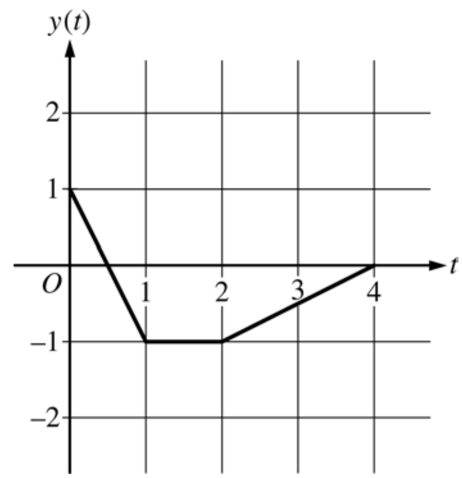
AP® CALCULUS BC
2016 SCORING GUIDELINES

Question 2



At time t, the position of a particle moving in the xy-plane is given by the parametric functions $(x(t), y(t))$, where $\frac{dx}{dt} = t^2 + \sin\left(3t^2\right)$. The graph of y, consisting of three line segments, is shown in the figure above. At $t = 0$, the particle is at position $(5, 1)$.

(a) Find the position of the particle at $t = 3$.

(b) Find the slope of the line tangent to the path of the particle at $t = 3$.

(c) Find the speed of the particle at $t = 3$.

(d) Find the total distance traveled by the particle from $t = 0$ to $t = 2$.

(a) $x(3) = x(0) + \int_0^3 x'(t)\,dt = 5 + 9.377035 = 14.377$

$y(3) = -\frac{1}{2}$

The position of the particle at $t = 3$ is $(14.377, -0.5)$.

$3 : \begin{cases} 1 : \text{integral} \\ 1 : \text{uses initial condition} \\ 1 : \text{answer} \end{cases}$

(b) $\text{Slope} = \frac{y'(3)}{x'(3)} = \frac{0.5}{9.956376} = 0.05$

1 : slope

(c) $\text{Speed} = \sqrt{(x'(3))^2 + (y'(3))^2} = 9.969$ (or 9.968)

$2 : \begin{cases} 1 : \text{expression for speed} \\ 1 : \text{answer} \end{cases}$

(d) $$\begin{aligned} \text{Distance} &= \int_0^2 \sqrt{(x'(t))^2 + (y'(t))^2}\,dt \\ &= \int_0^1 \sqrt{(x'(t))^2 + (-2)^2}\,dt + \int_1^2 \sqrt{(x'(t))^2 + 0^2}\,dt \\ &= 2.237871 + 2.112003 = 4.350 \text{ (or } 4.349) \end{aligned}$$

$3 : \begin{cases} 1 : \text{expression for distance} \\ 1 : \text{integrals} \\ 1 : \text{answer} \end{cases}$

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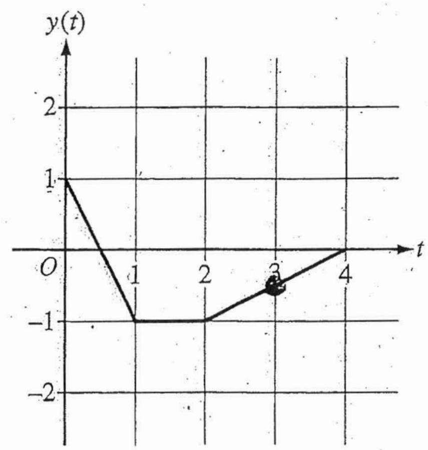
2A1



2. At time t, the position of a particle moving in the xy-plane is given by the parametric functions $(x(t), y(t))$, where $\frac{dx}{dt} = t^2 + \sin(3t^2)$. The graph of y, consisting of three line segments, is shown in the figure above. At $t = 0$, the particle is at position $(5, 1)$.

(a) Find the position of the particle at $t = 3$.

$t=3 \quad y = -0.5$

$\boxed{t=3 \quad (14.377, -0.5)}$

$\int_0^3 (t^2 + \sin(3t^2))\,dt = x(3) - x(0)$

$5 + \int_0^3 (t^2 + \sin(3t^2))\,dt = 14.377$

(b) Find the slope of the line tangent to the path of the particle at $t = 3$.

