Student Performance Q&A:

2013 AP® Calculus AB and Calculus BC Free-Response Questions

The following comments on the 2013 free-response questions for AP® Calculus AB and Calculus BC were written by the Chief Reader, Stephen Kokoska of Bloomsburg University, Bloomsburg, Pa. They give an overview of each free-response question and of how students performed on the question, including typical student errors. General comments regarding the skills and content that students frequently have the most problems with are included. Some suggestions for improving student performance in these areas are also provided. Teachers are encouraged to attend a College Board workshop to learn strategies for improving student performance in specific areas.

Question AB1/BC1

What was the intent of this question?

This problem provided information related to the amount of gravel at a processing plant during an eight-hour period. The function G, given by $G(t) = 90 + 45\cos\left(\frac{t^2}{18}\right)$, models the rate, in tons per hour, at which gravel arrives at the plant. The problem also states that gravel is processed at a constant rate of 100 tons per hour. In part (a) students were asked to find $G'(5)$, the derivative of G at time $t = 5$. This value is negative, so students should have interpreted the absolute value of this number as the rate at which the rate of arrival of gravel at the plant is decreasing, in tons per hour per hour, at time $t = 5$. In part (b) students were asked to find the total amount of unprocessed gravel arriving at the plant over the 8-hour workday. Students should have evaluated the definite integral $\int_0^8 G(x)\,dx$, recognizing that integrating the rate at which gravel arrives over a time interval gives the net amount of gravel that arrived over that time interval. Part (c) asked whether the amount of unprocessed gravel at the plant is increasing or decreasing at time $t = 5$. Students determined whether the rate at which unprocessed gravel is arriving is greater than the rate at which gravel is being processed, i.e. whether $G(5) > 100$. Part (d) asked students to determine the maximum amount of unprocessed gravel at the plant during this workday. Because the amount of unprocessed gravel at the plant at time t is given by $A(t) = 500 + \int_0^t (G(s) - 100)\,ds$, students needed to identify the critical points of this function (where $G(t) = 100$) and to determine the global maximum on the interval $[0, 8]$. This could have been done by observing that there is a unique critical point on the interval, which is a maximum, and determining the amount of unprocessed gravel at the plant at that time, or by computing the amount of unprocessed gravel at this critical point and at the endpoints for comparison.

How well did students perform on this question?

In general, students did not perform as well on this problem as they have on the first problem of the exam in recent years. The context of the problem seemed to inhibit some students. The mean score was 2.57 for AB students and 3.87 for BC students out of a possible 9 points.

Most students found parts (b) and (c) easier than parts (a) and (d). The context of the problem was the biggest issue. In part (b) several students included the initial condition in the expression for the total amount of unprocessed gravel that arrives at the plant. In part (c) several students were unable to determine the comparison necessary to answer the question. Students needed to only compare $G(5)$, the rate of arrival of gravel at time $t = 5$, to 100, the rate of processing of gravel. Several students confused amounts with rates.

Students who were unable to determine the correct derivative, or comparison, to consider in part (c) often continued with an incorrect expression in part (d). Those students who correctly found the maximum value on the interval often did not provide justification or only justification for a local maximum.

What were common student errors or omissions?

In part (a) some students provided incorrect units or no units, were unable to provide a correct interpretation of $G'(5)$, or forgot to indicate that this expression represented change at time $t = 5$. Some students indicated this expression referred to an interval of time.

In part (b) several students incorrectly included the initial amount of gravel in the net increase. These students added 500 to the correct answer. In part (c) students made a variety of errors in finding an expression for the rate of change in the amount of unprocessed gravel. Common errors included $G(5)$, $G'(5)$, $G'(5) - 100$, and $G(5) - 500$.

In part (d) several students used incorrect presentations for the amount function, or the derivative of this function, from part (c). Most students who obtained a correct value for the maximum amount of gravel provided the integral setup. However, several students did not provide the mathematical expression that led to their answer. Few students were able to provide a complete, correct justification for a unique critical point. Many students provided only a local argument.

Based on your experience of student responses at the AP® Reading, what message would you like to send to teachers that might help them to improve the performance of their students on the exam?

Continued work and practice with a variety of contextual problems is very important. Students should learn to recognize a typical “rate in-rate out” problem. They should notice whether the given functions represent amounts or rates. Students should have experience in both the candidates test and the unique critical point approaches for finding a global maximum or global minimum on an interval. The importance of showing work, and in particular showing the mathematical setups in Part A, the calculator-required portion of the exam, should continue to be stressed. Emphasize general communication skills and correct use of standard mathematical notation as well.

