Question 6

15 points total | **Distribution of points**

(a) 2 points

For the correct substitution of the values of μ_0, I, and r into the correct expression for the magnetic field around a wire — 1 point

$$B = \frac{\mu_0 I}{2\pi r} = \frac{\left(4\pi \times 10^{-7}\ \text{T}\bullet\text{m/A}\right)(65\ \text{A})}{(2\pi)(0.025\ \text{m})}$$

For the correct answer — 1 point

$$B = 5.2 \times 10^{-4}\ \text{T}$$

(b)

i. 4 points

For an expression of equilibrium between the magnetic and gravitational forces on the wire — 1 point

$$W = F_B$$

For correct expressions for the magnetic and gravitational forces — 1 point

$$mg = BIL$$

For a correct expression for the current in terms of the linear density — 1 point

$$I = \left(\frac{m}{L}\right)\frac{g}{B}$$

For correctly substituting the answer from part (a) into the above equation to solve for the current — 1 point

$$I = \left(5.6 \times 10^{-3}\ \text{kg/m}\right)\frac{\left(9.8\ \text{m/s}^2\right)}{\left(5.2 \times 10^{-4}\ \text{T}\right)}$$

$I = 106\ \text{A}$ (or 108 A using $g = 10\ \text{m/s}^2$)

ii. 1 point

For selecting "To the right" — 1 point

(c) 2 points

For stating that the wire would move upward — 1 point

For stating that the wire would <u>accelerate</u> with a correct justification — 1 point

<u>Example</u>

As wire Y is moved closer to X, the magnetic field from X will increase. This will create an upward net force on Y that will accelerate Y upward toward wire X.

(d) 1 point

Question 6 (continued)

	Distribution of points
For stating that the direction of the current in one of the wires must change	1 point

Examples

- The current must be reversed in one wire, but not in the other.
- Reverse the current in wire X.
- Change the direction of the current in wire Y.

(e)

i. 3 points

For a correct statement of either Faraday's law, or the motional emf formula	1 point

$\mathcal{E} = B\upsilon L$ OR $\mathcal{E} = -\dfrac{\Delta\phi}{\Delta t}$ (since $\mathcal{E} = -B\dfrac{\Delta A}{\Delta t} = -B\upsilon L$, with the minus sign irrelevant since the question asks for a magnitude)

For a correct expression for, or determination of, the value of the magnetic field	1 point

Method 1) Taking half the value determined in part (a), because the wires are now twice as far apart

$$B = \frac{1}{2}\left(5.2 \times 10^{-4}\ \text{T}\right) = 2.6 \times 10^{-4}\ \text{T}$$

Method 2) Using the formula for the magnetic field around a wire

$$B = \frac{\mu_0 I}{2\pi r} = \frac{\left(4\pi \times 10^{-7}\ \text{T}\bullet\text{m/A}\right)(65\ \text{A})}{(2\pi)(0.050\ \text{m})} = 2.6 \times 10^{-4}\ \text{T}$$

For substituting *correct* values of L and υ and *any* value of B into a correct expression	1 point

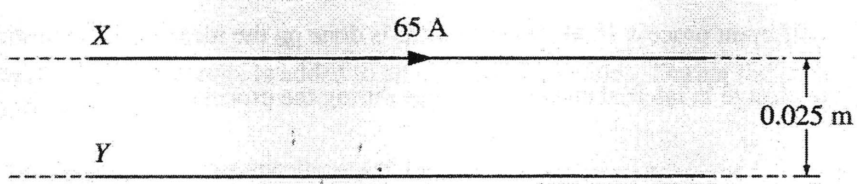
$$\mathcal{E} = (2.6 \times 10^{-4}\ \text{T})(3.0\ \text{m/s})(1.2\ \text{m}) = 9.4 \times 10^{-4}\ \text{V}$$

ii. 2 points

For selecting "The right end"	1 point
For a correct justification	1 point

Example

Using the right-hand rule, positive charges moving upward in the magnetic field, which is out of the page, would experience a magnetic force to the right. Thus, the right end of wire Y develops a positive charge and the left end develops a negative charge.



