

AP[®] STATISTICS

2013 SCORING GUIDELINES

Question 1

Intent of Question

The primary goals of this question were to assess a student's ability to (1) use a stem-and-leaf plot to answer a question about a distribution of data; (2) identify and compute an appropriate confidence interval after checking the necessary conditions; and (3) interpret the interval in the context of the data.

Solution

Part (a):

Four of the 23 crows in the sample had a lead level greater than 6.0 ppm. Therefore, the proportion of crows in the sample that were classified as unhealthy is $\frac{4}{23} \approx 0.174$.

Part (b):

Step 1: Identifies the appropriate confidence interval (by name or by formula) and checks appropriate conditions.

The appropriate procedure is a one-sample t -interval for a population mean.

Conditions: 1. The sample is randomly selected from the population.
2. The population has a normal distribution, or the sample size is large.

The first condition is met because we were told that the crows were randomly selected. The sample size of 23 is not considered large, so we need to examine the sample data to assess whether it is reasonable to assume that the population distribution of lead levels for all crows in this region is normal. The stem-and-leaf plot shows no strong skewness or outliers, so we will consider the second condition to be met.

Step 2: Correct mechanics

A 95% confidence interval for the population mean μ is given by: $\bar{x} \pm t^* \frac{s}{\sqrt{n}}$. The critical value for 95% confidence, based on $23 - 1 = 22$ degrees of freedom, is $t^* = 2.074$. The 95% confidence interval for μ is therefore

$$4.90 \pm 2.074 \times \frac{1.12}{\sqrt{23}} \approx 4.90 \pm 0.484,$$

which is the interval (4.416, 5.384) ppm.

Using the raw data rather than the given summary statistics, the 95% confidence interval for μ is (4.411, 5.3803).

Step 3: Interpretation

We can be 95% confident that the population mean lead level among all crows in this region is between 4.416 and 5.384 parts per million.

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Question 1 (continued)

Scoring

This question is scored in four sections. Section 1 consists of part (a); section 2 consists of part (b), step 1; section 3 consists of part (b), step 2; and section 4 consists of part (b), step 3. Each section is scored as essentially correct (E), partially correct (P), or incorrect (I).

Section 1 is scored as follows:

Essentially correct (E) if the response provides the correct numerical answer, as a decimal or as a fraction, with work shown.

Partially correct (P) if the response provides the correct numerical answer as a decimal but does not show the fraction that produced the answer, *OR* shows a fraction with the correct numerator but an incorrect denominator, *OR* shows the correct fraction but computes an incorrect answer.

Incorrect (I) if the response does not meet the criteria for E or P.

Section 2 is scored as follows:

Essentially correct (E) if the response identifies a one-sample t -interval for a population mean (either by name or formula) *AND* also checks both the random sampling and the normality/large sample condition correctly.

Note: Any reasonable comment about the normality displayed in the stem-and-leaf plot (or another appropriately sketched plot) is acceptable.

Partially correct (P) if the response identifies the correct procedure *AND* checks only one of the two conditions correctly *OR* does not identify the correct procedure but does check both conditions correctly.

Incorrect (I) if the response identifies the correct procedure but does not check conditions correctly *OR* does not identify the correct procedure and checks at most one condition correctly.

Section 3 is scored as follows:

Essentially correct (E) if the response gives the correct confidence interval. Supporting work is not required, but if included, it must be correct.

Partially correct (P) if the response gives an incorrect but reasonable confidence interval with appropriate supporting work shown *OR* gives a correct confidence interval with incorrect (but appropriate) supporting work shown.

Note: If the response identifies a one-sample z -interval as the correct procedure in Section 2, then the response earns a P in Section 3 if this interval is calculated correctly.

Incorrect (I) if the response makes use of an inappropriate procedure for a confidence interval about a population mean.

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Question 1 (continued)

Section 4 is scored as follows:

Essentially correct (E) if the response gives a reasonable interpretation of the interval that includes four elements:

1. Estimating a mean
2. Inference about a population
3. 95% confidence
4. Context (lead level/ppm and crows).

Partially correct (P) if the response gives a reasonable interpretation of the interval that includes both of the first two elements and one of the last two elements;

OR

if the response gives a correct interpretation of the confidence level in context (lead level/ppm and crows) but does not attempt to interpret the confidence interval.