Question AB2

What was the intent of this question?

This problem presented students with a particle in rectilinear motion during the time interval $0 \le t \le 5$. The particle's position at time $t = 0$ is given, and the velocity function v is provided. Part (a) asked students to determine the times when the speed of the particle is 2, which required determining where the velocity function is ± 2 or where the absolute value of the velocity function is 2. In part (b) students were asked to provide an integral expression for the position $s(t)$ and then to use this expression to find the position of the particle at time $t = 5$. Students should have recognized that the position is given by $s(t) = s(0) + \int_0^t v(x)\, dx$ and then evaluated $s(5)$ to determine the position at time $t = 5$. Part (c) asked students to determine all times t, $0 \le t \le 5$, at which the particle changes direction. Students needed to determine where $v(t)$ changes sign. In part (d) students were asked whether the speed of the particle is increasing or decreasing at time $t = 4$. Students should have evaluated both the velocity and the acceleration functions at time $t = 4$. Since $v(4) < 0$ and $a(4) < 0$, the speed of the particle is increasing.

How well did students perform on this question?

This problem was a very typical particle motion problem. The mean score was 2.55 out of a possible 9 points.

In part (a) several students earned no points because they only considered the velocity. These students did not correctly use speed, the absolute value of velocity. In part (b) many students were unable to write the correct expression for $s(t)$. However, students were successful at presenting an expression for $s(5)$ and evaluating this expression on a calculator.

In part (c) most students were able to present a conceptual understanding of how to identify locations where the particle changes direction. However, they were often unable to provide the correct justification for these values. In part (d) many students answered this question correctly by examining the signs of the velocity and acceleration. However, several students answered this question by considering only the acceleration or only the velocity.

What were common student errors or omissions?

In part (a) many students did not consider the absolute value of the velocity function. These students solved the equation $v(t) = 2$ and obtained only one value for t. In part (b) many students had difficulty writing an expression for the position function $s(t)$. These students often included an indefinite integral. However, most students were able to write a correct expression for $s(5)$ and compute this value using the calculator. Some students did not include the initial condition.

In part (c) most students did consider the equation $v(t) = 0$ in order to locate the times when the particle changes direction. Of those students who considered this equation, most were able to correctly find the two values of t. Many students were unable to provide a correct justification. Some students still rely on sign charts only as justification. In part (d) some students stated a general rule for determining when speed is increasing without applying this concept to the specific problem. Some students attempted to find $v'(t)$ analytically. Many students incorrectly considered only one quantity, either $v(4)$ or $a(4)$, to determine if speed was increasing or decreasing. Students attempted to answer this question by evaluating velocity and acceleration at a variety of integer values.

Based on your experience of student responses at the AP® Reading, what message would you like to send to teachers that might help them to improve the performance of their students on the exam?

Students need to understand the relationship between velocity and speed. Teachers should remind students that even though sign charts are a wonderful organizational tool, there must be a related written interpretation accompanying this notation. Clear, concise mathematical communication is very important. Students should not refer to functions using the words "it," "the function," or "the graph."

Students need more practice writing an expression for a function that involves an integral. Students should practice with the Fundamental Theorem of Calculus in the form $f(b) = f(a) + \int_a^b f'(x)\,dx.$ Teachers and students must be familiar with the use of the graphing calculator in the AP® Calculus course and on the AP® Calculus exam. Students must be comfortable using the calculator for the four required capabilities on the exam.

Question AB3/BC3

What was the intent of this question?

