Question 6

Intent of Question

The primary goals of this question were to assess a student's ability to (1) summarize information provided in a time plot that involves trend components; (2) perform calculations related to a summary statistic not previously studied; and (3) compare and contrast information conveyed by the summary statistics with the data.

Solution

Part (a):

The Western Pacific Ocean had more typhoons than the Eastern Pacific Ocean in all but one of these years. The average seems to have been about 31 typhoons per year in the Western Pacific Ocean, which is higher than the average of about 19 typhoons per year in the Eastern Pacific Ocean. The Western Pacific Ocean also saw more variability (in number of typhoons per year) than the Eastern Pacific Ocean; for example, the range of the frequencies for the Western Pacific is about 21 typhoons and only 10 typhoons for the Eastern Pacific.

Part (b):

The Western Pacific Ocean had a decreasing trend in number of typhoons per year over this time period, especially from about 2001 through 2010. In contrast, the Eastern Pacific Ocean was fairly consistent in the number of typhoons per year over this time period, with a slight increasing trend in the later years from 2005 through 2010.

Part (c):

The four-year moving average for the year 2010 in the Western Pacific Ocean is

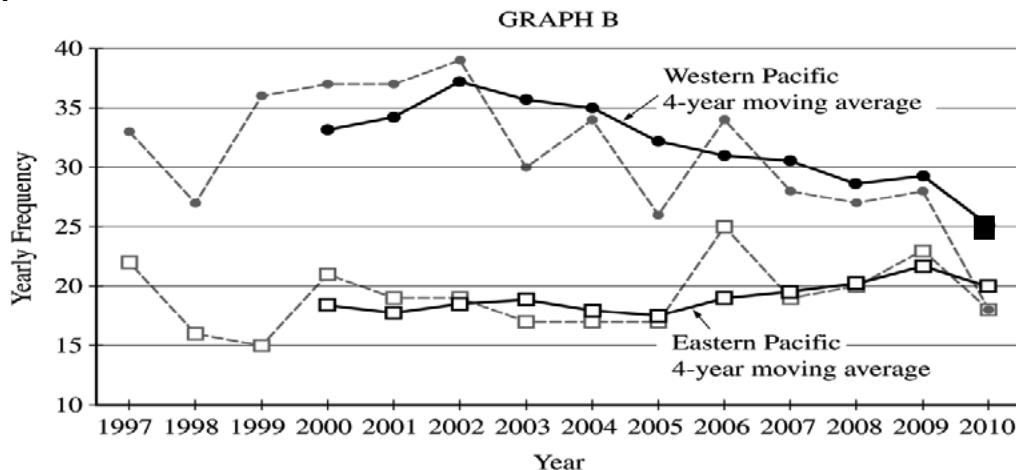$$\frac{28 + 27 + 28 + 18}{4} = 25.25.$$

The value is written in the table as follows.

:	:	:	:	:
2008	20	20.25	27	28.75
2009	23	21.75	28	29.25
2010	18	20.00	18	**25.25**

Part (d):

GRAPH B

Part (e):

(i) The overall trends across this time period were more apparent with the moving averages than with the original frequencies. The moving averages reduce variability, making more apparent the overall decreasing trend in number of typhoons in the Western Pacific Ocean and the slight increasing trend in the number of typhoons in the Eastern Pacific Ocean.

(ii) The year-to-year variability in number of typhoons is less apparent with the moving averages than with the original frequencies.

Scoring

This question is scored in four sections: Section 1 consists of part (a); section 2 consists of part (b); section 3 consists of parts (c) and (d); and section 4 consists of part (e). Each of the four sections is scored as essentially correct (E), partially correct (P), or incorrect (I).

Section 1 is scored as follows:

Essentially correct (E) if the response correctly includes the following three components:
1. A comparison of the center or location of the two distributions of frequencies.
2. A comparison of the variability of the two distributions of frequencies.
3. Comments in context.

Partially correct (P) if the response correctly includes two of the three components listed above.

Incorrect (I) if the response does not meet the criteria for E or P.

Notes:
- Comparison of frequencies for just a few years of the two regions is not a comparison of the distributions of the frequencies of the two regions.
- Comparisons of the two endpoints in the 1997 and 2010 for both regions were not sufficient for component 2.