$t=3 \rightarrow$

$y = -0.5$

$x = 14.377$

$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\frac{y(4)-y(2)}{4-2}}{3^2 + \sin(3(3^2))}$

$\frac{dy}{dx} = \frac{\frac{0+1}{2}}{9 + \sin 27} = \frac{1}{2} \cdot \frac{1}{9 + \sin 27} = \boxed{0.050}$

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2A2

(c) Find the speed of the particle at $t = 3$.

$$SPEED = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$$

$$SPEED = \sqrt{(9 + \sin 27)^2 + \left(\frac{1}{2}\right)^2}$$

$$SPEED \text{ @ } t=3 \Rightarrow \boxed{9.969 \text{ u/time}}$$

(d) Find the total distance traveled by the particle from $t = 0$ to $t = 2$.

$$TD = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$$

$$TD = \int_0^1 \sqrt{(t^2 + \sin(3t^2))^2 + (-2)^2} \; +$$

$$\int_1^2 \sqrt{(t^2 + \sin(3t^2))^2 + (0)^2}$$

$$\boxed{TD = 4.350 \text{ units}}$$

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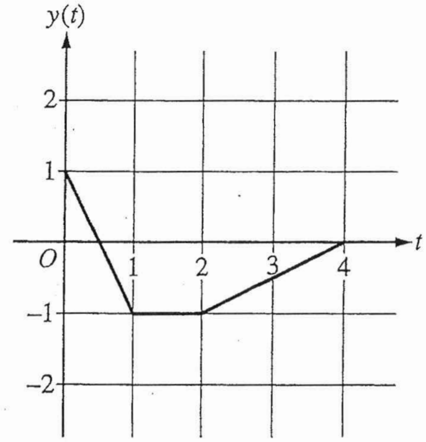
2B1



2. At time t, the position of a particle moving in the xy-plane is given by the parametric functions $(x(t), y(t))$, where $\frac{dx}{dt} = t^2 + \sin(3t^2)$. The graph of y, consisting of three line segments, is shown in the figure above. At $t = 0$, the particle is at position $(5, 1)$.

(a) Find the position of the particle at $t = 3$.

$t = 3 \rightarrow y = -0.5$ from graph

$x = \int_0^3 (t^2 + \sin(3t^2))\,dt + 5$

$x = 14.377$

$(14.377, \ -0.5)$

(b) Find the slope of the line tangent to the path of the particle at $t = 3$.

$\frac{dy}{dx} = \frac{dy}{dt} / \frac{dx}{dt}$

$\left.\frac{dy}{dt}\right|_{t=3} = \frac{0-(-1)}{4-2} = \frac{1}{2}$

$\left.\frac{dx}{dt}\right|_{x=3} = 3^2 + \sin(3 \cdot 3^2) = 9.956376$

$\frac{dy}{dx} = \frac{9.956376}{\frac{1}{2}} = 19.913$

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2B2

(c) Find the speed of the particle at $t = 3$.

$$s\Big|_{t=3} = \sqrt{\left(\frac{dx}{dt}\Big|_{t=3}\right)^2 + \left(\frac{dy}{dt}\Big|_{t=3}\right)^2}$$

$$= \sqrt{(4.956376)^2 + (3)^2}$$

$$= 4.969$$

(d) Find the total distance traveled by the particle from $t = 0$ to $t = 2$.

$$d = \int_0^2 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$$d = \int_0^1 \sqrt{(t + \sin(3t^2))^2 + (-2t+1)^2}\, dt + \int_1^2 \sqrt{(t+\sin(3t^2))^2 + (-1)^2}\, dt$$

$$d = 3.377$$

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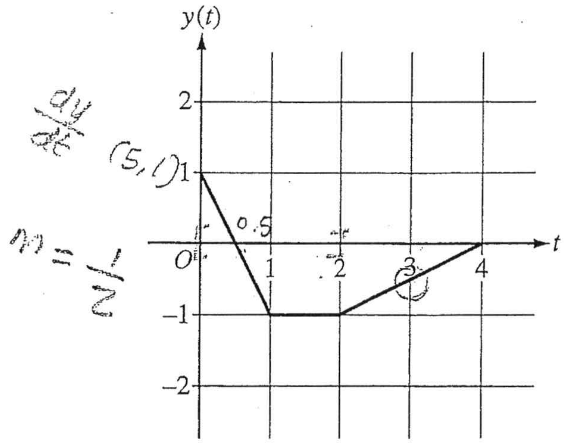

2. At time t, the position of a particle moving in the xy-plane is given by the parametric functions $(x(t), y(t))$, where $\frac{dx}{dt} = t^2 + \sin(3t^2)$. The graph of y, consisting of three line segments, is shown in the figure above. At $t = 0$, the particle is at position $(5, 1)$.

