
AP Calculus BC

Sample Student Responses and Scoring Commentary

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**AP[®] CALCULUS BC
2017 SCORING GUIDELINES**

Question 5

(a) $f'(x) = \frac{-3(4x - 7)}{(2x^2 - 7x + 5)^2}$

$$f'(3) = \frac{(-3)(5)}{(18 - 21 + 5)^2} = -\frac{15}{4}$$

(b) $f'(x) = \frac{-3(4x - 7)}{(2x^2 - 7x + 5)^2} = 0 \Rightarrow x = \frac{7}{4}$

The only critical point in the interval $1 < x < 2.5$ has x -coordinate $\frac{7}{4}$.

f' changes sign from positive to negative at $x = \frac{7}{4}$.

Therefore, f has a relative maximum at $x = \frac{7}{4}$.

(c)
$$\begin{aligned} \int_5^{\infty} f(x) dx &= \lim_{b \rightarrow \infty} \int_5^b \frac{3}{2x^2 - 7x + 5} dx = \lim_{b \rightarrow \infty} \int_5^b \left(\frac{2}{2x - 5} - \frac{1}{x - 1} \right) dx \\ &= \lim_{b \rightarrow \infty} \left[\ln(2x - 5) - \ln(x - 1) \right]_5^b = \lim_{b \rightarrow \infty} \left[\ln \left(\frac{2x - 5}{x - 1} \right) \right]_5^b \\ &= \lim_{b \rightarrow \infty} \left[\ln \left(\frac{2b - 5}{b - 1} \right) - \ln \left(\frac{5}{4} \right) \right] = \ln 2 - \ln \left(\frac{5}{4} \right) = \ln \left(\frac{8}{5} \right) \end{aligned}$$

(d) f is continuous, positive, and decreasing on $[5, \infty)$.

The series converges by the integral test since $\int_5^{\infty} \frac{3}{2x^2 - 7x + 5} dx$ converges.

— OR —

$$\frac{3}{2n^2 - 7n + 5} > 0 \text{ and } \frac{1}{n^2} > 0 \text{ for } n \geq 5.$$

Since $\lim_{n \rightarrow \infty} \frac{\frac{3}{2n^2 - 7n + 5}}{\frac{1}{n^2}} = \frac{3}{2}$ and the series $\sum_{n=5}^{\infty} \frac{1}{n^2}$ converges,

the series $\sum_{n=5}^{\infty} \frac{3}{2n^2 - 7n + 5}$ converges by the limit comparison test.

2 : $f'(3)$

2 : $\begin{cases} 1 : x\text{-coordinate} \\ 1 : \text{relative maximum} \\ \text{with justification} \end{cases}$

3 : $\begin{cases} 1 : \text{antiderivative} \\ 1 : \text{limit expression} \\ 1 : \text{answer} \end{cases}$

2 : answer with conditions

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NO CALCULATOR ALLOWED

5A

5. Let f be the function defined by $f(x) = \frac{3}{2x^2 - 7x + 5}$.

(a) Find the slope of the line tangent to the graph of f at $x = 3$.

$$f(x) = 3(2x^2 - 7x + 5)^{-1}$$

$$f'(x) = -3(2x^2 - 7x + 5)^{-2}(4x - 7)$$

$$f'(3) = \frac{-3(4(3) - 7)}{(2(3)^2 - 7(3) + 5)^2} = \frac{-15}{(18 - 21 + 5)^2} = \frac{-15}{4}$$

(b) Find the x -coordinate of each critical point of f in the interval $1 < x < 2.5$. Classify each critical point as the location of a relative minimum, a relative maximum, or neither. Justify your answers.

$$0 = f'(x)$$

$$0 = \frac{-3(4x - 7)}{(2x^2 - 7x + 5)^2}$$

$$(2x - 5)(2x - 1)$$

$$\rightarrow 0 = -3(4x - 7)$$

$$x = \frac{7}{4}$$

$$\rightarrow 0 = (2x^2 - 7x + 5)^2$$

$$0 = (2x^2 - 7x + 5)$$

$$0 = (2x - 5)(x - 1)$$

$$x = \frac{5}{2} \quad x = 1$$

$x = \frac{7}{4}$ is a relative maximum because $f'(x)$ goes from positive to negative

