AP[®] CALCULUS AB 2013 SCORING GUIDELINES

Question 2

A particle moves along a straight line. For $0 \le t \le 5$, the velocity of the particle is given by

 $v(t) = -2 + (t^2 + 3t)^{6/5} - t^3$, and the position of the particle is given by s(t). It is known that s(0) = 10.

- (a) Find all values of t in the interval $2 \le t \le 4$ for which the speed of the particle is 2.
- (b) Write an expression involving an integral that gives the position s(t). Use this expression to find the position of the particle at time t = 5.
- (c) Find all times t in the interval $0 \le t \le 5$ at which the particle changes direction. Justify your answer.
- (d) Is the speed of the particle increasing or decreasing at time t = 4? Give a reason for your answer.

(a)	Solve $ v(t) = 2$ on $2 \le t \le 4$. t = 3.128 (or 3.127) and $t = 3.473$	2 : $\begin{cases} 1 : \text{ considers } v(t) = 2\\ 1 : \text{ answer} \end{cases}$
(b)	$s(t) = 10 + \int_0^t v(x) dx$ $s(5) = 10 + \int_0^5 v(x) dx = -9.207$	$2: \begin{cases} 1: s(t) \\ 1: s(5) \end{cases}$
(c)	v(t) = 0 when $t = 0.536033$, $3.317756v(t)$ changes sign from negative to positive at time $t = 0.536033$. v(t) changes sign from positive to negative at time $t = 3.317756$. Therefore, the particle changes direction at time $t = 0.536$ and time $t = 3.318$ (or 3.317).	3 : $\begin{cases} 1 : \text{considers } v(t) = 0 \\ 2 : \text{answers with justification} \end{cases}$
(d)	v(4) = -11.475758 < 0, $a(4) = v'(4) = -22.295714 < 0The speed is increasing at time t = 4 because velocity and acceleration have the same sign.$	2 : conclusion with reason

2. A particle moves along a straight line. For $0 \le t \le 5$, the velocity of the particle is given by

 $v(t) = -2 + (t^2 + 3t)^{6/5} - t^3$, and the position of the particle is given by s(t). It is known that s(0) = 10.

(a) Find all values of t in the interval $2 \le t \le 4$ for which the speed of the particle is 2.

1v(+)=2 25+54 t= 3.128, 3.473

(b) Write an expression involving an integral that gives the position s(t). Use this expression to find the position of the particle at time t = 5.

5(t) = (0 + of v(x))dx $S(5) = 10 + \int^{5} V(x) dx = (-9.207)$

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© 2013 The College Board. Visit the College Board on the Web: www.collegeboard.org. t = 0.536, 5.318the pertucle changes direction at 0.536 because v(t) = 0for (0, 0.536) and v(t) > 0 for (0.536, 3.318). The particle theoryes direction at 3.308 because v(t) > 0 for (0.536)3.318 and v(t) < 0 for t > 3.318

(c) Find all times t in the interval $0 \le t \le 5$ at which the particle changes direction. Justify your answer.

(d) Is the speed of the particle increasing or decreasing at time t = 4? Give a reason for your answer.

V14)40 a(4) = v'(4) - 0The speed is increasing at t=4 because both v (4) and a (4) = v'(4) cere negative.

