AP® Physics C: Electricity and Magnetism
2014 Free-Response Questions

TABLE OF INFORMATION, EFFECTIVE 2012

CONSTANTS AND CONVERSION FACTORS

Proton mass, $m_p = 1.67 \times 10^{-27}$ kg	Electron charge magnitude, $e = 1.60 \times 10^{-19}$ C
Neutron mass, $m_n = 1.67 \times 10^{-27}$ kg	1 electron volt, $1 \text{ eV} = 1.60 \times 10^{-19}$ J
Electron mass, $m_e = 9.11 \times 10^{-31}$ kg	Speed of light, $c = 3.00 \times 10^8$ m/s
Avogadro's number, $N_0 = 6.02 \times 10^{23}$ mol^{-1}	Universal gravitational constant, $G = 6.67 \times 10^{-11}$ m^3/kg•s^2
Universal gas constant, $R = 8.31$ J/(mol•K)	Acceleration due to gravity at Earth's surface, $g = 9.8$ m/s^2
Boltzmann's constant, $k_B = 1.38 \times 10^{-23}$ J/K	

1 unified atomic mass unit,	$1 \text{ u} = 1.66 \times 10^{-27} \text{ kg} = 931 \text{ MeV}/c^2$
Planck's constant,	$h = 6.63 \times 10^{-34} \text{ J•s} = 4.14 \times 10^{-15} \text{ eV•s}$
	$hc = 1.99 \times 10^{-25} \text{ J•m} = 1.24 \times 10^3 \text{ eV•nm}$
Vacuum permittivity,	$\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/\text{N•m}^2$
Coulomb's law constant,	$k = 1/4\pi\epsilon_0 = 9.0 \times 10^9 \text{ N•m}^2/\text{C}^2$
Vacuum permeability,	$\mu_0 = 4\pi \times 10^{-7}$ (T•m)/A
Magnetic constant,	$k' = \mu_0/4\pi = 1 \times 10^{-7}$ (T•m)/A
1 atmosphere pressure,	$1 \text{ atm} = 1.0 \times 10^5 \text{ N/m}^2 = 1.0 \times 10^5 \text{ Pa}$

UNIT SYMBOLS									
	meter,	m	mole,	mol	watt,	W	farad,	F
	kilogram,	kg	hertz,	Hz	coulomb,	C	tesla,	T
	second,	s	newton,	N	volt,	V	degree Celsius,	°C
	ampere,	A	pascal,	Pa	ohm,	Ω	electron-volt,	eV
	kelvin,	K	joule,	J	henry,	H		

PREFIXES

Factor	Prefix	Symbol
10^9	giga	G
10^6	mega	M
10^3	kilo	k
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p

VALUES OF TRIGONOMETRIC FUNCTIONS FOR COMMON ANGLES

θ	0°	30°	37°	45°	53°	60°	90°
$\sin\theta$	0	$1/2$	$3/5$	$\sqrt{2}/2$	$4/5$	$\sqrt{3}/2$	1
$\cos\theta$	1	$\sqrt{3}/2$	$4/5$	$\sqrt{2}/2$	$3/5$	$1/2$	0
$\tan\theta$	0	$\sqrt{3}/3$	$3/4$	1	$4/3$	$\sqrt{3}$	∞

The following conventions are used in this exam.

I. Unless otherwise stated, the frame of reference of any problem is assumed to be inertial.

II. The direction of any electric current is the direction of flow of positive charge (conventional current).

III. For any isolated electric charge, the electric potential is defined as zero at an infinite distance from the charge.

