or writing a reversal in the integrand. Some students used an incorrect axis of rotation. Some students used an incorrect constant for the washer method; the most common errors were 1 and $2\pi$. There were some algebra errors and parentheses presentation errors.

In part (c), most students recognized the need to differentiate $xe^{x^2}$. Many students did not differentiate this function correctly. Many students were unable to recall the formula for arc length. Some students omitted the length of the vertical line segment in the final answer. There were some algebra errors.

*Based on your experience of student responses at the AP® Reading, what message would you like to send to teachers that might help them to improve the performance of their students on the exam?*

Teachers should provide more practice in finding the area bounded by two curves. Students should understand that when integrating with respect to $x$, if $f(x) \geq g(x),$ then the integrand should be $f(x) - g(x).$ Students must present an integral that leads to an antiderivative. Teachers should caution students to use the correct axis of rotation, use parentheses where appropriate, and properly apply differentiation rules, especially related to transcendental functions. Teachers must continue to stress communication skills. Students must label their work and carefully organize their presentation of work to ensure their solutions will be accurately scored.

## Question BC6

*What was the intent of this question?*

In this problem students were given a Taylor series for a function $f$ about $x = 1.$ In part (a) students were asked to find the radius of convergence of this Taylor series. It was expected that students would use the ratio test to determine that the radius of convergence is $\frac{1}{2}.$ In part (b) students needed to differentiate the series term-by-term to find the first three nonzero terms and the general term of the Taylor series for $f'.$ In part (c) students were told that the Taylor series for $f'$ is a geometric series. Students needed to know that finding the sum of that series requires dividing the first term of the series by the difference of 1 and the common ratio. This results in $f'(x) = \frac{2}{2x - 1}.$ Students were also asked to find $f$. This required integrating $f'(x)$ to find $f(x) = \ln|2x - 1| + C.$ In order to evaluate the constant of integration, students needed to use the initial condition that $f(1) = 0$ which yields $f(x) = \ln|2x - 1|$ for $|x - 1| < \frac{1}{2}.$

*How well did students perform on this question?*

The mean score on this problem was 3.10 out of 9 possible points. This was good for a series question that was the last problem on the exam. Twenty-two percent of students did not earn any points on this problem. Most students presented good work in parts (a) and (b). Part (c) was difficult for most students.

*What were common student errors or omissions?*

In part (a) many students did not use correct or consistent notation. Absolute value signs and limit notations appeared, and then were omitted in subsequent steps. Some students lost a factor of 2, and some students reported an interval rather than a radius of convergence. Some students also made algebra errors in working with the absolute value.

In part (b) many students forgot to write the general term of the series or wrote an incorrect exponent in the general term. Some students included $n!$ in the denominator of the general term.

Many students had trouble answering part (c). Many students could not identify the constant ratio, and many made algebra errors when simplifying the expression for $f'(x)$. Some students omitted a constant of integration and therefore did not address the initial condition. The most common incorrect answer for $f'$ was

$f'(x) = \dfrac{2}{3 - 2x}$.

*Based on your experience of student responses at the AP® Reading, what message would you like to send to teachers that might help them to improve the performance of their students on the exam?*

Students need more practice manipulating and working with power series. Series is an important "big idea" for the Calculus BC course. Students need time to process the concepts and techniques associated with power series so that they can apply what they have learned. For example, students should have multiple opportunities to differentiate and antidifferentiate power series. If the root test is used instead of the ratio test, students must learn when the root test is appropriate to determine an interval of convergence.

Students should be encouraged to use good notation and do so consistently. For example, a student should not write $\lim\limits_{n\to\infty}\left|\dfrac{2^{n+1}(x-1)^{n+1}}{2^n(x-1)^n}\cdot\dfrac{n}{n+1}\right| = \left|(x-1)\cdot\dfrac{2n}{n+1}\right| = 2|x-1|$, dropping the limit symbol prior to evaluating the limit. Students should be reminded not to confuse the Taylor series for a function and a specific Taylor polynomial. For example, $f'(x) \neq 2 - 4(x-1) + 8(x-1)^2$. Students need more work with geometric series. They should be able to express a function of the form $\dfrac{a}{k+bx}$ as a geometric series and be able to express a series of the form $\sum a(x-c)^n$ as $\dfrac{a}{k+bx}$.