- The values of the mean (31 Western, 19.14 Eastern), median (31.5 Western, 19 Eastern), range (21 Western, 10 Eastern), or standard deviation (5.74 Western, 2.82 Eastern) were *not necessary* for components 1 or 2.
- Values that were not approximately equal to the mean, median, range, or standard deviation were <u>not</u> sufficient (for example, 17 is the value of the time series of the midpoint of the frequencies of the Eastern Pacific Ocean for the years 2003 and 2004 and not the median of the frequencies for the Eastern Pacific Ocean).
- Shape is typically required when describing distributions; however, it will not be necessary in this rubric for a response to be essentially correct in part (a). It is difficult to identify the shape of the distribution from Graph A (without creating separate graphs for the distributions of the frequencies).

Section 2 is scored as follows:

Essentially correct (E) if the response correctly comments on the following two components:
1. The overall trend for the Western Pacific Ocean.
2. The relatively constant (or slightly increasing) overall trend for the Eastern Pacific.

Partially correct (P) if the response correctly comments on only one of these overall trends
OR
if the response lists the changes of frequencies over time for both regions and does not specifically describe an overall trend for both regions.

Incorrect (I) if the response does not meet the criteria for E or P.

Notes:
- In distinguishing between a trend and a listing of changes in frequencies over time, at most two time segments can be used in describing a trend for either region.
- Comparison of the two endpoints in the 1997 and 2010 for a region is not sufficient for identifying an overall trend.
- Identifying the overall trends in the regions is an investigative part of this question. Thus, the extent to which the response appropriately describes the overall trends in part (b) can be used for the holistic scoring of this problem.

Section 3 is scored as follows:

Essentially correct (E) if the response includes the following four components:
1. Shows appropriate calculations.
2. Has the value of 25.25.
3. The calculated value in part (c) is correctly placed in the table in part (c).
4. The calculated value in part (c) is appropriately placed in the graph in part (d).

Partially correct (P) if the response includes two or three of the four components listed above.

Incorrect (I) if the response does not meet the criteria for E or P.

Notes:
- In completing the graph, it is not necessary to draw the line connecting the calculated moving average in part (c) for 2010 to the moving average value for 2009.
- An incorrect value in part (c) can be used to obtain component 3 or component 4.
- An appropriately placed value on the graph for component 4 requires the placement on the graph close to 2010 and at or slightly above the 25 yearly frequency line.
- The severity of the mistakes resulting in a partially correct response can be used in holistic scoring for this question.

Section 4 is scored as follows:

Essentially correct (E) if the response includes the following three components:
1. Correctly mentions overall trend in regards to the moving averages graph in at least one region.
2. Correctly identifies some characteristic of the data that is less apparent by not having individual frequencies of the typhoons.
3. Correctly links at least one of the previous two components to the time plot in this specific question.

Partially correct (P) if the response has exactly two of the three components.

Incorrect (I) if the response does not meet the criteria for E or P.

Notes:
- Stating that the frequencies were not determinable from the graph of moving averages is not sufficient for describing a less apparent characteristic of the data.
- If the response in part (e) can be applied to *any* plot of 4-year moving averages, then component 3 (linkage to the time plot in this specific question) is not achieved.
- Because moving averages decrease variability, which may make identifying trends easier, clear communication (for example identifying direction of the trend) in describing the trend or the decreased variability for these data can be used for the holistic scoring of this question.

Each essentially correct (E) section counts as 1 point, and a partially correct (P) section counts as ½ point.