(a) Find the position of the particle at $t = 3$.

$(5,1)$ $t=0$

$\frac{dx}{dt} = (3)^2 + \sin(27)$

$= 9.956$

$x = 5 + 9.956$ $\quad y = 1 + \frac{1}{2}$

$\boxed{(14.956, 1.5)}$

(b) Find the slope of the line tangent to the path of the particle at $t = 3$.

$\frac{dx}{dt} = t^2 + \sin(3t^2)$

$= 9 + \sin(27)$

$\frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{dy}{dx}$

$\frac{dy}{dx} = \frac{0.5}{9 + \sin 27} = \boxed{0.050}$

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2C2

(c) Find the speed of the particle at $t = 3$.

$$\text{speed} = \sqrt{(x'(t))^2 + (y'(t))^2}$$
$$= \sqrt{(9 + \sin 27)^2 + (0.5)^2}$$
$$= 9.969$$

(d) Find the total distance traveled by the particle from $t = 0$ to $t = 2$.

$$\int_0^2 (t^2 + \sin(3t^2)) + \int_0^2 y(t)$$

$$2.960 + (0.25 + 0.25 + 1)$$
$$2.960 + 1.50$$
$$= \boxed{3.460}$$

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Question 2

Overview

In this problem students were given information about the motion of a particle in the *xy*-plane. The position of the particle is defined as $(x(t),\, y(t)),$ where $\frac{dx}{dt} = t^2 + \sin\left(3t^2\right).$ The graph of y, consisting of three line segments, and the position of the particle at $t = 0$ were also given. In part (a) students needed to find the position of the particle at $t = 3.$ Students were expected to evaluate $5 + \int_0^3 \left(t^2 + \sin\left(3t^2\right)\right) dt$ using the calculator to find $x(3).$ The value of $y(3)$ can be read from the graph. In part (b) students needed to compute the slope of the line tangent to the particle's path at $t = 3$ by evaluating $\frac{y'(3)}{x'(3)}.$ The value for $y'(3)$ is found by computing the slope of the line segment from $t = 2$ to $t = 4,$ and the value for $x'(3)$ is found using the calculator. In part (c) students needed to find the speed of the particle at $t = 3.$ Students were expected to write the expression for speed, $\sqrt{(x'(3))^2 + (y'(3))^2}$, and use the calculator to compute this value. In part (d) students needed to find the total distance traveled from $t = 0$ to $t = 2.$ Although the general formula for total distance traveled is $\int_0^2 \sqrt{(x'(t))^2 + (y'(t))^2}\, dt,$ students were expected to write the total distance traveled as the sum of two integrals since $y'(t) = -2$ from $t = 0$ to $t = 1,$ and $y'(t) = 0$ from $t = 1$ to $t = 2.$ The expression $\int_0^1 \sqrt{(x'(t))^2 + (-2)^2}\, dt + \int_1^2 \sqrt{(x'(t))^2 + (0)^2}\, dt$ is evaluated using the calculator.

Sample: 2A
Score: 9

The response earned all 9 points.

Sample: 2B
Score: 6

The response earned 6 points: 3 points in part (a), no points in part (b), 2 points in part (c), and 1 point in part (d). In part (a) the student's work is correct. In part (b) the student calculates $\frac{dx}{dy}$ instead of $\frac{dy}{dx}$ and did not earn the point. In part (c) the student's work is correct. In part (d) the student presents a single integral for total distance traveled and earned the first point. The student attempts to present the total distance traveled as the sum of two integrals. The $\frac{dy}{dt}$ in each integrand is incorrect. The student did not earn any other points.

Sample: 2C
Score: 3

The response earned 3 points: no points in part (a), 1 point in part (b), 2 points in part (c), and no points in part (d). In part (a) the student evaluates $x'(3)$ instead of $x(3).$ The student was not eligible to earn any points. In parts (b) and (c) the student's work is correct. In part (d) the student does not present a correct integral expression for total distance traveled. The student was not eligible for any points.