AP Physics 1: Algebra-Based
Sample Student Responses and Scoring Commentary

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Free Response Question 3
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General Notes About 2018 AP Physics Scoring Guidelines

1. The solutions contain the most common method of solving the free-response questions and the allocation of points for this solution. Some also contain a common alternate solution. Other methods of solution also receive appropriate credit for correct work.

2. The requirements that have been established for the paragraph-length response in Physics 1 and Physics 2 can be found on AP Central at https://secure-media.collegeboard.org/digitalServices/pdf/ap/paragraph-length-response.pdf.

3. Generally, double penalty for errors is avoided. For example, if an incorrect answer to part (a) is correctly substituted into an otherwise correct solution to part (b), full credit will usually be awarded. One exception to this may be cases when the numerical answer to a later part should be easily recognized as wrong, e.g., a speed faster than the speed of light in vacuum.

4. Implicit statements of concepts normally receive credit. For example, if use of the equation expressing a particular concept is worth 1 point, and a student’s solution embeds the application of that equation to the problem in other work, the point is still awarded. However, when students are asked to derive an expression, it is normally expected that they will begin by writing one or more fundamental equations, such as those given on the exam equation sheet. For a description of the use of such terms as “derive” and “calculate” on the exams, and what is expected for each, see “The Free-Response Sections — Student Presentation” in the AP Physics; Physics C: Mechanics, Physics C: Electricity and Magnetism Course Description or “Terms Defined” in the AP Physics 1: Algebra-Based Course and Exam Description and the AP Physics 2: Algebra-Based Course and Exam Description.

5. The scoring guidelines typically show numerical results using the value $g = 9.8 \text{ m/s}^2$, but the use of $10 \text{ m/s}^2$ is of course also acceptable. Solutions usually show numerical answers using both values when they are significantly different.

6. Strict rules regarding significant digits are usually not applied to numerical answers. However, in some cases answers containing too many digits may be penalized. In general, two to four significant digits are acceptable. Numerical answers that differ from the published answer due to differences in rounding throughout the question typically receive full credit. Exceptions to these guidelines usually occur when rounding makes a difference in obtaining a reasonable answer. For example, suppose a solution requires subtracting two numbers that should have five significant figures and that differ starting with the fourth digit (e.g., 20.295 and 20.278). Rounding to three digits will lose the accuracy required to determine the difference in the numbers, and some credit may be lost.
The disk shown above spins about the axle at its center. A student’s experiments reveal that, while the disk is spinning, friction between the axle and the disk exerts a constant torque on the disk.

(a) LO / SP: 3.A.1.1 / 1.5, 2.2; 3.F.1.1 / 1.4; 3.F.2.1 / 6.4; 4.D.2.1 / 1.2, 1.4

4 points

At time $t = 0$ the disk has an initial counterclockwise (positive) angular velocity $\omega_0$. The disk later comes to rest at time $t = t_1$.

i. On the grid at left below, sketch a graph that could represent the disk’s angular velocity as a function of time $t$ from $t = 0$ until the disk comes to rest at time $t = t_1$.

ii. On the grid at right below, sketch the disk’s angular acceleration as a function of time $t$ from $t = 0$ until the disk comes to rest at time $t = t_1$.

Example graphs:

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Example graphs:  (a)(i)  (a)(ii)
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i. 2 points

For a curve that has an angular velocity of $+\omega_0$ at time $t = 0$ and decreases to zero at time $t = t_1$ 1 point

For a curve that is a straight line with a negative slope showing the angular velocity approaching zero (can be a positive slope, if the initial angular velocity on the graph is negative) 1 point
Question 3 (continued)

(a) (continued)
ii. 2 points

| For a curve that is negative for the entire time | 1 point |
| For a curve that is a constant nonzero function | 1 point |