Side View

6. (15 points)

Two long, straight horizontal wires are near each other and parallel, with one directly above the other as shown in the figure. Wire X is fixed in place and connected to a battery (not shown) so that it carries a current of 65 A. Wire Y, which is part of a second circuit, is free to move vertically and is suspended at rest by the magnetic force between the wires. The mass per length of wire Y is 5.6×10^{-3} kg/m. Neglect effects from the parts of the circuits that are not shown.

(a) Calculate the magnitude of the magnetic field produced by wire X at the position of wire Y.

$$B = \frac{u_0}{2\pi} \cdot \frac{I}{r} = \frac{4\pi \times 10^{-7} (T \cdot m)/A}{2\pi} \cdot \frac{65 A}{0.025 m} = 5.2 \times 10^{-4} T$$

(b)

i. Calculate the magnitude of the current in wire Y.

$$mg = BIl$$

$$(5.6 \times 10^{-3} kg/m)(10 m/s^2) = (5.2 \times 10^{-4} T)(I)(1 m)$$

$$I = 107.7 A$$

ii. Indicate the direction of the current in wire Y.

____ To the left _X_ To the right ____ Neither left nor right, since there is no current

(c) Now wire Y is moved to a new position that is closer to wire X, but wire Y is still below wire X and is still carrying the same current as determined in part (b). Wire Y is released from rest. Describe the initial motion of wire Y. Justify your answer.

Wire Y will move toward wire X. This is because the magnetic field created by wire x increases as wire Y gets nearer to it. When wire Y is closer to wire x, the magnetic field is bigger, making the magnetic force bigger. Since the gravitational force never increases, the wire would have a net force toward wire X and move toward it.

(d) Suppose wire Y is moved to a position 0.025 m above wire X. What changes in current, if any, must occur to maintain equilibrium?

Wire Y should make its current move to the left. This is because the magnetic field by wire x above it is facing out of the page. Since we want the force to push the wire upward and counteract gravity to create equilibrium, according to the hand rule the current in the wire must move left.

(e) With wire Y still above wire X, the circuit connected to wire Y is removed. Wire Y, which is 1.2 m long, is then moved vertically up and away from wire X at a constant speed of 3.0 m/s.

i. Calculate the magnitude of the induced emf in wire Y when the wires are 0.050 m apart.

$$\mathcal{E} = B\ell v$$

$$\mathcal{E} = \left(\frac{\mu_0}{2\pi}\cdot\frac{I}{r}\right)(\ell)(v) = \left(\frac{4\pi\times10^{-7}(\text{T}\cdot\text{m})/\text{A}}{2\pi}\cdot\frac{65\text{A}}{0.050\text{m}}\right)(1.2\text{m})(3.0\text{m/s})$$

$$\mathcal{E} = 9.36\times10^{-4}\text{ V}$$

ii. Indicate which end of wire Y is at a higher electric potential.

X The left end ____ The right end

____ Neither end, since they are at the same electric potential

Justify your answer.

Current moves from high potential energy to low potential energy. Since the wire is moving away from the magnetic field, pointing out of the page below it, it will create its own magnetic field according to lenz' law to stop the change. In order for it to make a field out of the page, below it, according to the hand rule the current must move to the right, showing the hige potential energy is to the left.

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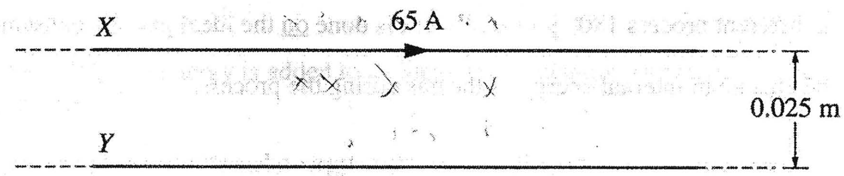

Side View

6. (15 points)

Two long, straight horizontal wires are near each other and parallel, with one directly above the other as shown in the figure. Wire X is fixed in place and connected to a battery (not shown) so that it carries a current of 65 A. Wire Y, which is part of a second circuit, is free to move vertically and is suspended at rest by the magnetic force between the wires. The mass per length of wire Y is 5.6×10^{-3} kg/m. Neglect effects from the parts of the circuits that are not shown.