In this problem, a table was provided giving values of a differentiable function C at selected times between $t = 0$ and $t = 6$ minutes, where $C(t)$ represents the amount of coffee, in ounces, in a cup at time t. Part (a) asked students to approximate the derivative of the function C at $t = 3.5$ and to indicate units of measure. Since $t = 3.5$ falls between values of t given in the table, students should have constructed a difference quotient using the temperature values across the smallest time interval containing $t = 3.5$ that is supported by the table. Students should have recognized this derivative as the rate at which the amount of coffee in the cup is increasing, in ounces per minute, at time $t = 3.5.$ Part (b) asked students whether there is a time $t,\ 2 \le t \le 4,$ at which $C'(t) = 2.$ Students should have recognized that the hypotheses for the Mean Value Theorem hold since C is differentiable and then applied this theorem to the function on the interval $[2, 4]$ to conclude that there must be such a time $t.$ Part (c) asked for an interpretation of $\frac{1}{6}\int_0^6 C(t)\,dt$ and a numeric approximation to this expression using a midpoint sum with three subintervals of equal length as indicated by the data in the table. Students should have recognized this expression as providing the average amount, in ounces, of coffee in the cup over the 6-minute time period. Students needed to use the values in the table at times $t = 1,\ t = 3,$ and $t = 5,$ with interval length 2, to compute this value. In part (d) students were given a symbolic expression for a function B that models the amount of coffee in the cup on the interval $0 \le t \le 6.$ Students were asked to use this model to determine the rate at which the amount of coffee in the cup is changing when time $t = 5.$ This was answered by computing the value $B'(5).$

How well did students perform on this question?

In general students performed well on this problem. Problems of this type have appeared on recent AP® Calculus exams and it seems evident that teachers are preparing students for this type of question. The mean score was 3.65 for AB students and 5.55 for BC students out of a possible 9 points.

Part (a) was a gentle entry part into the problem. Most students earned at least 1 point in this part. In part (b) many students computed two difference quotients and attempted to use the Intermediate Value Theorem to justify the answer. In part (c) some students presented other Riemann sums or incorrect midpoint sums. Many students linked unequal quantities or made simple arithmetic errors. Students did very well in part (d).

What were common student errors or omissions?

There were few errors in part (a). Most errors involved inverting the difference quotient, omitting units, or writing "ounces" for units. Several students also attempted a linear approximation. In part (b) the most common error presented was an appeal to the Intermediate Value Theorem. These students had estimated $C'(3.5)$ in part (a) and believed they should estimate $C'(2.5)$ also.

In part (c) student errors were both conceptual and computational. There were several simple arithmetic errors. Many students used incorrect widths in their midpoint sums, and several students presented other Riemann sums. Students who did not earn the interpretation point often presented incorrect units or omitted the word "average." In part (d) students who correctly used the chain rule and did not attempt to simplify the final answer did very well. Several students did not apply the chain rule correctly, and many students made arithmetic or algebraic errors in computing $B'(t)$.

Based on your experience of student responses at the AP® Reading, what message would you like to send to teachers that might help them to improve the performance of their students on the exam?

It is very important for students to have a solid mathematical background before taking calculus. Students frequently make arithmetic and algebraic errors while attempting to do calculus. In general students performed well on this tabular data problem and seemed well prepared for this type of question. Teachers should continue to incorporate these types of questions with tabular representations in their AP® Calculus courses.

Question AB4/BC4

What was the intent of this question?

This problem presented a function f that is defined and twice differentiable for all real numbers, and for which $f(8) = 4.$ The graph of $y = f'(x)$ on $[0, 8]$ is given, along with information about locations of horizontal tangent lines for the graph of f' and the areas of the regions between the graph of f' and the x-axis over this interval. Part (a) asked for all values of x in the interval $(0, 8)$ at which f has a local minimum. Students needed to recognize that this occurs where f' changes sign from negative to positive. Part (b) asked for the absolute minimum value of f on the interval $[0, 8].$ Students needed to use the information about the areas provided with the graph, as well as $f(8),$ to evaluate $f(x)$ at 0 and at the local minimum found in part (a). Part (c) asked for the open intervals on which the graph of f is both concave down and increasing. Students needed to recognize that this is given by intervals where the graph of f' is both decreasing and positive. Students were to determine these intervals from the graph. Part (d) introduced a new function g defined by $g(x) = (f(x))^3$ and the value $f(3) = -\frac{5}{2}$. Students were asked to find the slope of the line tangent to the graph of g at $x = 3$. Students needed to recognize that this slope is given by $g'(3)$. In order to determine this value, students needed to apply the chain rule correctly and read the value of $f'(3)$ from the graph.

How well did students perform on this question?

Students performed well in parts (a), (c), and (d). The mean score was 2.62 for AB students and 4.62 for BC students out of a possible 9 points.

This was a fairly straightforward question, and many students were able to correctly interpret the graph provided. There were some students who incorrectly used the point $(2, 1.5)$ on the graph to answer parts of this question.

What were common student errors or omissions?

