For \( t \geq 0 \), a particle moves along the \( x \)-axis. The velocity of the particle at time \( t \) is given by 
\[
v(t) = 1 + 2 \sin \left( \frac{t^2}{2} \right).
\]
The particle is at position \( x = 2 \) at time \( t = 4 \).

(a) At time \( t = 4 \), is the particle speeding up or slowing down?

(b) Find all times \( t \) in the interval \( 0 < t < 3 \) when the particle changes direction. Justify your answer.

(c) Find the position of the particle at time \( t = 0 \).

(d) Find the total distance the particle travels from time \( t = 0 \) to time \( t = 3 \).

(a) \( v(4) = 2.978716 > 0 \)
\[
v'(4) = -1.164000 < 0
\]
The particle is slowing down since the velocity and acceleration have different signs.

(b) \( v(t) = 0 \Rightarrow t = 2.707468 \)
\[
v(t) \text{ changes from positive to negative at } t = 2.707.
\]
Therefore, the particle changes direction at this time.

(c) \[
x(0) = x(4) + \int_{4}^{0} v(t) \, dt
\]
\[
= 2 + (-5.815027) = -3.815
\]

(d) \[
\text{Distance} = \int_{0}^{3} |v(t)| \, dt = 5.301
\]
2. For \( t \geq 0 \), a particle moves along the \( x \)-axis. The velocity of the particle at time \( t \) is given by
\[
v(t) = 1 + 2\sin \left( \frac{t^2}{2} \right).
\] The particle is at position \( x = 2 \) at time \( t = 4 \).

(a) At time \( t = 4 \), is the particle speeding up or slowing down?

\[
v(4) = 1 + 2\sin \left( \frac{4^2}{2} \right)
\]
\[
v(4) = 2.979
\]
\[
v'(t) = 2\cos \left( \frac{t^2}{2} \right) (t)
\]
\[
v'(4) = 2\cos \left( \frac{4^2}{2} \right) (4)
\]
\[
v'(4) = -1.64.
\]

slowing down because \( v(4) \) is positive and \( v'(4) \) is negative.

(b) Find all times \( t \) in the interval \( 0 < t < 3 \) when the particle changes direction. Justify your answer.

\[
1 + 2\sin \left( \frac{t^2}{2} \right) = 0
\]
\[
t = 2.707
\]

The particle changes direction one time at
\( t = 2.707 \) because \( v(t) = 0 \) and \( v(t) \) changes from positive to negative.
(c) Find the position of the particle at time $t = 0$.

\[ 2^4 \int_0^4 v(t) \, dt = 3.815 \]

(d) Find the total distance the particle travels from time $t = 0$ to time $t = 3$.

\[ \int_0^3 |v(t)| \, dt = 5.301 \]
2. For \( t \geq 0 \), a particle moves along the \( x \)-axis. The velocity of the particle at time \( t \) is given by 
\[
v(t) = 1 + 2\sin\left(\frac{t^2}{2}\right)\]
The particle is at position \( x = 2 \) at time \( t = 4 \).

(a) At time \( t = 4 \), is the particle speeding up or slowing down?

\[
\begin{align*}
v(4) &= 1 + 2\sin\left(\frac{4^2}{2}\right) = 2.979 \\
a(4) &= 2\cos\left(\frac{4^2}{2}\right) = -1.144
\end{align*}
\]

The particle is slowing down at \( t = 4 \) because \( a(4) \) and \( v(4) \) have different signs.