    4   **Complete Response**

    3   **Substantial Response**

    2   **Developing Response**

    1   **Minimal Response**

If a response is between two scores (for example, 2½ points), use a holistic approach to determine whether to score up or down, depending on the overall strength of the response and communication. See notes following sections 2, 3, and 4 for additional guidance in holistic scoring of this question.
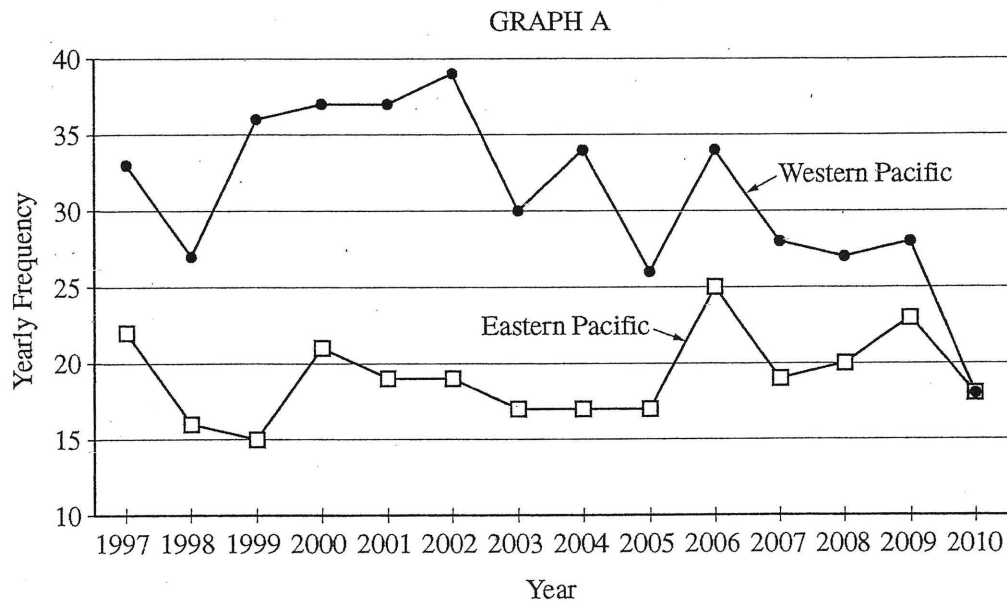
STATISTICS
SECTION II
Part B
Question 6
Spend about 25 minutes on this part of the exam.
Percent of Section II score—25

Directions: Show all your work. Indicate clearly the methods you use, because you will be scored on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.

6. Tropical storms in the Pacific Ocean with sustained winds that exceed 74 miles per hour are called typhoons. Graph A below displays the number of recorded typhoons in two regions of the Pacific Ocean—the Eastern Pacific and the Western Pacific—for the years from 1997 to 2010.

GRAPH A

GO ON TO THE NEXT PAGE.

(a) Compare the distributions of yearly frequencies of typhoons for the two regions of the Pacific Ocean for the years from 1997 to 2010.

The distribution of the western Pacific is centered higher than Eastern Pacific. There is more variability for western Pacific than for Eastern Pacific. The only time the two were the same was in 2010. The range of western is about 22 which is higher than Eastern rage of about 10.

(b) For each region, describe how the yearly frequencies changed over the time period from 1997 to 2010.

western: The yearly frequencies for western Pacific gradually got smaller over time.

Eastern: The yearly frequencies for western Pacific pretty much stayed the same, possibly getting slightly bigger.

