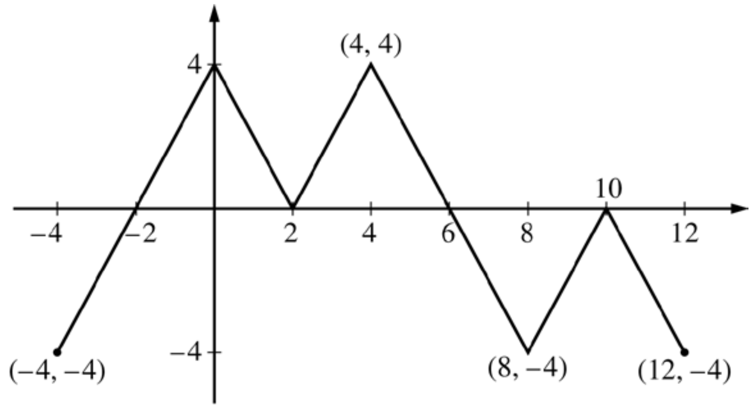
Question 3

The figure above shows the graph of the piecewise-linear function f. For $-4 \le x \le 12$, the function g is defined by $g(x) = \int_2^x f(t)\,dt$.



Graph of f

(a) Does g have a relative minimum, a relative maximum, or neither at $x = 10$? Justify your answer.

(b) Does the graph of g have a point of inflection at $x = 4$? Justify your answer.

(c) Find the absolute minimum value and the absolute maximum value of g on the interval $-4 \le x \le 12$. Justify your answers.

(d) For $-4 \le x \le 12$, find all intervals for which $g(x) \le 0$.

---

1 : $g'(x) = f(x)$ in (a), (b), (c), or (d)

(a) The function g has neither a relative minimum nor a relative maximum at $x = 10$ since $g'(x) = f(x)$ and $f(x) \le 0$ for $8 \le x \le 12$.

1 : answer with justification

(b) The graph of g has a point of inflection at $x = 4$ since $g'(x) = f(x)$ is increasing for $2 \le x \le 4$ and decreasing for $4 \le x \le 8$.

1 : answer with justification

(c) $g'(x) = f(x)$ changes sign only at $x = -2$ and $x = 6$.

x	$g(x)$
−4	−4
−2	−8
6	8
12	−4

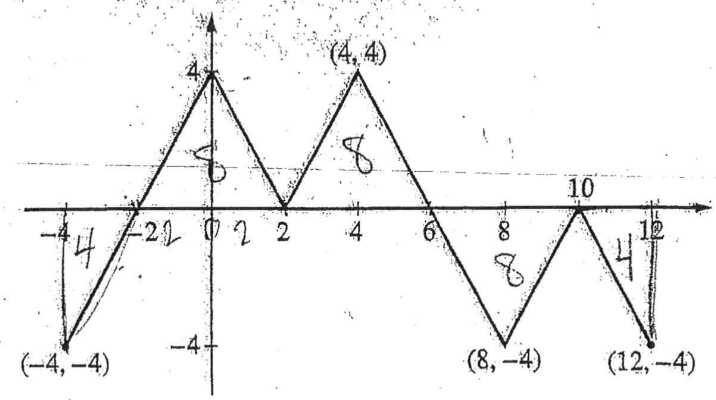
On the interval $-4 \le x \le 12$, the absolute minimum value is $g(-2) = -8$ and the absolute maximum value is $g(6) = 8$.

$$4 : \begin{cases} 1 : \text{considers } x = -2 \text{ and } x = 6 \text{ as candidates} \\ 1 : \text{considers } x = -4 \text{ and } x = 12 \\ 2 : \text{answers with justification} \end{cases}$$

(d) $g(x) \le 0$ for $-4 \le x \le 2$ and $10 \le x \le 12$.

2 : intervals

NO CALCULATOR ALLOWED



Graph of f

3. The figure above shows the graph of the piecewise-linear function f. For $-4 \le x \le 12$, the function g is defined by $g(x) = \int_2^x f(t)\,dt$.

(a) Does g have a relative minimum, a relative maximum, or neither at $x = 10$? Justify your answer.

$g'(x) = f(x)$

g does not have a relotive minimum or maximum at $x=10$ because $g'(x) = f(x)$ does not change sign at this point

(b) Does the graph of g have a point of inflection at $x = 4$? Justify your answer.

$g''(x) = f'(x)$

$f'(x) = g''(x)$ does change sign at $x=4$ so g does have a point of inflection at this point

Do not write beyond this border.

Continue problem 3 on page 15.

(c) Find the absolute minimum value and the absolute maximum value of g on the interval $-4 \le x \le 12$. Justify your answers.

