AP® Physics C: Electricity and Magnetism
2013 Free-Response Questions

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### TABLE OF INFORMATION DEVELOPED FOR 2012

<table>
<thead>
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<th>CONSTANTS AND CONVERSION FACTORS</th>
</tr>
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<tr>
<td>Proton mass, $m_p = 1.67 \times 10^{-27}$ kg</td>
</tr>
<tr>
<td>Neutron mass, $m_n = 1.67 \times 10^{-27}$ kg</td>
</tr>
<tr>
<td>Electron mass, $m_e = 9.11 \times 10^{-31}$ kg</td>
</tr>
<tr>
<td>Avogadro’s number, $N_0 = 6.02 \times 10^{23}$ mol⁻¹</td>
</tr>
<tr>
<td>Universal gas constant, $R = 8.31$ J/(mol·K)</td>
</tr>
<tr>
<td>Boltzmann’s constant, $k_B = 1.38 \times 10^{-23}$ J/K</td>
</tr>
</tbody>
</table>

| Electron charge magnitude, $e = 1.60 \times 10^{-19}$ C |
| 1 electron volt, $1$ eV = $1.60 \times 10^{-19}$ J |
| Speed of light, $c = 3.00 \times 10^8$ m/s |
| Universal gravitational constant, $G = 6.67 \times 10^{-11}$ m³/kg·s² |
| Acceleration due to gravity at Earth’s surface, $g = 9.8$ m/s² |

- $1$ unified atomic mass unit, $1$ u = $1.66 \times 10^{-27}$ kg = $931$ MeV/c²
- Planck’s constant, $h = 6.63 \times 10^{-34}$ J·s = $4.14 \times 10^{-15}$ eV·s
- $hc = 1.99 \times 10^{-25}$ J·m = $1.24 \times 10^3$ eV·nm
- Vacuum permittivity, $\varepsilon_0 = 8.85 \times 10^{-12}$ C²/N·m²
- Coulomb’s law constant, $k = 1/4\pi\varepsilon_0 = 9.0 \times 10^9$ N·m²/C²
- Vacuum permeability, $\mu_0 = 4\pi \times 10^{-7}$ (T·m)/A
- Magnetic constant, $k' = \mu_0/4\pi = 1 \times 10^{-7}$ (T·m)/A
- $1$ atmosphere pressure, $1$ atm = $1.0 \times 10^5$ N/m² = $1.0 \times 10^5$ Pa

### UNIT SYMBOLS

<table>
<thead>
<tr>
<th>UNIT SYMBOLS</th>
<th>meter, m</th>
<th>mole, mol</th>
<th>watt, W</th>
<th>farad, F</th>
<th>kilogram, kg</th>
<th>hertz, Hz</th>
<th>coulomb, C</th>
<th>tesla, T</th>
</tr>
</thead>
<tbody>
<tr>
<td>second, s</td>
<td>newton, N</td>
<td>volt, V</td>
<td>ohm, Ω</td>
<td>degree Celsius, °C</td>
<td>pascal, Pa</td>
<td>joule, J</td>
<td>henry, H</td>
<td></td>
</tr>
<tr>
<td>ampere, A</td>
<td>centi c</td>
<td>micro μ</td>
<td>nano n</td>
<td>electron-volt, eV</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>kelvin, K</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### PREFIXES

<table>
<thead>
<tr>
<th>Factor</th>
<th>Prefix</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^9$</td>
<td>giga G</td>
<td>G</td>
</tr>
<tr>
<td>$10^6$</td>
<td>mega M</td>
<td>M</td>
</tr>
<tr>
<td>$10^3$</td>
<td>kilo k</td>
<td>k</td>
</tr>
<tr>
<td>$10^{-2}$</td>
<td>centi c</td>
<td>c</td>
</tr>
<tr>
<td>$10^{-3}$</td>
<td>milli m</td>
<td>m</td>
</tr>
<tr>
<td>$10^{-6}$</td>
<td>micro μ</td>
<td>μ</td>
</tr>
<tr>
<td>$10^{-9}$</td>
<td>nano n</td>
<td>n</td>
</tr>
<tr>
<td>$10^{-12}$</td>
<td>pico p</td>
<td>p</td>
</tr>
</tbody>
</table>

### VALUES OF TRIGONOMETRIC FUNCTIONS FOR COMMON ANGLES

<table>
<thead>
<tr>
<th>$\theta$</th>
<th>0°</th>
<th>15°</th>
<th>30°</th>
<th>45°</th>
<th>60°</th>
<th>75°</th>
<th>90°</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sin \theta$</td>
<td>0</td>
<td>1/2</td>
<td>3/5</td>
<td>$\sqrt{2}/2$</td>
<td>4/5</td>
<td>$\sqrt{3}/2$</td>
<td>1</td>
</tr>
<tr>
<td>$\cos \theta$</td>
<td>1</td>
<td>$\sqrt{3}/2$</td>
<td>4/5</td>
<td>$\sqrt{2}/2$</td>
<td>3/5</td>
<td>1/2</td>
<td>0</td>
</tr>
<tr>
<td>$\tan \theta$</td>
<td>0</td>
<td>$\sqrt{3}/3$</td>
<td>3/4</td>
<td>1</td>
<td>4/3</td>
<td>$\sqrt{3}$</td>
<td>$\infty$</td>
</tr>
</tbody>
</table>

The following conventions are used in this exam.