Incorrect (I) if the response does not meet the criteria for an E or a P.

Each essentially correct (E) response counts as 1 point, and a partially correct (P) response counts as $\frac{1}{2}$ point.

- 4 Complete Response**
- 3 Substantial Response**
- 2 Developing Response**
- 1 Minimal Response**

If a response is between two scores (for example, $2\frac{1}{2}$ points), use a holistic approach to determine whether to score up or down, depending on the strength of the response and communication.

STATISTICS

SECTION II

Part A

Questions 1-5

Spend about 65 minutes on this part of the exam.

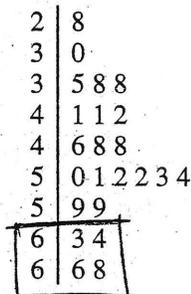
Percent of Section II score—75

1A1

Directions: Show all your work. Indicate clearly the methods you use, because you will be scored on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.

1. An environmental group conducted a study to determine whether crows in a certain region were ingesting food containing unhealthy levels of lead. A biologist classified lead levels greater than 6.0 parts per million (ppm) as unhealthy. The lead levels of a random sample of 23 crows in the region were measured and recorded. The data are shown in the stemplot below.

Lead Levels



Key: 2|8 = 2.8 ppm

- (a) What proportion of crows in the sample had lead levels that are classified by the biologist as unhealthy?

6.3, 6.4, 6.6, 6.8
favorable = 4
 $n = 23$

4/23

1A2

- (b) The mean lead level of the 23 crows in the sample was 4.90 ppm and the standard deviation was 1.12 ppm. Construct and interpret a 95 percent confidence interval for the mean lead level of crows in the region.

95% CI

$$\bar{x} \pm t^* \frac{s}{\sqrt{n}}$$

$$df = 22$$

$$4.90 \pm 2.074 \left(\frac{1.12}{\sqrt{23}} \right)$$

$$(4.4157, 5.3843)$$

Conditions:

- Normal distribution ~ the stemplot in the question appears to not be skewed and there are no outliers making t procedures valid when $15 < n < 30$
- Random sample ~ yes, given in question
- using s to estimate σ

I am 95% confident that the true mean lead level of crows in the region is between 4.4157 ppm and 5.3843 ppm.

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STATISTICS

SECTION II

Part A

Questions 1-5

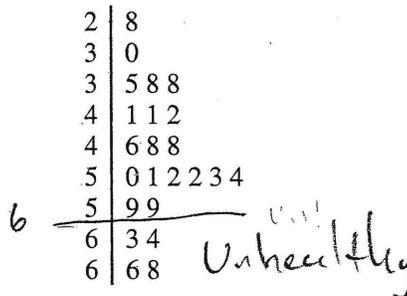
Spend about 65 minutes on this part of the exam.

Percent of Section II score—75

Directions: Show all your work. Indicate clearly the methods you use, because you will be scored on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.

1. An environmental group conducted a study to determine whether crows in a certain region were ingesting food containing unhealthy levels of lead. A biologist classified lead levels greater than 6.0 parts per million (ppm) as unhealthy. The lead levels of a random sample of 23 crows in the region were measured and recorded. The data are shown in the stemplot below.

Lead Levels

Key: $2|8 = 2.8$ ppm.

- (a) What proportion of crows in the sample had lead levels that are classified by the biologist as unhealthy?

Lead levels 6.3, 6.4, 6.6, 6.8 ppm are
unhealthy.

$$\frac{4 \text{ crows with unhealthy lead levels}}{23 \text{ total crows}} = .1739$$

- (b) The mean lead level of the 23 crows in the sample was 4.90 ppm and the standard deviation was 1.12 ppm. Construct and interpret a 95 percent confidence interval for the mean lead level of crows in the region.

$$\bar{x} = 4.9 \text{ ppm} \quad n = 23$$

$$s_x = 1.12 \text{ ppm}$$

one-sample t -test

Confidence Interval: $4.9 \pm 2.074 \left(\frac{1.12}{\sqrt{23}} \right)$

$$df = n - 1 = 22$$

$$= (4.416, 5.384)$$

I am 95% confident that the true mean lead level of the crow population is between 4.416 to 5.384.

