Question 1

15 points total

(a) 5 points

For drawing a Gaussian surface that is a cylinder, contained within and sharing the axis of the given cylinder 1 point

For using a correct statement of Gauss’s law 1 point

\[ \frac{q_{enc}}{\varepsilon_0} = \oint E \cdot dA \]

For a correct expression for \( q_{enc} \) including the expression for the volume of a cylinder

\[ q_{enc} = \rho V_{enc} = \rho \left( \pi r^2 L \right) \]

For a correct expression for the surface area of the sides of a cylinder

\[ A = 2\pi r L \]

Substitute expressions into Gauss’s law:

\[ \frac{\rho \left( \pi r^2 L \right)}{\varepsilon_0} = E \left( 2\pi r L \right) \]

For a correct answer 1 point

\[ E = \frac{\rho r}{2\varepsilon_0} \]

(b) 1 point

\[ \frac{q_{enc}}{\varepsilon_0} = \oint E \cdot dA \]

\[ q_{enc} = \rho V_{tot} = \rho \left( \pi R^2 L \right) \]

\[ \frac{\rho \left( \pi R^2 L \right)}{\varepsilon_0} = E \left( 2\pi r L \right) \]

For a correct answer 1 point

\[ E = \frac{\rho R^2}{2\varepsilon_0 r} \]
Question 1 (continued)

(c) 3 points

For a graph in the region $0 < r < R$ that is a straight line starting at the origin, and with a positive slope 1 point

For a graph in the region $R < r < 2R$ that is continuous, decreasing, concave upward, and not zero at $r = 2R$ 1 point

For labeling the maximum value of $E$ on the vertical axis, consistent with part (a) 1 point

(d) i. 3 points

For stating the correct relation between the magnitude of the potential difference and the integral of the electric field 1 point

\[ \Delta V = -\int_{0}^{R} E \cdot dr \quad \text{(negative sign not required)} \]

For substituting the expression for electric field from part (a) 1 point

\[ \Delta V = -\int_{0}^{R} \frac{\rho}{2\varepsilon_0} \cdot dr \]

For integrating with the proper limits or correctly evaluating the constant of integration 1 point

\[ V(R) - V(0) = -\frac{\rho}{2\varepsilon_0} \left[ \frac{r^2}{2} \right]_{0}^{R} = -\frac{\rho}{2\varepsilon_0} \left( \frac{R^2}{2} - 0 \right) = -\frac{\rho R^2}{4\varepsilon_0} \]

\[ |\Delta V| = \frac{\rho R^2}{4\varepsilon_0} \left( \text{leaving the answer } -\frac{\rho R^2}{4\varepsilon_0} \text{ is also acceptable} \right) \]

Note: Graphical integration could also be performed. For students taking this approach, 1 point was earned for evaluating the area for $0 < r < R$, 1 point for calculating the area of a triangle, and 1 point for a magnitude consistent with previous results.
Question 1 (continued)

ii. 1 point

For selecting “\( r = 0 \)”

(e) 2 points

For \( 0 < r < R \), drawing a horizontal graph with \( E = 0 \)

For \( R < r < 2R \), drawing a graph consistent with the graph in part (c) in that region

Distribution of points

1 point

1 point
Question 2

15 points total

(a)

i. 1 point

For selecting $B$  

ii. 1 point

For correctly drawing the voltmeter on the diagram in parallel with the capacitor  

(b)

i. 2 points

For stating two correct variables that would yield a linear graph  

Examples: $t$ and $\ln V$  

$t$ and $-\ln V$  

$t$ and $\ln\left(\frac{V}{V_0}\right)$, where $V_0 = 252$ V

Note: Students who do not come up with a correct pair of variables but who use Kirchhoff’s loop rule to analyze the circuit, or write an appropriate differential equation or exponential relationship may earn one point.

ii. 1 point

For filling in values in a blank table row consistent with part (b) i  

<table>
<thead>
<tr>
<th>$t$ (s)</th>
<th>6</th>
<th>18</th>
<th>30</th>
<th>42</th>
<th>54</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V$ (V)</td>
<td>252</td>
<td>74</td>
<td>33</td>
<td>10</td>
<td>6</td>
</tr>
<tr>
<td>$-\ln V$</td>
<td>$-6.57$</td>
<td>$-4.30$</td>
<td>$-3.50$</td>
<td>$-2.30$</td>
<td>$-1.79$</td>
</tr>
</tbody>
</table>
(c) 4 points