(b) LO / SP: 3.A.1.1 / 1.5, 2.2; 3.F.2.1 / 6.4; 4.D.1.1 / 1.2, 1.4; 4.D.2.1 / 1.2, 1.4; 4.D.3.1 / 2.2

3 points

The magnitude of the frictional torque exerted on the disk is \( \tau_0 \). Derive an equation for the rotational inertia \( I \) of the disk in terms of \( \tau_0, \omega_0, t_1 \), and physical constants, as appropriate.

| For using an equation for the rotational version of Newton’s second law | 1 point |
| For using an appropriate rotational kinematics equation \[ \alpha = \Delta \omega / \Delta t \] | 1 point |
| For a correct answer in terms of the listed quantities, derived from first principles \[ I = \frac{\tau_0 t_1}{\omega_0} \] | 1 point |

Note: This point is still earned if there is a minus sign, e.g., from using \(-\tau_0\) or \(-\omega_0\).

Alternate solution using angular momentum and rotational impulse:

| For defining and using angular momentum \( L = I\omega \) | 1 point |
| For using rotational impulse \( \Delta L = \tau \Delta t \) | 1 point |
| For a correct answer in terms of the listed quantities, derived from first principles \[ I = \frac{\tau_0 t_1}{\omega_0} \] | 1 point |

Note: This point is still earned if there is a minus sign, e.g., from using \(-\tau_0\) or \(-\omega_0\).
In another experiment, the disk again has an initial positive angular velocity $\omega_0$ at time $t = 0$. At time $t = \frac{1}{2} t_1$, the student starts dripping oil on the contact surface between the axle and the disk to reduce the friction. As time passes, more and more oil reaches that contact surface, reducing the friction even further.

i. On the grid at left below, sketch a graph that could represent the disk’s angular velocity as a function of time from $t = 0$ to $t = t_1$, which is the time at which the disk came to rest in part (a).

ii. On the grid at right below, sketch the disk’s angular acceleration as a function of time from $t = 0$ to $t = t_1$.

Example graphs:

![Graph](c)(i) ![Graph](c)(ii)

### Question 3 (continued) Distribution of points

(c) LO / SP: 3.A.1.1 / 1.5, 2.2; 3.A.3.1 / 6.4, 7.2; 3.F.1.1 / 1.4; 3.F.2.1 / 6.4; 4.D.1.1 / 1.2, 1.4; 4.D.2.1 / 1.2, 1.4

4 points

<table>
<thead>
<tr>
<th>Points</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>For curve with a clear change of slope or curvature at $\frac{1}{2} t_1$ and showing a decrease in speed thereafter</td>
</tr>
<tr>
<td>1</td>
<td>For a curve that indicates slowing at a decreasing rate between times $\frac{1}{2} t_1$ and $t_1$</td>
</tr>
<tr>
<td>1</td>
<td>For a curve that does not reach zero at or before time $t_1$</td>
</tr>
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</table>
**Question 3 (continued)**

**Distribution of points**

<table>
<thead>
<tr>
<th>ii.</th>
<th>1 point</th>
</tr>
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</table>
| For a curve with decreasing magnitude between times $\frac{1}{2}t_1$ and $t_1$  
Note: The acceleration may reach zero at or before time $t_1$. If so, it must remain zero for the remaining time. | 1 point |

<table>
<thead>
<tr>
<th>(d)</th>
<th>LO / SP: 3.F.2.1 / 6.4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 point</td>
<td></td>
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</table>

The student is trying to mathematically model the magnitude $\tau$ of the torque exerted by the axle on the disk when the oil is present at times $t > \frac{1}{2}t_1$. The student writes down the following two equations, each of which includes a positive constant ($C_1$ or $C_2$) with appropriate units.