(b) Find all times \( t \) in the interval \( 0 < t < 3 \) when the particle changes direction. Justify your answer.

\[
v(t) = 0
\]
\[
1 + 2\sin\left(\frac{t^2}{2}\right) = 0
\]
\[
t = 2.707468
\]

The particle changes direction at \( t = 2.707 \) because the velocity changes sign at that time.
(c) Find the position of the particle at time $t = 0$.

\[ \text{position} = \int v(t) \, dt \]

\[ p = \int 1 + 2 \sin \left( \frac{t^2}{2} \right) \, dt \]

\[ u = \frac{1}{2} t^2 \]

\[ \frac{1}{u} \, du = \frac{1}{2} \, dt \]

\[ p = \frac{1}{1} \int 1 + 2 \sin(u) \, du \]

\[ \frac{1}{u} \left( u - 2 \cos \left( \frac{t^2}{2} \right) \right) + C \]

\[ p = \frac{1}{t} \left( t - 2 \cos \left( \frac{t^2}{2} \right) \right) \]

(d) Find the total distance the particle travels from time $t = 0$ to time $t = 3$.

\[ \text{total distance} = \int_0^3 |v(t)| \, dt \]

\[ \int_0^3 \left| 1 + 2 \sin \left( \frac{t^2}{2} \right) \right| \, dt = 5.301 \]
2. For $t \geq 0$, a particle moves along the $x$-axis. The velocity of the particle at time $t$ is given by

$$v(t) = 1 + 2\sin\left(\frac{t^2}{2}\right).$$

The particle is at position $x = 2$ at time $t = 4$.

(a) At time $t = 4$, is the particle speeding up or slowing down?

**At time $t = 4$ the particle is slowing down**

(b) Find all times $t$ in the interval $0 < t < 3$ when the particle changes direction. Justify your answer.

$$v(t) = 1 + 2\sin\left(\frac{t^2}{2}\right) = 0$$

$$v'(t) = t + 2t^2\cos\left(\frac{t^2}{2}\right) = 0$$

**Particle changes direction at**

$t = 2.407$ and $t = 1.375$
(c) Find the position of the particle at time $t = 0$.

\[
\int_0^t \left( 1 + 2 \sin \left( \frac{t^2}{2} \right) \right) \, dt = -\int_0^t \left( 1 + 2 \sin \left( \frac{t^2}{2} \right) \right) \, dt = -5.815
\]

(d) Find the total distance the particle travels from time $t = 0$ to time $t = 3$.

\[
\int_0^3 \left| 1 + 2 \sin \left( \frac{t^2}{2} \right) \right| \, dt = 5.301
\]
Question 2

Overview

In this problem students were given information about a particle moving along the x-axis for time $t \geq 0$. The velocity of the particle is given as a trigonometric function, and the particle is at position $x = 2$ at time $t = 4$. In part (a) students needed to conclude that the particle is slowing down at $t = 4$ because $v(4)$ and $v'(4)$ have different signs. In part (b) students needed to determine when the particle changes direction in the interval $0 < t < 3$, and justify their answer. This required use of the calculator to solve $v(t) = 0$ on $0 < t < 3$. In part (c) students needed to apply the Fundamental Theorem of Calculus to find the position of the particle at time $t = 0$; i.e., $x(0) = x(4) - \int_0^4 v(t) \, dt$. The expression is evaluated using the calculator. In part (d) students needed to find the total distance the particle travels from $t = 0$ to $t = 3$. Students were expected to set up and evaluate $\int_0^3 |v(t)| \, dt$ (or an appropriate sum of definite integrals) using the calculator.

Sample: 2A

Score: 9

The response earned all 9 points.

Sample: 2B

Score: 6

The response earned 6 points: 2 points in part (a), 2 points in part (b), no points in part (c), and 2 points in part (d). In part (a) the student’s work is correct. The student is not required to explicitly state that $a(4) = v'(4)$. In part (b) the student’s work is correct. In part (c) the student is not working with a definite integral and did not earn the first point. The student was not eligible to earn the other 2 points. In part (d) the student’s work is correct.

Sample: 2C

Score: 3

The response earned 3 points: no points in part (a), no points in part (b), 1 point in part (c), and 2 points in part (d). In part (a) the student has a conclusion without a reason, so no points were earned. In part (b) the student reports two incorrect values of $t$. The student did not earn the first point and was not eligible for the second point. In part (c) the student earned the first point for a correct definite integral. The student does not use the initial condition and was not eligible to earn the other 2 points. In part (d) the student’s work is correct.