I. Unless otherwise stated, the frame of reference of any problem is assumed to be inertial.

II. The direction of any electric current is the direction of flow of positive charge (conventional current).

III. For any isolated electric charge, the electric potential is defined as zero at an infinite distance from the charge.
### Mechanics

- \( v = v_0 + at \)  
- \( x = x_0 + v_0 t + \frac{1}{2}at^2 \)  
- \( v^2 = v_0^2 + 2a(x - x_0) \)  
- \( \sum F = F_{net} = ma \)  
- \( F = \frac{dp}{dt} \)  
- \( J = \int F \cdot dt = \Delta p \)  
- \( p = mv \)  
- \( F_{frie} \leq \mu N \)  

#### Electric Field and Potential

- \( W = \int F \cdot dr \)  
- \( K = \frac{1}{2}mv^2 \)  
- \( W = \) work done on a system  
- \( P = \frac{dW}{dt} \)  
- \( P = F \cdot v \)  
- \( \Delta U_g = mgh \)  
- \( a_c = \frac{v^2}{r} = \omega^2 r \)  
- \( \tau = r \times F \)  
- \( \sum \tau = \tau_{net} = I \alpha \)  
- \( I = \int r^2 dm = \Sigma mr^2 \)  
- \( r_{cm} = \Sigma mr/\Sigma m \)  
- \( v = r \omega \)  
- \( L = r \times p = I \omega \)  
- \( K = \frac{1}{2}I \omega^2 \)  
- \( \omega = \omega_0 + at \)  
- \( \theta = \theta_0 + \omega_0 t + \frac{1}{2}at^2 \)  

### Electricity and Magnetism

- \( E = \frac{F}{q} \)  
- \( \oint E \cdot dA = \frac{Q}{\epsilon_0} \)  
- \( E = \frac{dV}{dr} \)  
- \( V = \frac{1}{4\pi \epsilon_0} \sum \frac{q_i}{r_i} \)  
- \( U_E = qV = \frac{1}{4\pi \epsilon_0} \frac{q_1 q_2}{r} \)  
- \( C = \frac{Q}{V} \)  
- \( C_p = \sum C_i \)  
- \( \frac{1}{C_s} = \sum \frac{1}{C_i} \)  
- \( I = \frac{dQ}{dt} \)  
- \( U_c = \frac{1}{2}QV = \frac{1}{2}CV^2 \)  
- \( d \mathbf{B} = \frac{\mu_0}{4\pi} \frac{I d\mathbf{\ell} \times \mathbf{r}}{r^3} \)  
- \( F = \int l \ d\mathbf{\ell} \times \mathbf{B} \)  
- \( B_s = \mu_0 n I \)  
- \( V = IR \)  
- \( R_s = \sum R_i \)  
- \( \frac{1}{R_p} = \sum \frac{1}{R_i} \)  
- \( \mathbf{F}_M = q \mathbf{v} \times \mathbf{B} \)  

\( A = \) area  
\( B = \) magnetic field  
\( C = \) capacitance  
\( d = \) distance  
\( E = \) electric field  
\( \mathbf{E} = \) emf  
\( F = \) force  
\( I = \) current  
\( J = \) current density  
\( N = \) normal force  
\( P = \) power  
\( Q = \) charge  
\( q = \) point charge  
\( R = \) resistance  
\( r = \) distance  
\( t = \) time  
\( U = \) potential or stored energy  
\( V = \) electric potential  
\( \nu = \) velocity or speed  
\( \rho = \) resistivity  
\( \phi_m = \) magnetic flux  
\( \kappa = \) dielectric constant
### GEOMETRY AND TRIGONOMETRY

- **Rectangle**
  \( A = bh \)

- **Triangle**
  \( A = \frac{1}{2}bh \)

- **Circle**
  \( A = \pi r^2 \)
  \( C = 2\pi r \)

- **Rectangular Solid**
  \( V = \ell wh \)

- **Cylinder**
  \( V = \pi r^2 \ell \)
  \( S = 2\pi r \ell + 2\pi r^2 \)

- **Sphere**
  \( V = \frac{4}{3}\pi r^3 \)
  \( S = 4\pi r^2 \)

- **Right Triangle**
  \( a^2 + b^2 = c^2 \)
  \( \sin \theta = \frac{a}{c} \)
  \( \cos \theta = \frac{b}{c} \)
  \( \tan \theta = \frac{a}{b} \)