$g'(x) = f(x) = 0 \quad x = -2 \quad x = 2$
$x = 6 \quad x = 10$

does not change sign at $x = 2$ and $x = 10$

x	$g(x)$
-4	$\int_2^{-4} f(t)\,dt = -8 + 4 = -4$
-2	$\int_2^{-2} f(t)\,dt = -8$
6	$\int_2^{6} f(t)\,dt = 8$
12	$\int_2^{12} f(t)\,dt = 8 - 8 - 4 = -4$

The absolute minimum value of g on the interval $-4 \le x \le 12$ is -8 and the absolute maximum value of g is 8

(d) For $-4 \le x \le 12$, find all intervals for which $g(x) \le 0$.

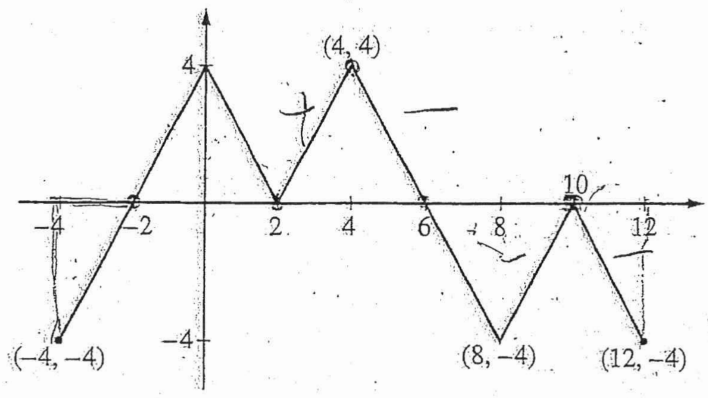
$g(x) = \int_2^x f(t)\,dt \le 0$

$g(x) = 0$ at $x = 2$ and $x = 10$

$g(x) \le 0$ in the intervals $-4 \le x \le 2$ and $10 \le x \le 12$

GO ON TO THE NEXT PAGE.

NO CALCULATOR ALLOWED



Graph of f

3. The figure above shows the graph of the piecewise-linear function f. For $-4 \le x \le 12$, the function g is defined by $g(x) = \int_2^x f(t)\,dt$.

(a) Does g have a relative minimum, a relative maximum, or neither at $x = 10$? Justify your answer.

$g'(x) = f(x)$

$g'(10) = f(10) = 0$

g is neither at x=10 bc g'(x) does not change from pos to neg or neg to pos at x = 10.

(b) Does the graph of g have a point of inflection at $x = 4$? Justify your answer.

$g''(x) = f'(x)$

$f'(4) = 0$

g has a poi at x=4 bc $g''(4) = 0$ and $g''(x) > 0$ when $2 \le x < 4$ and $g''(x) < 0$ when $4 < x < 8$.

Do not write beyond this border.

Continue problem 3 on page 15.

NO CALCULATOR ALLOWED

(c) Find the absolute minimum value and the absolute maximum value of g on the interval $-4 \le x \le 12$. Justify your answers.

$g'(x) = f(x) = 0$

$x = -2, 2, 6, 10$

x	$g(x)$
-4	$-\int_{-4}^{2} f(t)\,dt = -\left[(\frac{1}{2})(-4)(2) + (\frac{1}{2})(4)(4)\right] = -(-4+8) = -4$
-2	$-\int_{-2}^{2} f(t)\,dt = -\left[(\frac{1}{2})(4)(4)\right] = -8$
2	$\int_{2}^{2} f(t)\,dt = 0$
6	$\int_{2}^{6} f(t)\,dt = (\frac{1}{2})(4)(4) = 8$
10	$\int_{2}^{10} f(t)\,dt = 0$
12	$\int_{2}^{12} f(t)\,dt = (\frac{1}{2})(-4)(2) = -4$

abs max $\rightarrow x = 6$ abs min $\Rightarrow x = -2$

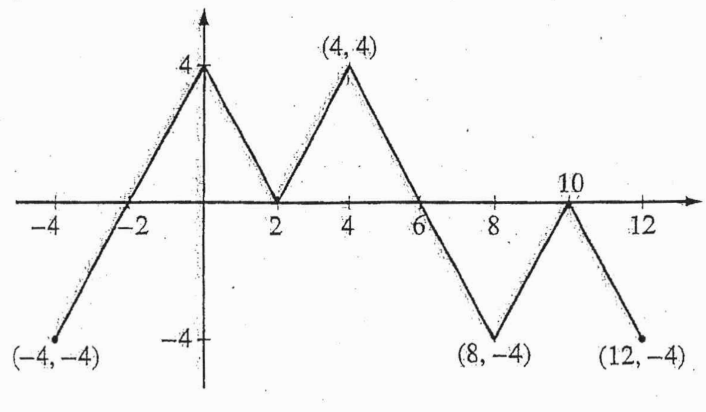
(d) For $-4 \le x \le 12$, find all intervals for which $g(x) \le 0$.

$\int_{2}^{x} f(t)\,dt \le 0$

$(10, 12)$

$(-4, 2)$

NO CALCULATOR ALLOWED



Graph of f

3. The figure above shows the graph of the piecewise-linear function f. For $-4 \le x \le 12$, the function g is defined by $g(x) = \int_2^x f(t)\,dt$.