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A moving average for data collected at regular time increments is the average of data values for two or more consecutive increments. The 4-year moving averages for the typhoon data are provided in the table below. For example, the Eastern Pacific 4-year moving average for 2000 is the average of 22, 16, 15, and 21, which is equal to 18.50.
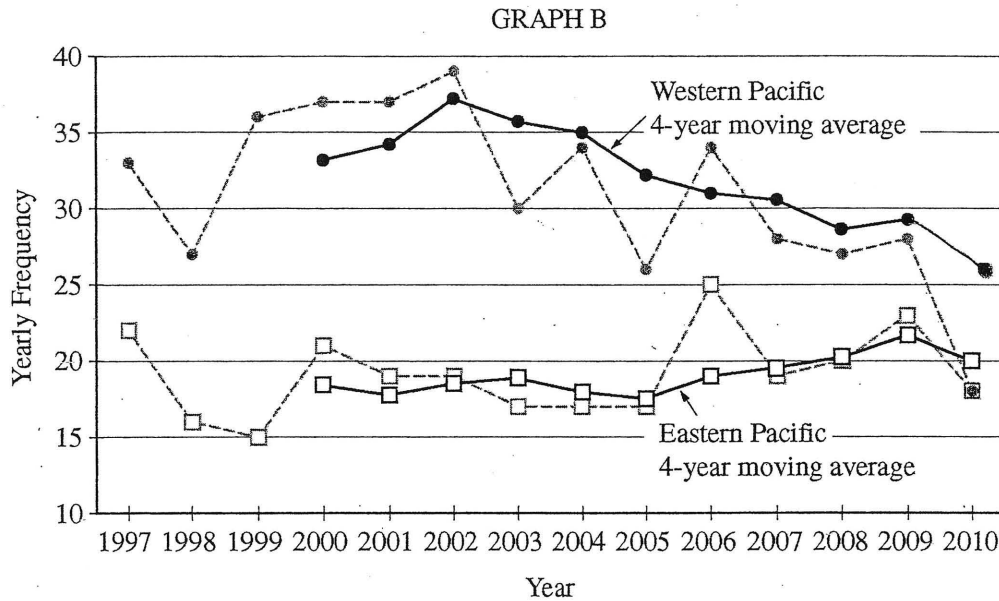
Year	Number of Typhoons in the Eastern Pacific	Eastern Pacific 4-year moving average	Number of Typhoons in the Western Pacific	Western Pacific 4-year moving average
1997	22		33	
1998	16		27	
1999	15		36	
2000	21	18.50	37	33.25
2001	19	17.75	37	34.25
2002	19	18.50	39	37.25
2003	17	19.00	30	35.75
2004	17	18.00	34	35.00
2005	17	17.50	26	32.25
2006	25	19.00	34	31.00
2007	19	19.50	28	30.50
2008	20	20.25	27	28.75
2009	23	21.75	28	29.25
2010	18	20.00	18	25.25

(c) Show how to calculate the 4-year moving average for the year 2010 in the Western Pacific. Write your value in the appropriate place in the table.

Take that year and the 3 previous years number of typhoons and get the average of it. That's the 4-year moving average.

$$\frac{18+28+27+28}{4} = 25.25$$

(d) Graph B below shows both yearly frequencies (connected by dashed lines) and the respective 4-year moving averages (connected by solid lines). Use your answer in part (c) to complete the graph.

GRAPH B

(e) Consider graph B.

i) What information is more apparent from the plots of the 4-year moving averages than from the plots of the yearly frequencies of typhoons?

The trends in the data are more apparent.
Western is becoming less frequent and
Eastern is becoming more frequent (not as
drastically as western).

ii) What information is less apparent from the plots of the 4-year moving averages than from the plots of the yearly frequencies of typhoons?

Big spikes in the data are less apparent
than in the regular yearly frequencies.

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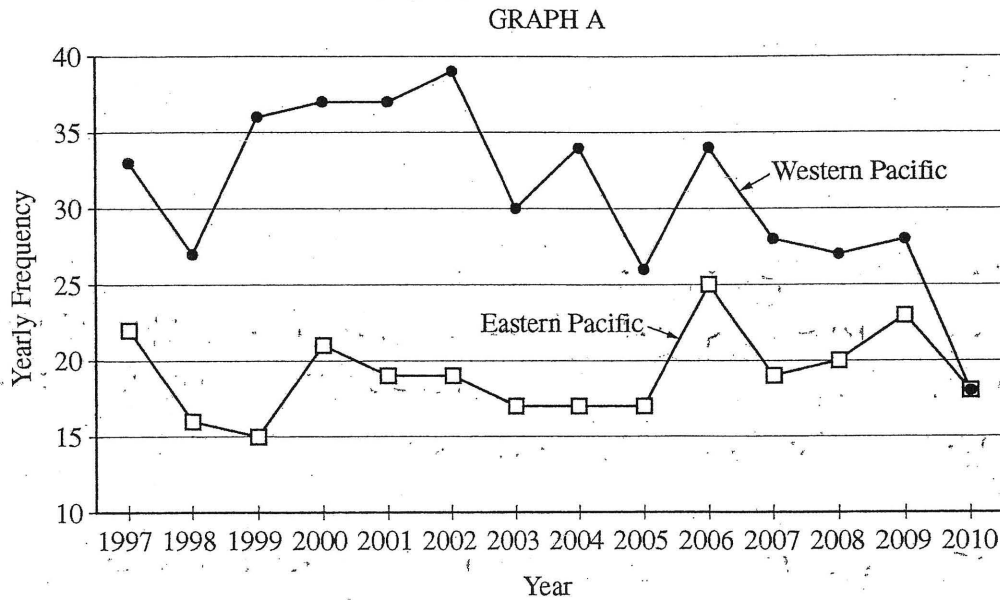
STATISTICS
SECTION II
Part B
Question 6
Spend about 25 minutes on this part of the exam.
Percent of Section II score—25

Directions: Show all your work. Indicate clearly the methods you use, because you will be scored on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.