### CALCULUS

\[
\frac{df}{dx} = \frac{df}{du} \frac{du}{dx}
\]

\[
\frac{d}{dx} (e^x) = e^x
\]

\[
\frac{d}{dx} (\ln x) = \frac{1}{x}
\]

\[
\frac{d}{dx} (\sin x) = \cos x
\]

\[
\frac{d}{dx} (\cos x) = -\sin x
\]

\[
\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1
\]

\[
\int e^x \, dx = e^x
\]

\[
\int \frac{dx}{x} = \ln |x|
\]

\[
\int \cos x \, dx = \sin x
\]

\[
\int \sin x \, dx = -\cos x
\]
E&M 1.

A very long, solid, nonconducting cylinder of radius $R$ has a positive charge of uniform volume density $\rho$. A section of the cylinder far from its ends is shown in the diagram above. Let $r$ represent the radial distance from the axis of the cylinder. Express all answers in terms of $r$, $R$, $\rho$, and fundamental constants, as appropriate.

(a) Using Gauss’s law, derive an expression for the magnitude of the electric field at a radius $r < R$. Draw an appropriate Gaussian surface on the diagram.

(b) Using Gauss’s law, derive an expression for the magnitude of the electric field at a radius $r > R$.

(c) On the axes below, sketch the graph of electric field $E$ as a function of radial distance $r$ for $r = 0$ to $r = 2R$. Explicitly label any intercepts, asymptotes, maxima, or minima with numerical values or algebraic expressions, as appropriate.

(d)

i. Derive an expression for the magnitude of the potential difference between $r = 0$ and $r = R$.

ii. Is the potential higher at $r = 0$ or $r = R$?

\[ r = 0 \quad r = R \]
(e) The nonconducting cylinder is replaced with a conducting cylinder of the same shape and same linear charge density. On the axes below, sketch the electric field $E$ as a function of $r$ for $r = 0$ to $r = 2R$. Explicitly label any intercepts, asymptotes, maxima, or minima with numerical values or algebraic expressions, as appropriate.
E&M 2.

In a lab, you set up a circuit that contains a capacitor C, a resistor R, a switch S, and a power supply, as shown in the diagram above. The capacitor is initially uncharged. The switch, which is initially open, can be moved to positions A or B.

(a)

i. Indicate the position to which the switch should be moved to charge the capacitor.

   __A  __B

ii. On the diagram, draw a voltmeter that is properly connected to the circuit in a manner that will allow the voltage to be measured across the capacitor.

After a long time you move the switch to discharge the capacitor, and your lab partner starts a stopwatch. You collect the following measurements of the voltage across the capacitor at various times.

<table>
<thead>
<tr>
<th>t (s)</th>
<th>6</th>
<th>18</th>
<th>30</th>
<th>42</th>
<th>54</th>
</tr>
</thead>
<tbody>
<tr>
<td>V(V)</td>
<td>252</td>
<td>74</td>
<td>33</td>
<td>10</td>
<td>6</td>
</tr>
</tbody>
</table>

You wish to determine the time constant \( \tau \) of the circuit from the slope of a linear graph.

(b)

i. Indicate two quantities you would plot to obtain a linear graph.

   i. Use the remaining rows in the table above, as needed, to record any quantities that you indicated that are not given. Label each row you use and include units.
(c) On the axes below, graph the data from the table that will produce a linear relationship. Clearly scale and label all axes including units, if appropriate. Draw a straight line that best fits your data points.

(d) From your line in part (c), obtain the value of the time constant $\tau$ of the circuit.

(e)

i. In the experiment, the capacitor $C$ had a capacitance of $1.50 \, \mu\text{F}$. Calculate an experimental value for the resistance $R$.

ii. On the axes in part (c), use a dashed line to sketch a possible graph if the capacitance was greater than $1.50 \, \mu\text{F}$ but the resistance $R$ was the same. Justify your answer.
E&M 3.

The figure above shows a circular loop of area 0.25 m$^2$ and resistance 12 $\Omega$ that lies in the plane of the page. A magnetic field of magnitude $B$ directed into the page exists in the area of the loop. The field varies with time $t$, as shown in the graph below.

(a)

i. Derive an expression for the magnitude of the induced emf in the loop as a function of time for the interval $t = 0$ s to $t = 8$ s.

ii. Calculate the magnitude of the induced current $I$ in the loop at time $t = 4$ s.
(b) i. Sketch a graph of the induced current $I$ in the loop as a function of time $t$ from $t = 0$ s to $t = 18$ s on the axes below, assuming that a counterclockwise (CCW) current is positive.

![Graph of induced current](image)

ii. For the time interval 12 s to 16 s, justify the direction of the current you have indicated in your graph.

(c) Calculate the total energy dissipated in the loop during the first 8 s shown.

STOP
END OF EXAM