In part (a) some students misinterpreted the graph of f' (provided) as the graph of f. There were some very vague justifications including references to "the function" or "the derivative." Several students presented sign charts as the only justification. In part (b) several students omitted the initial condition. Many students also had difficulty reporting the absolute minimum value. These students often reported $x = 0$ or the ordered pair $(0, -8)$ as the absolute minimum value.

Most students were able to find the intervals on which the graph of f was concave down and the intervals on which f was increasing. However, many students did not find the intersection of these sets. There were also sign charts presented by students without any interpretations. Several students also made similar vague references to "the function" or "the derivative." Some students found only one of the two intervals. In part (d) many students did not use the chain rule to find $g'(x)$. Some students who correctly found $g'(x)$ made arithmetic errors in computing $g'(3)$. Some students found the correct value for $g'(3)$ but continued to present an incorrect tangent line.

Based on your experience of student responses at the AP® Reading, what message would you like to send to teachers that might help them to improve the performance of their students on the exam?

Students should be very specific in their communication when making reference to a function. It is important to distinguish between an absolute minimum value, the x-coordinate at which this occurs, and the ordered pair on the graph of f. Arithmetic and algebra skills are very important for doing calculus. Students need practice writing precise justifications and explanations.

Question AB5

What was the intent of this question?

Students were given the graph of a region R bounded below by the graph of the function f and above by the graph of the function g, where $f(x) = 2x^2 - 6x + 4$ and $g(x) = 4\cos\left(\frac{1}{4}\pi x\right)$. In part (a) students were asked to find the area of R, requiring an appropriate integral setup and evaluation. Students needed to correctly evaluate $\int_0^2 (g(x) - f(x))\, dx$. Part (b) asked for an integral expression for the volume of the solid obtained by rotating the region R about the horizontal line $y = 4$. Students needed to set up an integral where the integrand represents a cross-sectional area of a circular disc with inner radius $(4 - g(x))$ and outer radius $(4 - f(x))$. This yielded the integral $\pi\int_0^2 \left[(4 - f(x))^2 - (4 - g(x))^2\right] dx$. Part (c) asked for an integral expression for the volume of the solid whose base is the region R and whose cross sections perpendicular to the x-axis are squares. Here the required integrand was $(g(x) - f(x))^2$.

How well did students perform on this question?

Most students presented significant work in all three parts of this problem. However, there were many students who presented algebra errors or misplaced or missing parentheses. The mean score was 4.14 out of a possible 9 points.

Throughout this problem, many students lost points for non-calculus errors, especially incorrect placement and missing parentheses. Many students had difficulty finding an antiderivative of the cosine function. Several students seemed confused by the graph provided, unable to identify the graphs of $y = f(x)$ and $y = g(x)$.

What were common student errors or omissions?

In part (a) the most common error was missing and/or misplaced parentheses. In addition, there were several students that clearly believed the graph of $y = g(x)$ was the lower curve, and the graph of $y = f(x)$ was the upper curve. Several students also struggled to find an antiderivative of the cosine function. These students usually had a sign error or an error in the constant $\frac{4}{\pi}$. Some students made errors in evaluating the cubic polynomial at $x = 2$.

In part (b) several students did not consider the line $y = 4$ in one or both of the radii. Many of these students reported their answer as $\pi\int_0^2 \left[(f(x))^2 - (g(x))^2\right] dx$. Some students omitted the square on the second term in the integrand.

Again in part (c) the most common error was misplaced and missing parentheses. Several students also included a constant π in the answer.

Based on your experience of student responses at the AP® Reading, what message would you like to send to teachers that might help them to improve the performance of their students on the exam?

When asked to present, but not evaluate, expressions representing certain quantities (e.g., volume), students may use the given function names. This will minimize possible errors. For example, in this question students could use $\int_0^2 (g(x) - f(x))\, dx$ instead of $\int_0^2 \left(4\cos\left(\frac{1}{4}\pi x\right) - \left(2x^2 - 6x + 4\right)\right) dx.$ Students should be careful to group expressions properly. For example, in this question $4\cos\left(\frac{1}{4}\pi x\right) - \left(2x^2 - 6x + 4\right)$ is correct, but $4\cos\left(\frac{1}{4}\pi x\right) - 2x^2 - 6x + 4$ is incorrect. Students need to practice with antidifferentiation by substitution of variables and with the washer method when rotating a region about a line that is not an axis. Present more problems involving volumes of regions with known cross sections other than disks and washers.

Question AB6

What was the intent of this question?