For correctly labeling the axes with variables and units consistent with part (b) 1 point
For properly scaling the axes 1 point
For correctly plotting data points consistent with part (b) 1 point
For drawing a line of best fit through linear data 1 point

(d) 2 points

Relate the time constant to the slope from the graph above
\[ \tau = \text{slope} \quad \text{(or 1/slope if axes on graph are swapped)} \]
For using points on the best fit line, not data points, to calculate the slope, or performing a linear regression 1 point
\[ \tau = \frac{(52 - 23) \text{ s}}{-1.6 - (-4.0)} \]
For calculating a correct time constant 1 point
\[ \tau = 12.1 \text{ s} \]
(e)

i. 2 points

For a correct expression relating the time constant to the resistance $\tau = RC$ 1 point

For correctly substituting into the correct equation 1 point

\[
R = \frac{\tau}{C} = \frac{(12.1 \text{ s})}{(1.5 \ \mu\text{F})} = 8.1 \ \text{M}\Omega
\]

ii. 2 points

For a dashed straight line with the same voltage at $t = 0$ as that of the solid line and a slope that is:

- steeper than that of the solid line, if $t$ is graphed as a function of $\pm \ln V$, as shown above
- shallower than that of the solid line, if instead $\pm \ln V$ is graphed as a function of $t$

For a correct justification for the change in the slope that relates the slope to either the time constant or its reciprocal 1 point
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Question 3

15 points total

(a)

i. 3 points

For using a correct expression of Faraday’s law
\[ E = -d\phi/dt \]

For correct substitution of area and magnetic field into Faraday’s law
\[ E = -A(dB/dt) = -A \left( d \left( 1.8e^{-0.05t} \right) \right)/dt \]

Taking the derivative and substituting values
\[ E = -(0.25 \text{ m}^2)(-0.05)(1.8)e^{-0.05t} \]

For a correct answer
\[ E = 0.0225e^{-0.05t} \text{ V} \]

Note: The negative sign is not needed in the calculations because the question asked for the magnitude of the emf.

ii. 1 point

Use Ohm’s law
\[ I = V/R \]

For using the expression from (a) in Ohm’s law with the correct time
\[ I = (0.0225)(e^{-0.05}(4 \text{ s})/12 \Omega \]

\[ I = 0.00154 \text{ A or 1.54 mA} \]

(b)

i. 4 points

For indicating a current that decays exponentially in the region \(0 \text{ s} < t < 8 \text{ s}\) 1 point

For indicating a current of zero in the regions \(8 \text{ s} < t < 12 \text{ s}\) and \(t > 16 \text{ s}\) 1 point

For indicating a current that is constant and nonzero in the region \(12 \text{ s} < t < 16 \text{ s}\) 1 point
For indicating a clockwise current in the regions $0 \, s < t < 8 \, s$ and $12 \, s < t < 16 \, s$  

ii. 3 points

For indicating that magnetic field is decreasing  
For indicating that an induced magnetic field opposes the change  
For indicating that the induced current must be clockwise to produce the induced magnetic field  

Example:  
The magnetic field is into the page and decreasing. From Lenz’s law, the new current must create a magnetic field to oppose this change. So the new current must create its own magnetic field that is into the page. Thus, according to the right hand rule, the current in the loop must be clockwise.

Note: 1 earned point was deducted for any incorrect statements made in conjunction with correct statements.

(c) 4 points

For indicating that energy is the integral of power over time  
$E = \int P \, dt$  
For using a correct expression of power  
$E = \int \frac{E^2}{R} \, dt$  
For a correct substitution from part (a)  
For using the proper limits on the integration or correctly evaluating a constant of integration

$E = \frac{8}{0} \left( \frac{0.0225e^{-0.05t}}{12 \, \Omega} \right)^2 \, dt = \left( \frac{0.0225}{12 \, \Omega} \right)^2 \int_0^8 e^{-0.10t} \, dt$  

$E = \left( \frac{4.22 \times 10^{-5}}{-0.10} \right) \left[ e^{-0.10t} \right]_0^8$  

$E = \left( -4.22 \times 10^{-4} \right) \left( e^{-(0.10)(8 \, s)} - e^0 \right)$  

$E = 2.32 \times 10^{-4} \, J$