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v(4) = 0

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. A particle mov	es along a stra	ight line. For $0 \leq$	$t \leq 5$, the velocity	of the partic	cle is given	by	
$v(t) = -2 + \left(t\right)$	$(t^{2}+3t)^{6/5}-t^{3}$, and the position	n of the particle is	given by $s(t)$. It is know	n that $s(0) = 10$.	
(a) Find all va $\beta = -\frac{1}{2}$ $\beta = -\frac{1}{2}$	lues of t in the $2 + (4^2 + 3^2)$ $-4 + (4^2 + 3^2)$	$\frac{1}{\sqrt{5}} = \frac{1}{\sqrt{3}}$	4 for which the s	peed of the p	article is 2.	×	
+=	5.140	+=	3.128	а., .,			
• •							
* *							
(b) Write an e	xpression invo	lving an integral	that gives the posit	ion $s(t)$. Use	e this expres	ssion to find the	
(b) Write an e position of	xpression invo the particle at	lving an integral time $t = 5$.	that gives the posit	ion $s(t)$. Use	e this expres	ssion to find the	
(b) Write an e position of	xpression invo the particle at $\int \underbrace{\left\langle \left[+\right] \right\rangle}$	lving an integral time $t = 5$.	that gives the posit $\frac{1}{\sqrt{(k)}}$	ion $s(t)$. Use	this expres	ssion to find the	ч
(b) Write an e position of	xpression invo the particle at $\int \int (+)$	lving an integral time $t = 5$. y = 10 + 10	that gives the positive $\frac{1}{4}$ $V(x) dx$	ion <i>s</i> (<i>t</i>). Use	e this expres	ession to find the	
(b) Write an e position of	xpression invo the particle at $\int \int \{+\}$	lving an integral time $t = 5$. j = 10 + 5 j = 10 + 5	that gives the positive $\frac{1}{10000000000000000000000000000000000$	ion $s(t)$. Use	this expres	ssion to find the	- -
(b) Write an e position of	xpression invo the particle at $\int \int \{+\}$	lving an integral time $t = 5$. y = 10 + 10 z = 10 + 10 z = 10 + 10 z = 10 + 10	that gives the positive $V(x) = V(x) = V(x)$	ion $s(t)$. Use	e this expres	ssion to find the	
(b) Write an e position of	xpression invo the particle at $\int \int \{t\}$	lving an integral time $t = 5$. y = 10 + 10 z = 10 + 10 z = 10 + 10	that gives the positive $V(X) dX$	ion <i>s</i> (<i>t</i>). Use	e this expres	asion to find the	
(b) Write an e position of	xpression invo the particle at $\int \int \{+\}$	lving an integral time $t = 5$. y = 10 + 10 z = 10 + 5 z = 10 + 5	that gives the positive $V(X) dX$	ion <i>s</i> (<i>t</i>). Use	e this expres	ssion to find the	
(b) Write an e position of	xpression invo the particle at $\int \int \{ t \}$	lving an integral time $t = 5$. j = 10 + 5 j = -9.	that gives the posit $\frac{1}{100}$ $\frac{V(k)}{200}$ $\frac{J(k)}{200}$	ion <i>s</i> (<i>t</i>). Use	e this expres	ssion to find the	
(b) Write an e position of	xpression invo the particle at $\int \int \{+\}$	lving an integral time $t = 5$. y = 10 + 10	that gives the positive $V(x) = V(x) = V(x)$	ion <i>s</i> (<i>t</i>). Use	e this expres	ession to find the	
(b) Write an e position of	xpression invo the particle at $\int \int \{t\}$	lving an integral time $t = 5$. y = 10 + 10 z = 10 + 10 z = 10 + 10	that gives the positive $\frac{1}{10000000000000000000000000000000000$	ion <i>s</i> (<i>t</i>). Use	e this expres	ossion to find the	

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2B, (c) Find all times t in the interval $0 \le t \le 5$ at which the particle changes direction. Justify your answer. (c) Find all lines i in the meridia of a prection when velocity changes ligh. The particle changes direction when velocity changes ligh. This occurs at time t=3.318 as the velocity changes from positive to hegatille. 0 = -> + (+2+3+) +2 +=3.318(d) Is the speed of the particle increasing or decreasing at time t = 4? Give a reason for your answer. $V(t) = -2 + (t^2 + 3t)^2 - t^2$ $\alpha(t) = 1.25\sqrt{t^{2}+3t}$ (2++3) -3+2 $V(4) = -2 + (16 + 12)^{12} - 64$ $v(4) = 1.25\sqrt{16 + 12}(8 + 3) - 48$ v(4) = -21 201 V(4)= -11.476 The speed of a porticle is increasing if acceleration and velocity have the same sign and decreasing if acceleration and velocity have different signs. The speed of the porticle is increasing at time t=4 because, at that point, both the vebcity and the acceleration are negative.