6. Tropical storms in the Pacific Ocean with sustained winds that exceed 74 miles per hour are called typhoons. Graph A below displays the number of recorded typhoons in two regions of the Pacific Ocean—the Eastern Pacific and the Western Pacific—for the years from 1997 to 2010.

GRAPH A

GO ON TO THE NEXT PAGE.

(a) Compare the distributions of yearly frequencies of typhoons for the two regions of the Pacific Ocean for the years from 1997 to 2010.

the distribution of the western pacific is slightly above the distribution of the Eastern pacific. Neither of the graphs have clusters. The strengths of the distributions are low since they don't follow a consecutive line. These scatterplots don't follow a shape.

(b) For each region, describe how the yearly frequencies changed over the time period from 1997 to 2010.

western pacific:
From 1997 to 1988 the frequency decrease till it began to increase till 1999. From 1999 to 2002, it slighty increased but roughly stayed the same. It decreased to 2003, increased to 2004, decreased to 2005 increased to 2006 and decreased to 2008 with a slight increase but then a dramatic dencrease to 2010 were the value meets up with the Eastern pacific.

Eastern pacific:
From 1997 to 1999 it decreares, increases to 2000. From 2000 to 2005 it moderately remains the same until it increases to 2006. 2006-2007 in decreases then increases to 2009. From 2009-2010 it decreases meeting the same value as western pacific.

GO ON TO THE NEXT PAGE.

A moving average for data collected at regular time increments is the average of data values for two or more consecutive increments. The 4-year moving averages for the typhoon data are provided in the table below. For example, the Eastern Pacific 4-year moving average for 2000 is the average of 22, 16, 15, and 21, which is equal to 18.50.

Year	Number of Typhoons in the Eastern Pacific	Eastern Pacific 4-year moving average	Number of Typhoons in the Western Pacific	Western Pacific 4-year moving average
1997	22		33	
1998	16		27	
1999	15		36	
2000	21	18.50	37	33.25
2001	19	17.75	37	34.25
2002	19	18.50	39	37.25
2003	17	19.00	30	35.75
2004	17	18.00	34	35.00
2005	17	17.50	26	32.25
2006	25	19.00	34	31.00
2007	19	19.50	28	30.50
2008	20	20.25	27	28.75
2009	23	21.75	28	29.25
2010	18	20.00	18	25.25

(c) Show how to calculate the 4-year moving average for the year 2010 in the Western Pacific. Write your value in the appropriate place in the table.
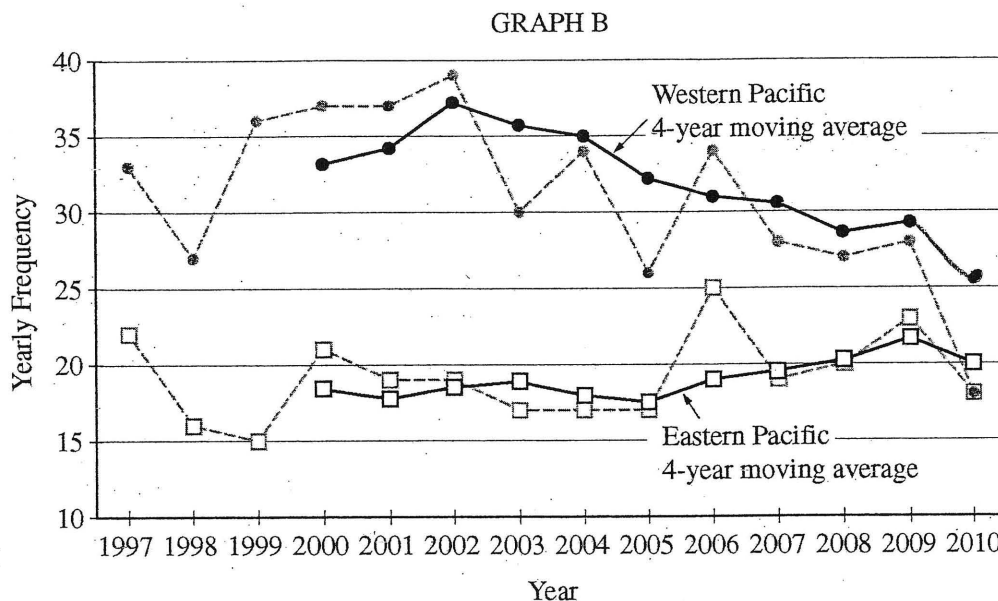
$$\frac{33+27+36}{3} = 32 \qquad \frac{33+27+36+37}{4}$$
$$= 33.25$$