1. $\tau = C_1\left(t - \frac{1}{2}t_1\right)$ (for $t > \frac{1}{2}t_1$)
2. $\tau = \frac{C_2}{\left(t + \frac{1}{2}t_1\right)}$ (for $t > \frac{1}{2}t_1$)

Which equation better mathematically models this experiment?  
___ Equation (1)  ___ Equation (2)

Briefly explain why the equation you selected is plausible and why the other equation is not plausible.

| Correct answer: “Equation (2)”  
Note: If the wrong selection is made, the explanation is not graded. |
| For stating that Equation (2) is plausible because the frictional torque decreases with increasing time, whereas in Equation (1) torque increases with increasing time | 1 point |

Examples:  
Equation (2) because $\tau$ decreases. In Equation (1), it doesn’t.  
Equation (2) is plausible because the frictional torque decreases as more oil reaches the contact surface over time. Equation (1) is not plausible because it shows friction increasing as more oil reaches the surface over time. |
Learning Objectives (LO)

LO 3.A.1.1: The student is able to express the motion of an object using narrative, mathematical, and graphical representations. [See Science Practices 1.5, 2.1, and 2.2]

LO 3.A.3.1: The student is able to analyze a scenario and make claims (develop arguments, justify assertions) about the forces exerted on an object by other objects for different types of forces or components of forces. [See Science Practices 6.4 and 7.2]

LO 3.F.1.1: The student is able to use representations of the relationship between force and torque. [See Science Practice 1.4]

LO 3.F.2.1: The student is able to make predictions about the change in the angular velocity about an axis for an object when forces exerted on the object cause a torque about that axis. [See Science Practice 6.4]

LO 4.D.1.1: The student is able to describe a representation and use it to analyze a situation in which several forces exerted on a rotating system of rigidly connected objects change the angular velocity and angular momentum of the system. [See Science Practices 1.2 and 1.4]

LO 4.D.2.1: The student is able to describe a model of a rotational system and use that model to analyze a situation in which angular momentum changes due to interaction with other objects or systems. [See Science Practices 1.2 and 1.4]

LO 4.D.3.1: The student is able to use appropriate mathematical routines to calculate values for initial or final angular momentum, or change in angular momentum of a system, or average torque or time during which the torque is exerted in analyzing a situation involving torque and angular momentum. [See Science Practice 2.2]
3. (12 points, suggested time 25 minutes)

The disk shown above spins about the axle at its center. A student’s experiments reveal that, while the disk is spinning, friction between the axle and the disk exerts a constant torque on the disk.

(a) At time $t = 0$ the disk has an initial counterclockwise (positive) angular velocity $\omega_0$. The disk later comes to rest at time $t = t_1$.

i. On the grid at left below, sketch a graph that could represent the disk’s angular velocity as a function of time $t$ from $t = 0$ until the disk comes to rest at time $t = t_1$.

ii. On the grid at right below, sketch the disk’s angular acceleration as a function of time $t$ from $t = 0$ until the disk comes to rest at time $t = t_1$.

(b) The magnitude of the frictional torque exerted on the disk is $\tau_0$. Derive an equation for the rotational inertia $I$ of the disk in terms of $\tau_0$, $\omega_0$, $t_1$, and physical constants, as appropriate.

\[ I = I\omega \]
\[ \frac{\Delta L}{\Delta t} = \tau \]
\[ L - \Delta L = 0 \]
\[ L = \Delta L \]
\[ L = \mathbf{r} \times \mathbf{F} \]
\[ I\omega_0 = \tau_0 \cdot t_1 \]

\[ I = \frac{\tau_0 \cdot t_1}{\omega_0} \]
(c) In another experiment, the disk again has an initial positive angular velocity $\omega_0$ at time $t = 0$. At time $t = \frac{1}{2} t_1$, the student starts dripping oil on the contact surface between the axle and the disk to reduce the friction. As time passes, more and more oil reaches that contact surface, reducing the friction even further.

i. On the grid at left below, sketch a graph that could represent the disk’s angular velocity as a function of time from $t = 0$ to $t = t_1$, which is the time at which the disk came to rest in part (a).

ii. On the grid at right below, sketch the disk’s angular acceleration as a function of time from $t = 0$ to $t = t_1$.

