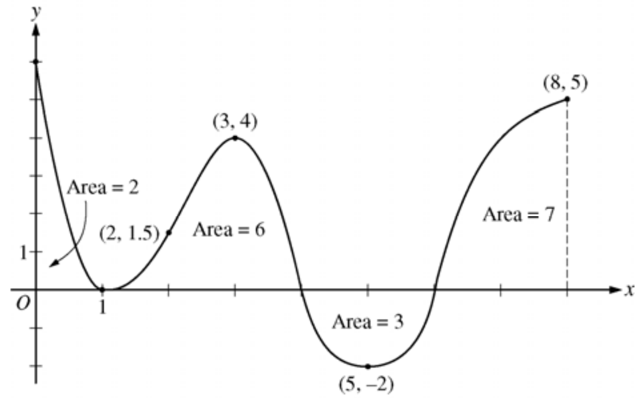
Question 4

The figure above shows the graph of f', the derivative of a twice-differentiable function f, on the closed interval $0 \le x \le 8$. The graph of f' has horizontal tangent lines at $x = 1$, $x = 3$, and $x = 5$. The areas of the regions between the graph of f' and the x-axis are labeled in the figure. The function f is defined for all real numbers and satisfies $f(8) = 4$.



Graph of f'

(a) Find all values of x on the open interval $0 < x < 8$ for which the function f has a local minimum. Justify your answer.

(b) Determine the absolute minimum value of f on the closed interval $0 \le x \le 8$. Justify your answer.

(c) On what open intervals contained in $0 < x < 8$ is the graph of f both concave down and increasing? Explain your reasoning.

(d) The function g is defined by $g(x) = (f(x))^3$. If $f(3) = -\frac{5}{2}$, find the slope of the line tangent to the graph of g at $x = 3$.

(a) $x = 6$ is the only critical point at which f' changes sign from negative to positive. Therefore, f has a local minimum at $x = 6$.	1 : answer with justification
(b) From part (a), the absolute minimum occurs either at $x = 6$ or at an endpoint. $f(0) = f(8) + \int_8^0 f'(x)\,dx$ $= f(8) - \int_0^8 f'(x)\,dx = 4 - 12 = -8$ $f(6) = f(8) + \int_8^6 f'(x)\,dx$ $= f(8) - \int_6^8 f'(x)\,dx = 4 - 7 = -3$ $f(8) = 4$ The absolute minimum value of f on the closed interval $[0, 8]$ is -8.	$3 : \begin{cases} 1 : \text{considers } x = 0 \text{ and } x = 6 \\ 1 : \text{answer} \\ 1 : \text{justification} \end{cases}$
(c) The graph of f is concave down and increasing on $0 < x < 1$ and $3 < x < 4$, because f' is decreasing and positive on these intervals.	$2 : \begin{cases} 1 : \text{answer} \\ 1 : \text{explanation} \end{cases}$
(d) $g'(x) = 3[f(x)]^2 \cdot f'(x)$ $g'(3) = 3[f(3)]^2 \cdot f'(3) = 3\left(-\frac{5}{2}\right)^2 \cdot 4 = 75$	$3 : \begin{cases} 2 : g'(x) \\ 1 : \text{answer} \end{cases}$

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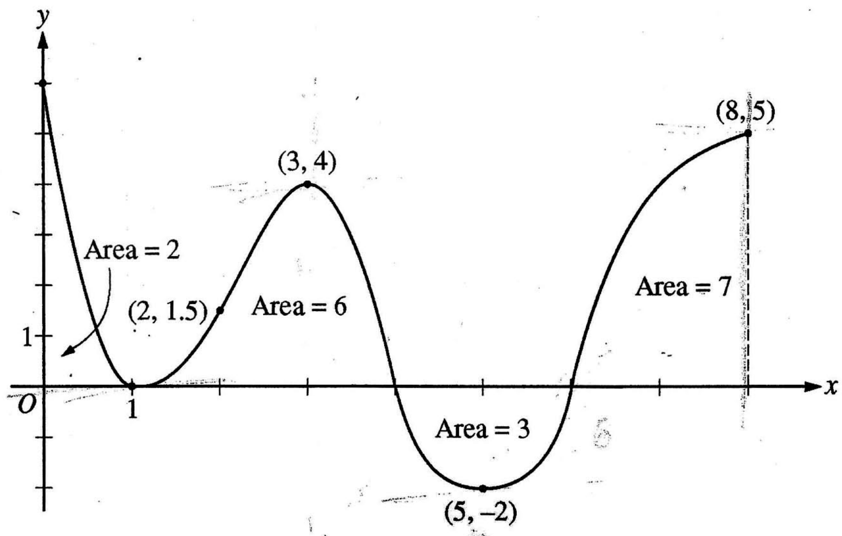

Graph of f'

4. The figure above shows the graph of f', the derivative of a twice-differentiable function f, on the closed interval $0 \le x \le 8$. The graph of f' has horizontal tangent lines at $x = 1$, $x = 3$, and $x = 5$. The areas of the regions between the graph of f' and the x-axis are labeled in the figure. The function f is defined for all real numbers and satisfies $f(8) = 4$.