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2. A particle moves along a straight line. For 0 ≤ t ≤ 5, the velocity of the particle is given by x y v(t) = -2 + (t² + 3t)^{6/5} - t³, and the position of the particle is given by s(t). It is known that s(0) = 10.
(a) Find all values of t in the interval 2 ≤ t ≤ 4 for which the speed of the particle is 2.

$$2 = -2 + (t^{2} + 3t)^{6/5}$$

$$0 = (t^{2} + 3t)^{6/5} - t^{3} - 4$$

$$\rightarrow graph$$

$$at t = 3.1276299$$

$$t = 3.128$$

(b) Write an expression involving an integral that gives the position s(t). Use this expression to find the position of the particle at time t = 5.

$$S(t) = \int V(t) \rightarrow -t^{2} + \int \left[\left(\frac{t^{3}}{3} + \frac{3t^{2}}{2} \right)^{n/5} - \frac{t^{4}}{4} + c \right]$$

$$I0 = 0 + \int \left[(0+0)^{n/5} - 0 + c \rightarrow c = 10 \right]$$

$$CS(t) = -25 + 6829.210659 - 156.25 + 10$$

$$S(t) = 6657.961$$

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(c) Find all times t in the interval $0 \le t \le 5$ at which the particle changes direction. Justify your answer.

The particle changes direction when the velocity changes sign $0 = -2 + (t^2 + 3t)^{6/5} - t^3 \rightarrow 9 voph$ v(0) =

(d) Is the speed of the particle increasing or decreasing at time t = 4? Give a reason for your answer.

 $V(4) = -2 + (4^2 + 3(4)^{6/5} - 4^3)$

V(4) = -11.47(e $0(t) = \frac{6}{2}(t^{2}+3t)^{1/5} \cdot (2t+3)^{2} - 3t^{2}$

a(4) = -22.296

The speed is increasing because acceleration and velocity are both negative.

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AP[®] CALCULUS AB 2013 SCORING COMMENTARY

Question 2

Overview

This problem presented students with a particle in rectilinear motion during the time interval $0 \le t \le 5$. The particle's position at time t = 0 is given, and the velocity function v is provided. Part (a) asked students to determine the times when the speed of the particle is 2, which required determining where the velocity function is ± 2 or where the absolute value of the velocity function is 2. In part (b) students were asked to provide an integral expression for the position s(t) and then to use this expression to find the position of the particle at time t = 5.

Students should have recognized that the position is given by $s(t) = s(0) + \int_{0}^{t} v(x) dx$ and then evaluated s(5)

to determine the position at time t = 5. Part (c) asked students to determine all times t, $0 \le t \le 5$, at which the particle changes direction. Students needed to determine where v(t) changes sign. In part (d) students were asked whether the speed of the particle is increasing or decreasing at time t = 4. Students should have evaluated both the velocity and the acceleration functions at time t = 4. Because v(4) < 0 and a(4) < 0, the speed of the particle is increasing.

Sample: 2A Score: 9

The student earned all 9 points.

Sample: 2B Score: 6

The student earned 6 points: no points in part (a), 2 points in part (b), 2 points in part (c), and 2 points in part (d). In part (a) the student's work is incorrect. In part (b) the student's work is correct. In part (c) the student earned the point for considering v(t) = 0 and 1 point for a single correct answer with correct justification. In part (d) the student's work is correct.

Sample: 2C Score: 3

The student earned 3 points: 1 point in part (c) and 2 points in part (d). In parts (a) and (b), the student's work is incorrect and did not earn any points. In part (c) the student earned the point for considering v(t) = 0. In part (d) the student's work is correct.