(d) The student is trying to mathematically model the magnitude $\tau$ of the torque exerted by the axle on the disk when the oil is present at times $t > \frac{1}{2} t_1$. The student writes down the following two equations, each of which includes a positive constant ($C_1$ or $C_2$) with appropriate units.

1. $\tau = C_1 \left( t - \frac{1}{2} t_1 \right)$ (for $t > \frac{1}{2} t_1$)

2. $\tau = \frac{C_2}{\left( t + \frac{1}{2} t_1 \right)}$ (for $t > \frac{1}{2} t_1$)

Which equation better mathematically models this experiment?

___ Equation (1)   ___ Equation (2)

Briefly explain why the equation you selected is plausible and why the other equation is not plausible.

According to equation 1, if the time continues to increase, torque will continue to increase but torque is supposed to decrease if we are oiling the system. Equation 2 shows that as $t$ increases, the torque gets closer to 0 which is what we want. Equation 1 also implies that torque can go to $\infty$ as time goes to $\infty$ which isn’t plausible in the problem’s context.
3. (12 points, suggested time 25 minutes)

The disk shown above spins about the axle at its center. A student’s experiments reveal that, while the disk is spinning, friction between the axle and the disk exerts a constant torque on the disk.

(a) At time \( t = 0 \) the disk has an initial counterclockwise (positive) angular velocity \( \omega_0 \). The disk later comes to rest at time \( t = t_1 \).

i. On the grid at left below, sketch a graph that could represent the disk’s angular velocity as a function of time \( t \) from \( t = 0 \) until the disk comes to rest at time \( t = t_1 \).

ii. On the grid at right below, sketch the disk’s angular acceleration as a function of time \( t \) from \( t = 0 \) until the disk comes to rest at time \( t = t_1 \).

(b) The magnitude of the frictional torque exerted on the disk is \( \tau_0 \). Derive an equation for the rotational inertia \( I \) of the disk in terms of \( \tau_0 \), \( \omega_0 \), \( t_1 \), and physical constants, as appropriate.
(c) In another experiment, the disk again has an initial positive angular velocity \( \omega_0 \) at time \( t = 0 \). At time \( t = \frac{1}{2} t_1 \), the student starts dripping oil on the contact surface between the axle and the disk to reduce the friction. As time passes, more and more oil reaches that contact surface, reducing the friction even further.

i. On the grid at left below, sketch a graph that could represent the disk’s angular velocity as a function of time from \( t = 0 \) to \( t = t_1 \), which is the time at which the disk came to rest in part (a).

ii. On the grid at right below, sketch the disk’s angular acceleration as a function of time from \( t = 0 \) to \( t = t_1 \).

(d) The student is trying to mathematically model the magnitude \( \tau \) of the torque exerted by the axle on the disk when the oil is present at times \( t > \frac{1}{2} t_1 \). The student writes down the following two equations, each of which includes a positive constant \((c_1 \text{ or } c_2)\) with appropriate units.

1. \( \tau = c_1 \left( t - \frac{1}{2} t_1 \right) \)  \( \text{for } t > \frac{1}{2} t_1 \)  

2. \( \tau = \frac{c_2}{\left( t + \frac{1}{2} t_1 \right)} \)  \( \text{for } t > \frac{1}{2} t_1 \)

Which equation better mathematically models this experiment?

___ Equation (1)  __ Equation (2)

Briefly explain why the equation you selected is plausible and why the other equation is not plausible.

Torque (\( \tau \)) is the rotational equivalent to the translational force (\( F \)).

If \( F \) is constant, the \( \tau = 1 \alpha \). Looking back at my answer for b, time would be in the denominator, and since everything else is constant in this scenario, the same would apply here. In addition, as the time goes over \( \frac{1}{2} t_1 \), torque decreases, and this would be statistically plausible with Equation (2).
3. (12 points, suggested time 25 minutes)

The disk shown above spins about the axle at its center. A student’s experiments reveal that, while the disk is spinning, friction between the axle and the disk exerts a constant torque on the disk.