(a) Find all values of x on the open interval $0 < x < 8$ for which the function f has a local minimum. Justify your answer.

$x = 6$

f has a local minimumat $x = 6$, because the graph of f ' changes from negative to positive at $X = 6$, so using the first derivative test and ~~the fat the at x=6~~, f has a critical number, at $X = 6$, f has a local minimum

(b) Determine the absolute minimum value of f on the closed interval $0 \le x \le 8$. Justify your answer.

local minimum = $X = 6$ $\qquad f(8) = 4$

$f(6) \rightarrow \int_6^8 f'(x)\,dx = 7 = f(8) - f(6) = 4 - f(6) \qquad f(6) = -3$

$f(0) \rightarrow \int_0^8 f'(x) = 12 = f(8) - f(0) = 4 - f(0) \qquad f(0) = -8$

The Absolute minimum value of f on the interval $0 \le x \le 8$ is -8 because it is the lowest value for f among the endpoints and critical numbers.

Continue problem 4 on page 17.

(c) On what open intervals contained in $0 < x < 8$ is the graph of f both concave down and increasing? Explain your reasoning.

$\hookrightarrow f'' < 0$

The open intervals where the graph of f is both concave down and increasing is $(0, 1) \cup (3, 4)$, or $0<x<1$ and $3<x<4$, because using the graph of f', when the graph of f' is positive and the slope of f' is negative, that means that f is increasing and f'' is negative, so f is both concave down and increasing.

(d) The function g is defined by $g(x) = (f(x))^3$. If $f(3) = -\frac{5}{2}$, find the slope of the line tangent to the graph of g at $x = 3$.

$$g'(x) = 3(f(x))^2 \cdot f'(x)$$

$$g'(3) = 3(f(3))^2 \cdot f'(3) = 3\left(-\frac{5}{2}\right)^2 \cdot (4)$$

$$= 3\left(\frac{25}{\cancel{4}}\right)\cancel{4} = 75$$

$$g'(3) = 75$$

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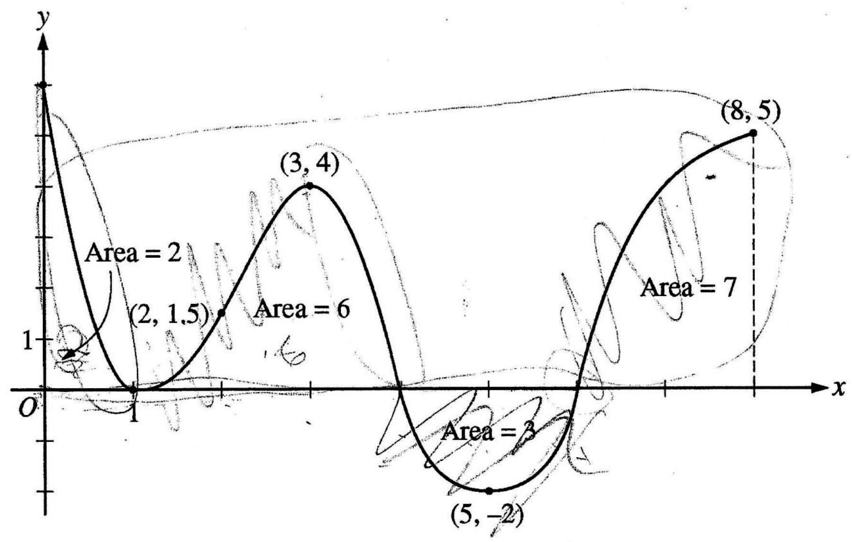

Graph of f'

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(a) Find all values of x on the open interval $0 < x < 8$ for which the function f has a local minimum. Justify your answer.

$x=6$ because the sign of f' changes from negative to positive.

(b) Determine the absolute minimum value of f on the closed interval $0 \le x \le 8$. Justify your answer.

x	$\int_0^x f'(x)dx$
0	0
8	4
1	neither
4	relmax
6	5

$x=8$ because that is where $f(x)$ is the smallest.

Continue problem 4 on page 17.

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(c) On what open intervals contained in $0 < x < 8$ is the graph of f both concave down and increasing? Explain your reasoning.

f(x) is concave down and increasing when f'' is negative and f' is positive this occurs (0,1) and (3,4).

(d) The function g is defined by $g(x) = (f(x))^3$. If $f(3) = -\frac{5}{2}$, find the slope of the line tangent to the graph of g at $x = 3$.