This problem presented students with a differential equation and defined $y = f(x)$ to be the particular solution to the differential equation passing through a given point. Part (a) asked students to write an equation for the line tangent to the graph of f at the given point, and then to use this tangent line to approximate $f(x)$ at a nearby value of x. Students needed to recognize that the slope of the tangent line is the value of the derivative, given in the differential equation, at the given point. Part (b) asked for the particular solution to the differential equation that passes through the given point. Students should have used the method of separation of variables to solve the differential equation.

How well did students perform on this question?

The mean score was 3.17 out of a possible 9 points. Several students had difficulty with using the given information to answer part (a). Nearly 30 percent of students did not earn any points on this question.

What were common student errors or omissions?

In part (a) several students did not substitute $(1, 0)$ into the given derivative. Some students worked to find the particular solution $y = f(x)$ through $(1, 0),$ then found the derivative and substituted. These students used the slope to find the tangent line and the approximation. This approach required more work, but is a valid solution technique. Some students attempted to find y'' and used this to compute a slope.

In part (b) most students attempted to solve this differential equation using the method of separation of variables, and most were able to correctly separate the variables. Some students incorrectly moved e^y into the numerator with dy rather than the denominator. Several students did not find the correct antiderivative of $\frac{1}{e^y}$. Some students incorrectly introduced a natural logarithmic function. Most students were able to correctly integrate the polynomial function. There were several errors involving the constant of integration. Some students simply did not include a constant. Some students included two constants, and some included the constant at an inappropriate point in simplification. Most students who were eligible used the initial condition properly. Several students made arithmetic and/or algebraic errors in solving for y. Some students incorrectly inserted absolute value symbols when solving for y.

Based on your experience of student responses at the AP® Reading, what message would you like to send to teachers that might help them to improve the performance of their students on the exam?

In part (a) most students understood the need to use the derivative to compute the slope of the tangent line. Unfortunately, several students misread the question or failed to recognize the derivative was given. Therefore, students need to carefully parse each question, and they need to communicate their work and solutions as clearly as possible. Students need to recognize that $\frac{dy}{dx}$ defined explicitly or implicitly can be used to find the slope of the tangent line.

Students need more practice with separating variables and working with antiderivatives, including ones such as $\int e^x \, dx$. Students need to understand when absolute values symbols are necessary and appropriate.

Question BC2

What was the intent of this question?

This problem provided the graphs of two curves defined by polar equations, along with values of θ at which the curves intersect. Part (a) asked students to find the area of the region common to the interiors of both graphs. This required students to divide the region into two subregions, bounded by arcs determined by the given values of θ, and then to apply the formula for polar area on each subregion to find the total area of the region. Part (b) described a particle moving along one of the curves and asked students to find the time when the x-coordinate of the particle is -1. Students needed to express the x-coordinate of the particle in terms of the angle θ, express θ in terms of time t, and, setting the resulting expression for x in terms of t equal to -1, solve for t in the desired time interval. Part (c) asked students to find the position vector of the particle in terms of time t, which required an expression for y in terms of t, and then to find the velocity vector at a given time. The final step required finding the numerical derivative of each expression in the position vector at the given time.

How well did students perform on this question?

Most students were able to set up an area formula involving polar curves. Students were able to write appropriate expressions in terms of t, solve an equation numerically, and find a numerical derivative. The mean score was 3.44 out of a possible 9 points.

What were common student errors or omissions?

In part (a) some students were not able to identify the correct region. Many students presented an integral expression that represented the region between the circle and limaçon from $\frac{\pi}{6}$ to $\frac{5\pi}{6}$ and went no further. Other students found the area of the region bounded outside by the limaçon and radially by the stated limits $\frac{\pi}{6}$ to $\frac{5\pi}{6}$ but neglected to add the circular region from $\frac{5\pi}{6}$ to $\frac{13\pi}{6}$, either by integration or by using a geometric approach.

In part (b) several students did not apply the polar to rectangular conversion formula $x = r\cos\theta$. In part (c) many students were unsure of how to present the components of the position vector and whether any additional computations were needed. Several students did not know how to find $v(1.5)$. These students did not differentiate the components of the position vector with respect to t. Some students did not use the calculator to compute $v(1.5)$. These students attempted to find an analytic derivative of each component.

Based on your experience of student responses at the AP® Reading, what message would you like to send to teachers that might help them to improve the performance of their students on the exam?