(a) At time \( t = 0 \) the disk has an initial counterclockwise (positive) angular velocity \( \omega_0 \). The disk later comes to rest at time \( t = t_1 \).

i. On the grid at left below, sketch a graph that could represent the disk’s angular velocity as a function of time \( t \) from \( t = 0 \) until the disk comes to rest at time \( t = t_1 \).

ii. On the grid at right below, sketch the disk’s angular acceleration as a function of time \( t \) from \( t = 0 \) until the disk comes to rest at time \( t = t_1 \).

(b) The magnitude of the frictional torque exerted on the disk is \( \tau_0 \). Derive an equation for the rotational inertia \( I \) of the disk in terms of \( \tau_0 \), \( \omega_0 \), \( t_1 \), and physical constants, as appropriate.

\[
I = \frac{\tau_0}{\alpha} = \frac{\tau_0}{\omega_0 + \alpha t} = \frac{\tau_0}{\omega_0} - \frac{\tau_0}{\omega_0} \frac{t}{\omega_0}
\]
(c) In another experiment, the disk again has an initial positive angular velocity \( \omega_0 \) at time \( t = 0 \). At time \( t = \frac{1}{2} t_1 \), the student starts dripping oil on the contact surface between the axle and the disk to reduce the friction. As time passes, more and more oil reaches that contact surface, reducing the friction even further.

i. On the grid at left below, sketch a graph that could represent the disk’s angular velocity as a function of time from \( t = 0 \) to \( t = t_1 \), which is the time at which the disk came to rest in part (a).

ii. On the grid at right below, sketch the disk’s angular acceleration as a function of time from \( t = 0 \) to \( t = t_1 \).

(d) The student is trying to mathematically model the magnitude \( \tau \) of the torque exerted by the axle on the disk when the oil is present at times \( t > \frac{1}{2} t_1 \). The student writes down the following two equations, each of which includes a positive constant \( (c_1 \text{ or } c_2) \) with appropriate units.

\[
\begin{align*}
(1) & \quad \tau = c_1 \left( t - \frac{1}{2} t_1 \right) \quad (\text{for } t > \frac{1}{2} t_1) \\
(2) & \quad \tau = \frac{c_2}{t + \frac{1}{2} t_1} \quad (\text{for } t > \frac{1}{2} t_1)
\end{align*}
\]

Which equation better mathematically models this experiment?

\[\text{___ Equation (1) } \Box \text{ Equation (2)}\]

Briefly explain why the equation you selected is plausible and why the other equation is not plausible.

\[t + \frac{1}{2} t_1 \text{ is the time that } \tau \text{ applied on the disk for when the oil is present after } \frac{1}{2} t, \text{ seconds which is the point that they started to apply oil. The first equation show the time } t - \frac{1}{2} t, \text{ which is } \frac{1}{2} t \text{ before the real time that was when they did not apply the oil.}\]

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Overview


The responses to this question were expected to demonstrate the following:

- An understanding of motion graphs, both angular velocity and angular acceleration, for a rotating object as it slows down.
- The ability to relate torque, angular acceleration, and rotational inertia in the rotational form of Newton’s second law or, alternatively, the same relationship expressed by connecting angular momentum and angular impulse.
- The ability to show how motion graphs would change under the condition of a nonconstant net torque.
- An understanding of general mathematical rules as they relate to physical behavior by recognizing where a time variable must appear in an equation to match the described behavior within a graph.