$x_1 = 3$
$y_1 = -\frac{5}{2}$
$\boxed{m = -75}$

$g'(x) = 3(f(x))^2 \cdot f'(x)$
$g'(3) = 3(-\frac{5}{2})^2 \cdot 4$
$g'(3) = -\frac{75}{\cancel{4}}\ \cancel{4}$
$g'(3) = -\frac{300}{4}$
$g'(3) = -75$

$\begin{array}{r} 75 \\ \times 4 \\ \hline 300 \end{array}$

$4 \overline{)300}$ = 075

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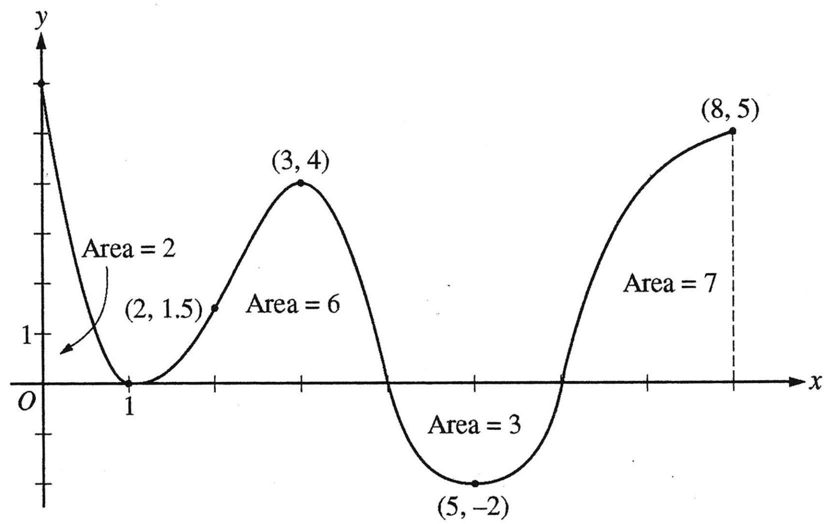

Graph of f'

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(a) Find all values of x on the open interval $0 < x < 8$ for which the function f has a local minimum. Justify your answer.

Min of f is when f' changes from (-) to (+)

Min @ X=6

(b) Determine the absolute minimum value of f on the closed interval $0 \le x \le 8$. Justify your answer.

X	$\int_0^8 f$
0	6
1	0
3	4
5	-2
8	5

$\boxed{\text{Min of -2 at } X=5}$

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Continue problem 4 on page 17.

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(c) On what open intervals contained in $0 < x < 8$ is the graph of f both concave down and increasing? Explain your reasoning.

F is concave down when f'' is (-)

F is increasing when f' is (+)

Both from (0,1), u (3,4)

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(d) The function g is defined by $g(x) = (f(x))^3$. If $f(3) = -\frac{5}{2}$, find the slope of the line tangent to the graph of g at $x = 3$.

$m = g'(x) = (f'(3))^3 = 4^3 = 48$

$x = 3$

$g(3) = (f(3))^3$

$\frac{-5^3}{3}$

$\frac{125}{9} = y$

16
$\times 3$
48

$\boxed{y - \frac{125}{9} = 48(x-3)}$

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Question 4

Overview

This problem described a function f that is defined and twice differentiable for all real numbers, and for which $f(8) = 4$. The graph of $y = f'(x)$ on $[0, 8]$ is given, along with information about locations of horizontal tangent lines for the graph of f' and the areas of the regions between the graph of f' and the x-axis over this interval. Part (a) asked for all values of x in the interval $(0, 8)$ at which f has a local minimum. Students needed to recognize that this occurs where f' changes sign from negative to positive. Part (b) asked for the absolute minimum value of f on the interval $[0, 8]$. Students needed to use the information about the areas provided with the graph, as well as $f(8)$, to evaluate $f(x)$ at 0 and at the local minimum found in part (a). Part (c) asked for the open intervals on which the graph of f is both concave down and increasing. Students needed to recognize that this is given by intervals where the graph of f' is both decreasing and positive. Students were to determine these intervals from the graph. Part (d) introduced a new function g defined by $g(x) = (f(x))^3$, and included that $f(3) = -\frac{5}{2}$. Students were asked to find the slope of the line tangent to the graph of g at $x = 3$. Students needed to recognize that this slope is given by $g'(3)$. In order to determine this value, students needed to apply the chain rule correctly and read the value of $f'(3)$ from the graph.

Sample: 4A
Score: 9

The student earned all 9 points.

Sample: 4B
Score: 6

The student earned 6 points: 1 point in part (a), 1 point in part (b), 2 points in part (c), and 2 points in part (d). In part (a) the student's work is correct. In part (b) the student earned the point for considering $x = 0$ and $x = 6$. The student does not report a correct answer and was not eligible for the justification point. In part (c) the student's work is correct. In part (d) the student earned the 2 points for $g'(x)$ but did not earn the answer point.

Sample: 4C
Score: 3

The student earned 3 points: 1 point in part (a) and 2 points in part (c). In part (a) the student's work is correct. In part (b) the student does not consider $x = 0$ and $x = 6$, does not find the answer, and is not eligible for the justification point. In part (c) the student's work is correct. In part (d) the student makes a chain rule error in the derivative and did not earn the answer point.