Teachers need to prepare students not only to compute polar areas by direct integration, but also to compute polar areas by assembling parts of the region and using geometry. In addition, the relationships among polar, rectangular, parametric, and vector-valued functions should be explored with students so that they know how to transition from one to another, as well as how to handle operations within those forms. Teachers should focus on the operations that students can perform using the calculator to save students from tedious symbolic operations that are subject to calculus, algebraic, or arithmetic errors.

Question BC5

What was the intent of this question?

This problem presented students with a differential equation and defined $y = f(x)$ to be the particular solution to the differential equation satisfying a given initial condition. Part (a) asked students to compute a limit of an expression involving $f(x)$, which required students to apply L'Hospital's Rule. Part (b) asked students to use Euler's method with two steps of equal size to approximate $f(x)$ at a value near the point given by the initial condition. Part (c) asked for the particular solution to the differential equation satisfying the given initial condition. Students should have used the method of separation of variables to solve the differential equation.

How well did students perform on this question?

In general, students did very well on this problem. The mean score was 5.59 out of a possible 9 points.

What were common student errors or omissions?

In general students attempted all three parts of this question. In part (a) several students tried to apply L'Hospital's Rule without limit notation or with incorrect notation after the initial presentation of a correct limit. Some students attempted to solve the differential equation in part (a) in order to find $f(x)$. These students then used this function to compute the limit. Other students imported their solution from part (c) in order to compute the limit. Several students did not address the variable y in the expression for the derivative and found the limit to be $2y^2$.

In part (b) some students who were able to demonstrate two Euler steps made arithmetic errors. Several students presented decimal answers, possibly to avoid the use of computations involving fractions. In part (c) several students presented an antiderivative of $\frac{1}{y^2}$ using logarithms or as expressions involving y^{-3}.

Based on your experience of student responses at the AP® Reading, what message would you like to send to teachers that might help them to improve the performance of their students on the exam?

Correct limit notation must be emphasized and used throughout the AP® Calculus course. Many students were able to present precise, labeled tables associated with Euler's method. However, many students continue to present vague, unlabeled tables. Without explicit arithmetic procedures, it is difficult to determine if a student has correctly applied Euler's method. Students must remember the constant of integration when solving a differential equation. There is no "universal logarithm rule" for finding antiderivatives, as demonstrated by students incorrectly using a logarithm in the antiderivative of $\frac{1}{y^2}$.

Question BC6

What was the intent of this question?

This problem described a function f known to have derivatives of all orders at $x = 0.$ In part (a) information was provided about the value of the function at $x = 0,$ as well as about the value of its first-degree Taylor polynomial about $x = 0$ at the point $x = \frac{1}{2}.$ Students needed to use this information to verify that $f'(0) = 2.$ In part (b) students were given information about the second and third derivatives of f at $x = 0.$ $x = 0.$ Students needed to use this additional information to find the third-degree Taylor polynomial for f about $x = 0.$ In part (c) a new function h was defined in terms of f. Students needed to use information provided about $h(0),$ as well as information already provided about f, to find the third-degree Taylor polynomial for h about $x = 0.$

How well did students perform on this question?

Students performed well on this question, especially because this was the last question on the exam. The mean score was 5.14 out of a possible 9 points.

Part (a) was a straightforward problem, but students did not perform as well as expected. Student work in part (b) was very good. Most students attempted to solve part (c) by using the second method presented on the scoring guideline.

What were common student errors or omissions?

In part (a) several students simply did not know how to start the problem. In part (b) some students attempted to simplify a correct answer and made an arithmetic or algebraic error or both. Some students presented only the third-degree term, not the entire third-degree Taylor polynomial. In part (c) it appears that some students misread or misunderstood the expression $h'(x) = f(2x)$ or did not understand how to use the derivative of h. These students dropped the derivative notation in an attempt to solve the problem. Several students made chain rule errors and some students did not carefully distinguish between the function h and its Taylor polynomial. Some students included a fourth-degree term in the Taylor polynomial.

Based on your experience of student responses at the AP® Reading, what message would you like to send to teachers that might help them to improve the performance of their students on the exam?

Students need more practice and familiarity with Taylor polynomials and series. Taylor polynomials and series are important parts of the Calculus BC course. Students should be careful to distinguish between the original given function, its Taylor series, and its Taylor polynomials. Notation is very important in this case. Students must continue to show the work that leads to an answer and label expressions that appear in their work.