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(c) Using the identity that $\frac{3}{2x^2 - 7x + 5} = \frac{2}{2x - 5} - \frac{1}{x - 1}$, evaluate $\int_5^{\infty} f(x) dx$ or show that the integral diverges.

$$\int_5^{\infty} f(x) dx = \int_5^{\infty} \left(\frac{2}{2x - 5} - \frac{1}{x - 1} \right) dx$$

$$\lim_{R \rightarrow \infty} \int_5^R \left(\frac{2}{2x - 5} - \frac{1}{x - 1} \right) dx = \lim_{R \rightarrow \infty} \left[\ln|2x - 5| - \ln|x - 1| \right]_5^R$$

$$= \lim_{R \rightarrow \infty} \left(\ln|2(R) - 5| - \ln|R - 1| \right) - \left(\ln|2(5) - 5| - \ln|5 - 1| \right)$$

$$= \lim_{R \rightarrow \infty} \left(\ln \left| \frac{2R - 5}{R - 1} \right| - \left(\ln \frac{2(5) - 5}{5 - 1} \right) \right)$$

$$= \lim_{R \rightarrow \infty} \ln \frac{2R - 5}{R - 1} - \lim_{R \rightarrow \infty} \ln \frac{2(5) - 5}{5 - 1} = \ln(2) - \ln\left(\frac{5}{4}\right)$$

(d) Determine whether the series $\sum_{n=5}^{\infty} \frac{3}{2n^2 - 7n + 5}$ converges or diverges. State the conditions of the test

used for determining convergence or divergence.

Since $f(x)$ is continuous, decreasing, and positive on the interval $[5, \infty)$, the integral test can be used.

Integral test

$$\int_5^{\infty} \frac{3}{2x^2 - 7x + 5} dx = \ln(2) - \ln\left(\frac{5}{4}\right)$$

$\int_5^{\infty} \frac{3}{2x^2 - 7x + 5} dx$ converges to a value, $\sum_{n=5}^{\infty} \frac{3}{2n^2 - 7n + 5}$ also converges by the integral test.

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NO CALCULATOR ALLOWED

5B₁

5. Let f be the function defined by $f(x) = \frac{3}{2x^2 - 7x + 5}$.

(a) Find the slope of the line tangent to the graph of f at $x = 3$.

$$f'(x) = \frac{-(3)(4x - 7)}{(2x^2 - 7x + 5)^2}$$

$$f'(3) = \frac{-3(12 - 7)}{(18 - 21 + 5)^2} = \frac{-3(5)}{2^2} = \boxed{-\frac{15}{4}}$$

(b) Find the x -coordinate of each critical point of f in the interval $1 < x < 2.5$. Classify each critical point as the location of a relative minimum, a relative maximum, or neither. Justify your answers.

$$f'(x) = \frac{-12x + 21}{(2x^2 - 7x + 5)^2}$$

$$\begin{aligned} -12x + 21 &= 0 \\ x &= \frac{21}{12} = \frac{7}{4} \end{aligned}$$

$$(2x^2 - 7x + 5) = 0$$

$$(2x - 5)(x - 1) = 0$$

$$x = \frac{5}{2} \quad x = 1$$

$x = \frac{7}{4}$ is a relative maximum because f' changes from positive to negative at that point

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NO CALCULATOR ALLOWED

5B₂

- (c) Using the identity that $\frac{3}{2x^2 - 7x + 5} = \frac{2}{2x - 5} - \frac{1}{x - 1}$, evaluate $\int_5^{\infty} f(x) dx$ or show that the integral diverges.

$$\lim_{b \rightarrow \infty} \int_5^b \frac{2}{2x - 5} dx - \int_5^b \frac{1}{x - 1} dx$$

$$\lim_{b \rightarrow \infty} \left[\ln|2x - 5| - \ln|x - 1| \right]_5^b$$

$$\lim_{b \rightarrow \infty} (\ln|2b - 5| - \ln|b - 1|) - \ln 5 - \ln 4 \quad \text{DNE}$$

diverges

- (d) Determine whether the series $\sum_{n=5}^{\infty} \frac{3}{2n^2 - 7n + 5}$ converges or diverges. State the conditions of the test used for determining convergence or divergence.