(a) Calculate the magnitude of the magnetic field produced by wire X at the position of wire Y.

$$B = \frac{\mu_0}{2\pi}\frac{I}{r}$$

$$B = \frac{4\pi \times 10^{-7}}{2\pi}\frac{(65)}{.025} \quad \boxed{B = 5.2 \times 10^{-4}\ T}$$

(b)

i. Calculate the magnitude of the current in wire Y.

$$B = \frac{\mu_0}{2\pi}\frac{I}{r} \qquad I = \frac{5.2 \times 10^{-4}(.025)}{\frac{\mu_0}{2\pi}}$$

$$\frac{Br}{\frac{\mu_0}{2\pi}} = I \qquad \boxed{I = 65\ A}$$

ii. Indicate the direction of the current in wire Y.

____ To the left ✓ To the right ____ Neither left nor right, since there is no current

(c) Now wire Y is moved to a new position that is closer to wire X, but wire Y is still below wire X and is still carrying the same current as determined in part (b). Wire Y is released from rest. Describe the initial motion of wire Y. Justify your answer.

The wire will begin to move away from wire X because based on the Right hand rule, the current is to the right and the B-field is into the paper, then the velocity is towards the bottom of the page.

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(d) Suppose wire Y is moved to a position 0.025 m above wire X. What changes in current, if any, must occur to maintain equilibrium?

The current must flow to the left so that the B field created by wire Y opposes the B field created by the current in wire X.

(e) With wire Y still above wire X, the circuit connected to wire Y is removed. Wire Y, which is 1.2 m long, is then moved vertically up and away from wire X at a constant speed of 3.0 m/s.

i. Calculate the magnitude of the induced emf in wire Y when the wires are 0.050 m apart.

$$\mathcal{E} = B\ell v$$

$$\mathcal{E} = (2.6\times10^{-4})(1.2)(3.0)$$

$$\boxed{\mathcal{E} = 9.36\times10^{-4}\ \text{V}}$$

$$B = \frac{\mu_0}{2\pi}\frac{I}{r}$$

$$B = \frac{4\pi\times10^{-7}}{2\pi}\frac{(65)}{.050}$$

$$B = 2.6\times10^{-4}\ \text{T}$$

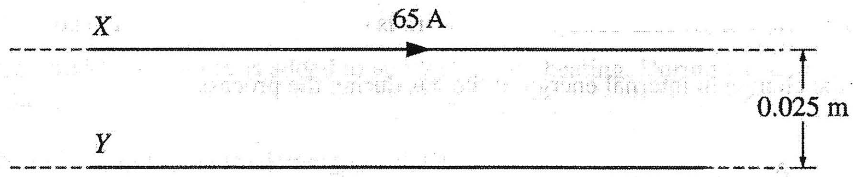
ii. Indicate which end of wire Y is at a higher electric potential.

____ The left end ____ The right end

✓ Neither end, since they are at the same electric potential

Justify your answer.

They are at the same electric potential because the entire length of the wire has the same current and is the same distance from wire X.



Side View

6. (15 points)

Two long, straight horizontal wires are near each other and parallel, with one directly above the other as shown in the figure. Wire X is fixed in place and connected to a battery (not shown) so that it carries a current of 65 A. Wire Y, which is part of a second circuit, is free to move vertically and is suspended at rest by the magnetic force between the wires. The mass per length of wire Y is 5.6×10^{-3} kg/m . Neglect effects from the parts of the circuits that are not shown.

(a) Calculate the magnitude of the magnetic field produced by wire X at the position of wire Y.