(a) Does g have a relative minimum, a relative maximum, or neither at $x = 10$? Justify your answer.

$g'(x) = f(x)$

Since $g'(x) = f(x)$, the graph of g has a relative maximum at $x=10$ because the graph of f increaces before $x = 10$ and decreaces after $x=10$ and $x=10$ is a critical point.

(b) Does the graph of g have a point of inflection at $x = 4$? Justify your answer.

$g''(x) = f'(x)$

Since $g''(x) = f'(x)$, the graph of g has an inflection point at $x=4$ because the graph of f increaces before $x= 4$ and decreaces after $x= 4$.

Do not write beyond this border.

Continue problem 3 on page 15.

NO CALCULATOR ALLOWED

(c) Find the absolute minimum value and the absolute maximum value of g on the interval $-4 \le x \le 12$. Justify your answers.

$g'(x) = f(x) = 0$

$x = -2, 2, 6, 10$

absolute maximum at $x = 10$ and absolute minimum at $x = 2$

. The absolute value for both extremas are 0 since

It is found by $g'(x) = f(x) = 0$.

(d) For $-4 \le x \le 12$, find all intervals for which $g(x) \le 0$.

$g(x)$ is decreasing when $g'(x)$ is ≤ 0 and $g'(x)$ is ≤ 0

since $g'(x) = f(x)$ and $g''(x) = f'(x)$, we know that

$6 < x < 10$ and $10 < x < 12$ are the only intervals

where both $g(x)$ and $g''(x)$, which is $f(x)$ and $f'(x)$,

are decreasing (having the same sign).

Do not write beyond this border.

Question 3

Overview

In this problem students were given the graph of f, a piecewise-linear function defined on the interval $[-4, 12]$. A second function g is defined by $g(x) = \int_2^x f(t)\, dt$. In part (a) students needed to determine whether g has a relative minimum, a relative maximum, or neither at $x = 10$, and justify their answer. Using the Fundamental Theorem of Calculus, students needed to recognize that $g'(x) = f(x)$ for all x in the interval $[-4, 12]$. Since $g'(10) = f(10) = 0$ and $f(x) \le 0$ for $[8, 12]$, the First Derivative Test may be applied to conclude that there is no relative extremum at $x = 10$. In part (b) students needed to determine whether the graph of g has a point of inflection at $x = 4$, and justify their answer. Since $g'(x) = f(x)$, the graph of g has a point of inflection at $x = 4$ because f changes from increasing to decreasing at $x = 4$. In part (c) students needed to find the absolute minimum value and the absolute maximum value of g on $[-4, 12]$. Since $g'(x) = f(x)$, students were expected to find relative extrema of g by identifying x-values where f changes sign. The absolute extrema occur either at the endpoints of the interval or at the relative extrema. By comparing the values of g at the four candidate x-values, students choose and justify the absolute extrema. Properties of the definite integral and the relation of the definite integral to accumulated area must be used to find the values of g. In part (d) students needed to find all intervals in $[-4, 12]$ for which $g(x) \le 0$. This part also required properties of the definite integral and the relation of the definite integral to accumulated area.

Sample: 3A
Score: 9

The response earned all 9 points. The student earned the $g'(x) = f(x)$ point in part (a). In part (a) the student earned the point with justification "$g'(x) = f(x)$ does not change sign at this point." In part (b) the student earned the point with justification "$f'(x) = g''(x)$ does change sign at $x = 4$." In part (c) the student identifies the absolute minimum and absolute maximum values with a candidates test that uses the necessary critical points. In part (d) the student gives the two correct closed intervals.

Sample: 3B
Score: 6

The response earned 6 points: 1 point for $g'(x) = f(x)$, 1 point in part (a), no points in part (b), 3 points in part (c), and 1 point in part (d). The student earned the $g'(x) = f(x)$ point in part (a). In part (a) the student earned the point with justification "$g'(x)$ does not change from pos to neg or neg to pos at $x = 10$." In part (b) the student gives the correct answer but includes an incorrect statement that $g''(4) = 0$. In part (c) the student earned the first 2 points. The student does not identify the absolute minimum as -8 or the absolute maximum as 8. The student earned 1 of the 2 answers with justification points. In part (d) the student does not include the endpoints of the intervals, so 1 point was earned.

Sample: 3C
Score: 3

The response earned 3 points: 1 point for $g'(x) = f(x)$, no points in part (a), 1 point in part (b), 1 point in part (c), and no points in part (d). The student earned the $g'(x) = f(x)$ point in part (a). In part (a) the student has

Question 3 (continued)

an incorrect answer. In part (b) the student's work is correct. In part (c) the student earned the first point by identifying $x = -2$ and $x = 6$ in the second line. The student earned no other points. In part (d) the student has an incorrect interval $(6, 10)$ that has no values where $g(x) \le 0$.