$$\frac{3}{2n^2 - 7n + 5} = \frac{\frac{3}{1}}{2n - 5} + \frac{\frac{3}{5/2}}{n - 1}$$

$$= \int \frac{3}{2n - 5} + \frac{\frac{6}{5}}{n - 1}$$

$$\frac{3}{2} \ln|2n - 5| + \frac{6}{5} \ln|n - 1| \Big|_5^b$$

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NO CALCULATOR ALLOWED

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5. Let f be the function defined by $f(x) = \frac{3}{2x^2 - 7x + 5}$. 8-14+5

(a) Find the slope of the line tangent to the graph of f at $x = 3$.

$$f'(x) = \frac{-3(4x-7)}{(2x^2-7x+5)^2}$$

$$m_T |_{x=3} = \frac{-3(12-7)}{(2 \cdot 9 - 7 \cdot 3 + 5)^2} = \frac{-15}{4}$$

(b) Find the x -coordinate of each critical point of f in the interval $1 < x < 2.5$. Classify each critical point as the location of a relative minimum, a relative maximum, or neither. Justify your answers.

$$\begin{aligned} -3(4x-7) &= 0 \\ -12x + 21 &= 0 \\ x &= \frac{21}{12} \end{aligned}$$

$$f' \quad \begin{array}{c} + \quad - \\ \left| \quad \right| \\ 1 \quad \frac{21}{12} \quad 2 \end{array}$$

relative maximum because
 f is increasing to the left
 and decreasing to the right.

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NO CALCULATOR ALLOWED

5C2

- (c) Using the identity that $\frac{3}{2x^2 - 7x + 5} = \frac{2}{2x - 5} - \frac{1}{x - 1}$, evaluate $\int_5^{\infty} f(x) dx$ or show that the integral diverges.

$$\int_5^{\infty} \left(\frac{2}{2x-5} - \frac{1}{x-1} \right) dx$$

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- (d) Determine whether the series $\sum_{n=5}^{\infty} \frac{3}{2n^2 - 7n + 5}$ converges or diverges. State the conditions of the test used for determining convergence or divergence.

$$= \sum_{n=5}^{\infty} \frac{3}{2n^2}$$

By test for Divergence $\sum_{n=5}^{\infty} \frac{3}{2n^2 - 7n + 5}$ diverges.

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Question 5

Overview

In this problem the function f is defined by $f(x) = \frac{3}{2x^2 - 7x + 5}$. In part (a) students were asked to find the slope of the line tangent to the graph of f at $x = 3$. Students needed to differentiate f and find the slope of the line tangent to the graph of f at $x = 3$ by evaluating $f'(3)$. [LO 2.3B/EK 2.3B1] In part (b) students were asked to find the x -coordinate of each critical point of f in the interval $1 < x < 2.5$ and to classify each critical point as the location of a relative minimum, a relative maximum, or neither, justifying these classifications. Students should have observed that f is differentiable on $1 < x < 2.5$ and found that $f'(x) = 0$ has just one solution, $x = \frac{7}{4}$, in this interval. Then students needed to determine that f has a relative maximum at $x = \frac{7}{4}$ by noting that f' changes sign from positive to negative at the critical point $x = \frac{7}{4}$. [LO 2.2A/EK 2.2A1] In part (c) students were given the partial fraction decomposition for $f(x)$ and asked to evaluate $\int_5^{\infty} f(x) dx$ or to show that the integral diverges. Students should have expressed the given improper integral as a limit of proper integrals, $\lim_{b \rightarrow \infty} \int_5^b f(x) dx$, and used the partial fraction decomposition for $f(x)$ to find that $\int_5^b f(x) dx = \ln(2b - 5) - \ln(b - 1) - (\ln 5 - \ln 4) = \ln\left(\frac{2b - 5}{b - 1}\right) - \ln\left(\frac{5}{4}\right)$. Applying limit theorems, students needed to take the limit of this expression as $b \rightarrow \infty$ to find that the improper integral converges to $\ln\left(\frac{8}{5}\right)$. [LO 1.1C/EK 1.1C1-1.1C2, LO 3.2D/EK 3.2D1-3.2D2] In part (d) students were asked to determine whether the series $\sum_{n=5}^{\infty} f(n)$ converges or diverges, stating the conditions of the test used for this determination. Students needed to combine the results of part (c) with the integral test or use a limit comparison test to the convergent p -series $\sum_{n=5}^{\infty} \frac{1}{n^2}$ to find that the series $\sum_{n=5}^{\infty} f(n)$ converges. For either test, students should have observed that the necessary conditions hold, namely that f is continuous, positive, and decreasing on $[5, \infty)$. [LO 4.1A/EK 4.1A6] This problem incorporates the following Mathematical Practices for AP Calculus (MPACs): reasoning with definitions and theorems, connecting concepts, implementing algebraic/computational processes, building notational fluency, and communicating.