Sample: P1 Q3 A
Score: 12

The graph of angular velocity in part (a)(i) earned 2 points: 1 point for having the correct initial angular velocity of $+\omega_0$ at time $t = 0$ and decreasing to zero at $t = t_1$ and 1 point for being a straight line approaching zero. The graph of angular acceleration in part (a)(ii) earned 2 points: 1 point for being negative the entire time and 1 point for being a nonzero constant. Part (b) earned 3 points for a solution using angular momentum and rotational impulse. The response earned 1 point for defining and using angular momentum, 1 point for using rotational impulse, and 1 point for a correct answer in terms of the listed quantities. The graph of angular velocity in part (c)(i) earned 3 points: 1 point for showing a clear change of curvature at $\frac{1}{2}t_1$ with a decrease in angular speed after $\frac{1}{2}t_1$, 1 point for indicating that the disk is slowing at a decreasing rate between times $\frac{1}{2}t_1$ and $t_1$, and 1 point for a curve that does not reach zero. The graph of angular acceleration in part (c)(ii) earned 1 point for indicating a decreasing magnitude between times $\frac{1}{2}t_1$ and $t_1$. Part (d) earned 1 point because the response selects the correct equation, and the response indicates that Equation 2 correctly shows that the frictional torque decreases with increasing time.

Sample: P1 Q3 B
Score: 7

The graph of angular velocity in part (a)(i) earned 2 points: 1 point for having the correct initial angular velocity of $+\omega_0$ at time $t = 0$ and decreasing to zero at $t = t_1$ and 1 point for being a straight line approaching zero. The graph of angular acceleration in part (a)(ii) earned 2 points: 1 point for being negative the entire time and 1 point for being a nonzero constant. Part (b) earned 1 of 3 points for using the rotational version of Newton’s second law. The response does not use an appropriate rotational kinematics equation or have a correct answer in terms of the listed quantities, so the remaining 2 points in part (b) were not earned. The graph of angular velocity in part (c)(i) earned 1 of 3 points for showing a clear change of slope or curvature at $\frac{1}{2}t_1$ with a decrease in angular speed after $\frac{1}{2}t_1$. The graph did not earn 1 point because it indicates slowing at an increasing rate (i.e., the slope...
Question 3 (continued)

gets steeper) between times $\frac{1}{2}t_1$ and $t_1$. The graph did not earn the remaining 1 point because it reaches zero at time $t_1$. The graph of angular acceleration in part (c)(ii) earned no points because it indicates an increasing magnitude between times $\frac{1}{2}t_1$ and $t_1$, the time interval over which the oil causes a decrease in magnitude.

Part (d) earned 1 point for selecting Equation 2 and indicating that torque decreases as time increases beyond $\frac{1}{2}t_1$, consistent with Equation 2.

Sample: P1 Q3 C
Score: 5

The graph of angular velocity in part (a)(i) earned 1 of 2 points for having the correct initial angular velocity of $+\omega_0$ at time $t = 0$ and decreasing to zero at $t = t_1$, but it did not earn 1 point because it is not a straight line.

The graph of angular acceleration in part (a)(ii) earned 1 of 2 points for being negative the entire time, but it did not earn 1 point because it was not constant. Part (b) earned 2 of 3 points: 1 point for using the rotational version of Newton’s second law and 1 point for using an appropriate rotational kinematics equation to substitute for the angular equation. The response did not earn 1 point for a correct answer, because $\tau_0$ was never substituted for the general torque variable in the derivation. The graph of angular velocity in part (c)(i) earned 1 of 3 points for showing a clear change of slope or curvature at $\frac{1}{2}t_1$ with a decrease in angular speed after $\frac{1}{2}t_1$. The graph did not earn 1 point because it indicates slowing at a constant rate (i.e., the slope is constant) between times $\frac{1}{2}t_1$ and $t_1$. The graph did not earn the remaining 1 point because it reaches zero at time $t_1$. The graph of angular acceleration in part (c)(ii) earned no points because the magnitude is constant between times $\frac{1}{2}t_1$ and $t_1$, the time interval over which the oil causes a decrease in magnitude. Part (d) earned no points because, although Equation 2 is correctly selected, there is no mention of whether the torque magnitude increases or decreases for either equation.