STATISTICS

SECTION II

Part A

Questions 1-5

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Spend about 65 minutes on this part of the exam.

Percent of Section II score—75

Directions: Show all your work. Indicate clearly the methods you use, because you will be scored on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.

1. An environmental group conducted a study to determine whether crows in a certain region were ingesting food containing unhealthy levels of lead. A biologist classified lead levels greater than 6.0 parts per million (ppm) as unhealthy. The lead levels of a random sample of 23 crows in the region were measured and recorded. The data are shown in the stemplot below.

Lead Levels

2		8
3		0
3		5 8 8
4		1 1 2
4		6 8 8
5		0 1 2 2 3 4
5		9 9
6		3 4
6		6 8

Key: 2|8 = 2.8 ppm

- (a) What proportion of crows in the sample had lead levels that are classified by the biologist as unhealthy?

6.0 ppm
were
unhealthy

6.3, 6.4, 6.6, 6.8

$\frac{4}{23} = .1739$ of the crows
had lead levels considered
unhealthy

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- (b) The mean lead level of the 23 crows in the sample was 4.90 ppm and the standard deviation was 1.12 ppm. Construct and interpret a 95 percent confidence interval for the mean lead level of crows in the region.

$$CI = 4.9 \pm 1.96 \left(\frac{1.12}{\sqrt{23}} \right) \quad 95\% = 1.96$$

$$4.9 \pm .4577$$
$$= (4.442, 5.358)$$

I am 95% confident that the true mean lead content in ppm lies between 4.442 and 5.358.

AP[®] STATISTICS

2013 SCORING COMMENTARY

Question 1

Overview

The primary goals of this question were to assess a student's ability to (1) use a stem-and-leaf plot to answer a question about a distribution of data; (2) identify and compute an appropriate confidence interval after checking the necessary conditions; and (3) interpret the interval in the context of the data.

Sample: 1A

Score: 4

In part (a) the student gives the correct proportion (as a fraction) of crows in the sample that are classified as unhealthy. Therefore, section 1 was scored as essentially correct. In part (b) the student correctly identifies a one-sample t -interval for a population mean by formula. The student also checks the random sampling condition and normality/large sample condition correctly. As a result, section 2 was scored as essentially correct. Because the student computes the 95 percent confidence interval correctly, section 3 was scored as essentially correct. Finally, the student provides an appropriate interpretation of the confidence interval that includes all four required elements: estimating a mean, inference about a population, 95 percent confidence, and context. Therefore, section 4 was scored as essentially correct. Because all four sections were scored as essentially correct, the response earned a score of 4.

Sample: 1B

Score: 3

In part (a) the student correctly calculates the proportion of crows with unhealthy lead levels. As a result, section 1 was scored as essentially correct. Because the student does not check any conditions in part (b), section 2 was scored as incorrect. However, the student calculates the 95 percent confidence interval correctly with supporting work shown. Consequently, section 3 was scored as essentially correct. The student gives an appropriate interpretation of the confidence interval with all four required elements, so section 4 was scored as essentially correct. Because three sections were scored as essentially correct and one section was scored as incorrect, the response earned a score of 3.

Sample: 1C

Score: 2

In part (a) the student gives the desired proportion with appropriate supporting work, so section 1 was scored as essentially correct. In part (b) the student includes a formula for a one-sample z -interval with numbers substituted, which is not the correct procedure. Because neither the random sampling nor the normality/large sample condition is checked, section 2 was scored as incorrect. The student calculates the z -interval correctly, so section 3 was scored as partially correct. One element - context - is missing from the student's interpretation of the interval. No mention is made of crows, the population of interest. The other three elements are all present in what is otherwise an appropriate confidence interval interpretation. As a result, section 4 was scored as partially correct. Because one section was scored as essentially correct, two sections were scored as partially correct, and one section was scored as incorrect, the response earned a score of 2.