Sample: 5A

Score: 9

The response earned all 9 points: 2 points in part (a), 2 points in part (b), 3 points in part (c), and 2 points in part (d). In part (a) the student earned both points by presenting a correct expression for $f'(x)$ and evaluating it correctly at $x = 3$, arriving at the value of $-\frac{15}{4}$. The student does not need to simplify the numerical expression $\frac{-3(4(3) - 7)}{(2(3)^2 - 7(3) + 5)^2}$ to earn the points. The student chooses to simplify and does so correctly. In part (b) the student earned both points with the sentence at the bottom. In this sentence the x -coordinate is correctly identified,

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Question 5 (continued)

the critical point is correctly classified as a relative maximum, and the student's justification is correct. In part (c) the student earned the antiderivative point with the correct expression presented on the right side of the second line. The limit expression point was earned with the expression on the left side of the second line. The student earned the answer point at the end of the last line. The expression $\ln(2) - \ln\left(\frac{5}{4}\right)$ does not need to be simplified.

In part (d) the student earned both points for using the integral test to conclude that the given series converges and for stating the conditions for the integral test on the left side of the page.

Sample: 5B

Score: 6

The response earned 6 points: 2 points in part (a), 2 points in part (b), 2 points in part (c), and no points in part (d). In part (a) the student earned both points by presenting a correct expression for $f'(x)$ and evaluating it correctly

at $x = 3$, arriving at the value of $-\frac{15}{4}$. The student does not need to simplify the numerical expression

$\frac{-3(12 - 7)}{(18 - 21 + 5)^2}$ to earn the points. The student chooses to simplify and does so correctly. In part (b) the student

earned both points with the boxed sentence in the lower left corner. In this sentence the x -coordinate is correctly identified, the critical point (which is identified in the work in the upper right corner) is correctly classified as a relative maximum, and the student's justification is correct. In part (c) the student earned the antiderivative point in the second line. The limit expression point was earned in the first line. The student does not determine the value of the limit correctly and did not earn the answer point. In part (d) the student does not draw any conclusion about the convergence of the given series, so no points were earned.

Sample: 5C

Score: 3

The response earned 3 points: 2 points in part (a), 1 point in part (b), no points in part (c), and no points in part (d). In part (a) the student earned both points by presenting a correct expression for $f'(x)$ and evaluating it correctly at $x = 3$, arriving at the value of $-\frac{15}{4}$. The student does not need to simplify the numerical expression

$\frac{-3(12 - 7)}{(2 \cdot 9 - 7 \cdot 3 + 5)^2}$ to earn the points. The student chooses to simplify and does so correctly. In part (b) the

student earned the first point for identifying $x = \frac{21}{12}$ as the x -coordinate of the critical point. The student does not provide sufficient justification for the classification of a relative maximum. The justification discusses the behavior of f as "increasing to the left and decreasing to the right" rather than focusing on the sign change of f'

at $x = \frac{21}{12}$. Thus, the second point was not earned. In part (c) the student does not evaluate the integral, so no points were earned. In part (d) the student makes an incorrect claim about the convergence of the series without any support for the conclusion. No test is used, and no conditions are stated. The student did not earn any points.