$$\frac{33+27}{2} = 30$$

33

$$\boxed{\frac{28+27+28+18}{4} = 25.25}$$

GO ON TO THE NEXT PAGE.

(d) Graph B below shows both yearly frequencies (connected by dashed lines) and the respective 4-year moving averages (connected by solid lines). Use your answer in part (c) to complete the graph.

GRAPH B

(e) Consider graph B.

i) What information is more apparent from the plots of the 4-year moving averages than from the plots of the yearly frequencies of typhoons?

The 4-year moving averages follow a straighter lines and demonstrate more clearly that the Eastern pacific is increasing while the western pacific is decreasing.

ii) What information is less apparent from the plots of the 4-year moving averages than from the plots of the yearly frequencies of typhoons?

What is less apparent are the changes they go through each year. Sometimes it increases sometimes it decreases. It relatively just shows the general trend.

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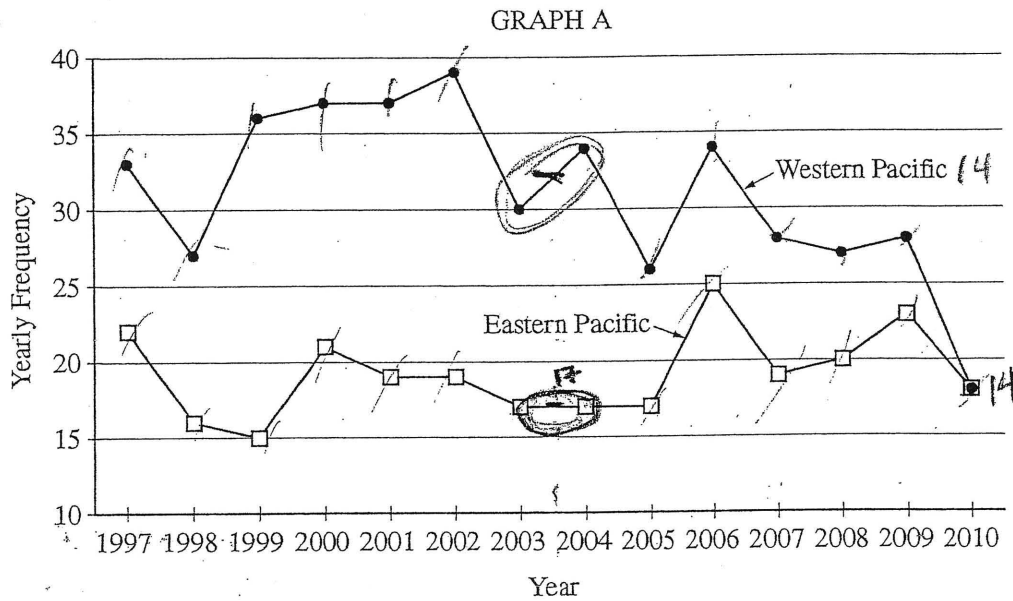
STATISTICS
SECTION II
Part B
Question 6
Spend about 25 minutes on this part of the exam.
Percent of Section II score—25

Directions: Show all your work. Indicate clearly the methods you use, because you will be scored on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.

6. Tropical storms in the Pacific Ocean with sustained winds that exceed 74 miles per hour are called typhoons. Graph A below displays the number of recorded typhoons in two regions of the Pacific Ocean—the Eastern Pacific and the Western Pacific—for the years from 1997 to 2010.