$B = ?$

$r = .025\,m$

$\frac{m}{L_Y} = 5.6 \times 10^{-3}$

$$\frac{\mu_0 I}{2\pi r} \qquad \frac{(4\times10^{-7}\,\frac{T}{m})(65A)}{2\pi\,(.025)m}$$

$$\boxed{B = 1.021 \times 10^{-6}\,T}$$

(b)

i. Calculate the magnitude of the current in wire Y.

$I_y = ?$

$B = 1.021 \times 10^{-6}$

$r = .025m$

No current

ii. Indicate the direction of the current in wire Y.

____ To the left ____ To the right _X_ Neither left nor right, since there is no current

(c) Now wire Y is moved to a new position that is closer to wire X, but wire Y is still below wire X and is still carrying the same current as determined in part (b). Wire Y is released from rest. Describe the initial motion of wire Y. Justify your answer.

Wire Y would move back to the position it was in ~~origin~~ before. It would be .025m away because the magnetic force would push the wire out.

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(d) Suppose wire Y is moved to a position 0.025 m above wire X. What changes in current, if any, must occur to maintain equilibrium?

No changes in current would occur because it's still R away from wire X.

(e) With wire Y still above wire X, the circuit connected to wire Y is removed. Wire Y, which is 1.2 m long, is then moved vertically up and away from wire X at a constant speed of 3.0 m/s.

i. Calculate the magnitude of the induced emf in wire Y when the wires are 0.050 m apart.

$\mathcal{E} = ?$

$l = 1.2\,m$

$v = 3\,m/s$

$r = .05\,m$

$B = 1.021 \times 10^{-6}$

$\mathcal{E} = Blv$

$(1.021 \times 10^{-6})(1.2\,m)(3)$

$\boxed{\mathcal{E} = 3.6756 \times 10^{-6}}$

ii. Indicate which end of wire Y is at a higher electric potential.

____ The left end __X__ The right end

____ Neither end, since they are at the same electric potential

Justify your answer.

The right end because the distance would be shorter to the wire.

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Question 6

Overview

This question assessed students' knowledge of magnetostatics and electromagnetic induction, including the right-hand rules.

Sample: B6-A
Score: 12

The correct substitution into Ampere's law and the correct answer earned full credit in part (a). Full credit was earned in part (b)(i) for a correct substitution into the equation for equilibrium and an answer consistent with part (a). The direction of the current is correctly indicated in part (b)(ii). One point was earned in part (c) for correctly stating that wire *Y* would move toward wire *X*, but does not make it explicitly clear that wire *Y* is accelerated. Full credit was earned in part (d) for stating that the direction of the current must change in one of the wires. Full credit was also earned in part (e)(i) for calculating the induced emf. No credit was earned in part (e)(ii) for incorrectly selecting "The left end." Because the selection was incorrect, the justification was not considered.

Sample: B6-B
Score: 7

Full credit was earned in part (a). No credit was earned in part (b)(i) because the student takes the answer to part (a) and uses Ampere's Law to recalculate the original current in wire *X*. Part (b)(ii) earned full credit because "To the right" was correctly selected. Credit was not earned in part (c) because the description of the motion of the wire is incorrect. Part (d) correctly indicates that the current in only one wire would change direction. Full credit was earned in part (e)(i). Part (e)(ii) earned no credit because "The right end" is not selected.

Sample: B6-C
Score: 4

One point was earned in part (a) for a correct substitution into Ampere's Law. The answer is not calculated correctly. No credit was earned in part (b)(i). Credit was not earned in part (b)(ii) because "To the right" is not selected. Part (c) earned no credit because the motion of the wire is described incorrectly. Credit was not earned in part (d) because no change in current is indicated as being necessary. Two points were awarded in part (e)(i) because the motional emf equation is used correctly but with an incorrect value for the magnetic field at the wire's new location. One point was earned in part (e)(ii) earned because "The right end" is correctly selected, but the justification is not correct.