GRAPH A

-14-

(a) Compare the distributions of yearly frequencies of typhoons for the two regions of the Pacific Ocean for the years from 1997 to 2010.

The western pacific is much more variable and spread from 1997 to 2010 than the Eastern Pacific is. The center of the W.P. is also almost nearly twice the size of the E.P. The W.P. graph also appears to be more skewed than the E.P. graph.

(b) For each region, describe how the yearly frequencies changed over the time period from 1997 to 2010.

W.P. From 1997 to 1998, there was a drastic drop in the frequency. 1998 - 1999 also experienced a drastic change, only it was positive. From 1999 - 2002, there is steady rise and it reaches the peak frequency in 2002. After 2002 until 2007, there are drastic frequency changes both positively and negatively. 2007 to 2009, the frequency is on a steady decline. After 2009, the frequency takes another drastic plumet.

E.P. 1997 to 1999 experiences a drop in frequency. 1999 to 2000 there is a drastic positive increase. After 2000, there is a steady decline into a plateau in the years 2003 - 2005. An enormous spike happens between the years 2005 and 2006, and then a drop in 2006-0 From 2007 to 2009, the frequency slowly increases until a final drop yet again in 2009. The peak in frequency appears in 2006.

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A moving average for data collected at regular time increments is the average of data values for two or more consecutive increments. The 4-year moving averages for the typhoon data are provided in the table below. For example, the Eastern Pacific 4-year moving average for 2000 is the average of 22, 16, 15, and 21, which is equal to 18.50.

Year	Number of Typhoons in the Eastern Pacific	Eastern Pacific 4-year moving average	Number of Typhoons in the Western Pacific	Western Pacific 4-year moving average
1997	22		33	
1998	16		27	
1999	15		36	
2000	21	18.50	37	33.25
2001	19	17.75	37	34.25
2002	19	18.50	39	37.25
2003	17	19.00	30	35.75
2004	17	18.00	34	35.00
2005	17	17.50	26	32.25
2006	25	19.00	34	31.00
2007	19	19.50	28	30.50
2008	20	20.25	27	28.75
2009	23	21.75	28	29.25
2010	18	20.00	18	27.06323

(c) Show how to calculate the 4-year moving average for the year 2010 in the Western Pacific. Write your value in the appropriate place in the table.

X	y
37	33.25
37	34.25
39	37.25
30	35.75
34	35.00
26	32.25
34	31.00
28	30.50
27	28.75
28	29.25

$\bar{X} = 32$ $\bar{y} = 32.725$

$\Sigma X = 320$ $\Sigma y = 327.25$

$\Sigma X^2 = 10444$ $\Sigma y^2 = 10782.6875$

$\Sigma xy = 10554.5$

$\beta_1 = \frac{SS_{XY}}{SS_{XX}} = \frac{\Sigma xy - \frac{(\Sigma x \cdot \Sigma y)}{n}}{\Sigma x^2 - \frac{(\Sigma x)^2}{n}}$

$= \frac{82.5}{204} = 0.4044118$

$\beta_0 = \bar{y} - (\beta_1 \bar{x})$

$= 32.725 - (0.4044118 \cdot 32)$

$= 19.7838224$

$\hat{y} = 19.7838224 + 0.4044118 X$
$\qquad\qquad\qquad\qquad\quad \uparrow$
$\qquad\qquad\qquad\qquad\quad 18$

$\hat{y} = 27.06323$

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(d) Graph B below shows both yearly frequencies (connected by dashed lines) and the respective 4-year moving averages (connected by solid lines). Use your answer in part (c) to complete the graph.

GRAPH B

(e) Consider graph B.

i) What information is more apparent from the plots of the 4-year moving averages than from the plots of the yearly frequencies of typhoons?

The overall trend in the frequency is more apparent from the plots of the 4-year averages. From 1997 to 2010, we can see that the average number of typoons per four year moving average is on the decline.

ii) What information is less apparent from the plots of the 4-year moving averages than from the plots of the yearly frequencies of typhoons?

The year to year spikes and drops in typoons is not apparent for the 4-year moving average. It is almost impossible to tell which years had more typoons and which had less when we are only talking about the overall trend.

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Question 6

Overview

The primary goals of this question were to assess a student's ability to (1) summarize information provided in a time plot that involves trend components; (2) perform calculations related to a statistic not previously studied; and (3) compare and contrast information conveyed by the summary statistics with the data.

Sample: 6A
Score: 4

In part (a) it is stated that the distribution of the Western Pacific Ocean is centered higher than the distribution of the Eastern Pacific Ocean. The greater variability in the Western Pacific Ocean frequencies is stated and illustrated with calculated values for the two ranges. Section 1 was scored as essentially correct. The appropriate trends for the frequencies for both the Western Pacific Ocean and the Eastern Pacific Ocean are given in part (b), and section 2 was scored as essentially correct. In parts (c) and (d) the correct 4-year moving average for 2010 is calculated, appropriate calculations are shown, and the calculated value is used to appropriately complete the table and Graph B for the Western Pacific Ocean moving averages. Section 3 was scored as essentially correct. In part (e) it is indicated that trends for both the Western Pacific Ocean and the Eastern Pacific Ocean are more apparent in the plots of 4-year moving averages than in the plots of the yearly frequencies. It is also indicated that big changes from year to year are less apparent in the plots of 4-year moving averages than in the plots of the yearly frequencies. The description of declining trend in the Western Pacific Ocean and the slight increasing trend in the Eastern Pacific Ocean provides good linkage of the response to Graph B. Section 4 was scored as essentially correct. Because all four sections were scored as essentially correct, the response earned a score of 4.

Sample: 6B
Score: 3

In part (a) it is indicated that the distribution of the Western Pacific Ocean is slightly higher than the distribution of the Eastern Pacific Ocean. There is no comparison of variability in the distributions of frequencies for the two regions. Thus, section 1 was scored as partially correct. A detailed description is given in part (b) for the yearly changes for the frequencies of typhoons in the Western Pacific Ocean and the Eastern Pacific Ocean, but the overall trends of declining frequencies in the Western Pacific Ocean and fairly stable frequencies for the Eastern Pacific Ocean are not identified. Section 2 was scored as partially correct. In parts (c) and (d) the correct 4-year moving average for 2010 is calculated, appropriate calculations are shown, and the calculated value is used to appropriately complete the table and Graph B for the Western Pacific Ocean moving averages. Section 3 was scored as essentially correct. In part (e) it is indicated that trends are illustrated more clearly for both the Western Pacific Ocean and the Eastern Pacific Ocean in the plots of 4-year moving averages than in the plots of the yearly frequencies. It is also indicated that the changes from year to year are less apparent in the plots of 4-year moving averages than in the plots of the yearly frequencies. The description of decreasing trend in the Western Pacific Ocean and the increasing trend in the Eastern Pacific Ocean provides good linkage of the response to Graph B. Hence, section 4 was scored as essentially correct. Because two sections were scored as essentially correct and two sections were scored as partially correct, the response earned a score of 3.

Sample: 6C
Score: 2

In part (a) it is correctly indicated that the distribution of frequencies for the Western Pacific Ocean is more variable than the distribution of frequencies for the Eastern Pacific Ocean. The comparison of centers for the two distributions is incorrect because the midpoints between the frequencies for 2003 and 2004 of the time series plots are incorrectly used as the medians for the frequencies of the two regions. Thus, section 1 was scored as partially correct. A detailed description is given in part (b) of the year-to-year changes for the frequencies of typhoons in the Western Pacific Ocean and the Eastern Pacific Ocean, but overall trends of declining frequencies in the Western Pacific Ocean and fairly stable frequencies for the Eastern Pacific Ocean are not identified. Section 2 was scored as partially correct. The 4-year moving average is calculated incorrectly, but the incorrect calculated value is appropriately used in completing the table and Graph B for the Western Pacific moving averages. Section 3 was scored as partially correct. In part (e) it is indicated that a declining trend was more apparent in the plots of 4-year moving averages than in the plots of the yearly frequencies. It is also indicated that the maximum and minimum frequencies could not be identified in the plots of 4-year moving averages but could be identified in the plots of the yearly frequencies. The identification of the decreasing trend in the Western Pacific Ocean and the stable (or slightly increasing) trend in the Eastern Pacific Ocean was missing. Hence, section 4 was scored as partially correct. Because all four sections were scored as partially correct, the response earned a score of 2.