ADVANCED PLACEMENT PHYSICS C EQUATIONS, EFFECTIVE 2012

MECHANICS

$v = v_0 + at$

$x = x_0 + v_0 t + \frac{1}{2}at^2$

$v^2 = v_0^2 + 2a(x - x_0)$

$\sum \mathbf{F} = \mathbf{F}_{net} = m\mathbf{a}$

$\mathbf{F} = \frac{d\mathbf{p}}{dt}$

$\mathbf{J} = \int \mathbf{F}\,dt = \Delta\mathbf{p}$

$\mathbf{p} = m\mathbf{v}$

$F_{fric} \le \mu N$

$W = \int \mathbf{F} \cdot d\mathbf{r}$

$K = \frac{1}{2}mv^2$

$P = \frac{dW}{dt}$

$P = \mathbf{F} \cdot \mathbf{v}$

$\Delta U_g = mgh$

$a_c = \frac{v^2}{r} = \omega^2 r$

$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$

$\sum \boldsymbol{\tau} = \boldsymbol{\tau}_{net} = I\boldsymbol{\alpha}$

$I = \int r^2 dm = \sum mr^2$

$\mathbf{r}_{cm} = \sum m\mathbf{r} / \sum m$

$v = r\omega$

$\mathbf{L} = \mathbf{r} \times \mathbf{p} = I\boldsymbol{\omega}$

$K = \frac{1}{2}I\omega^2$

$\omega = \omega_0 + \alpha t$

$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$

a = acceleration
F = force
f = frequency
h = height
I = rotational inertia
J = impulse
K = kinetic energy
k = spring constant
ℓ = length
L = angular momentum
m = mass
N = normal force
P = power
p = momentum
r = radius or distance
$\mathbf{r}$ = position vector
T = period
t = time
U = potential energy
v = velocity or speed
W = work done on a system
x = position
μ = coefficient of friction
θ = angle
τ = torque
ω = angular speed
α = angular acceleration
ϕ = phase angle

$\mathbf{F}_s = -k\mathbf{x}$

$U_s = \frac{1}{2}kx^2$

$x = x_{max}\cos(\omega t + \phi)$

$T = \frac{2\pi}{\omega} = \frac{1}{f}$

$T_s = 2\pi\sqrt{\frac{m}{k}}$

$T_p = 2\pi\sqrt{\frac{\ell}{g}}$

$\mathbf{F}_G = -\frac{Gm_1m_2}{r^2}\hat{\mathbf{r}}$

$U_G = -\frac{Gm_1m_2}{r}$

ELECTRICITY AND MAGNETISM

$F = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r^2}$

$\mathbf{E} = \frac{\mathbf{F}}{q}$

$\oint \mathbf{E} \cdot d\mathbf{A} = \frac{Q}{\epsilon_0}$

$E = -\frac{dV}{dr}$

$V = \frac{1}{4\pi\epsilon_0}\sum_i \frac{q_i}{r_i}$

$U_E = qV = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r}$

$C = \frac{Q}{V}$

$C = \frac{\kappa\epsilon_0 A}{d}$

$C_p = \sum_i C_i$

$\frac{1}{C_s} = \sum_i \frac{1}{C_i}$

$I = \frac{dQ}{dt}$

$U_c = \frac{1}{2}QV = \frac{1}{2}CV^2$

$R = \frac{\rho\ell}{A}$

$\mathbf{E} = \rho\mathbf{J}$

$I = Nev_d A$

$V = IR$

$R_s = \sum_i R_i$

$\frac{1}{R_p} = \sum_i \frac{1}{R_i}$

$P = IV$

$\mathbf{F}_M = q\mathbf{v} \times \mathbf{B}$

A = area
B = magnetic field
C = capacitance
d = distance
E = electric field
$\mathcal{E}$ = emf
F = force
I = current
J = current density
L = inductance
ℓ = length
n = number of loops of wire per unit length
N = number of charge carriers per unit volume
P = power
Q = charge
q = point charge
R = resistance
r = distance
t = time
U = potential or stored energy
V = electric potential
v = velocity or speed
ρ = resistivity
ϕ_m = magnetic flux
κ = dielectric constant

$\oint \mathbf{B} \cdot d\boldsymbol{\ell} = \mu_0 I$

$d\mathbf{B} = \frac{\mu_0}{4\pi}\frac{I\,d\boldsymbol{\ell} \times \mathbf{r}}{r^3}$

$\mathbf{F} = \int I\,d\boldsymbol{\ell} \times \mathbf{B}$

$B_s = \mu_0 nI$

$\phi_m = \int \mathbf{B} \cdot d\mathbf{A}$

$\mathcal{E} = \oint \mathbf{E} \cdot d\boldsymbol{\ell} = -\frac{d\phi_m}{dt}$

$\mathcal{E} = -L\frac{dI}{dt}$

$U_L = \frac{1}{2}LI^2$

GEOMETRY AND TRIGONOMETRY

Rectangle

$A = bh$

Triangle

$A = \frac{1}{2}bh$

Circle

$A = \pi r^2$

$C = 2\pi r$

Rectangular Solid

$V = \ell wh$

Cylinder

$V = \pi r^2 \ell$

$S = 2\pi r\ell + 2\pi r^2$

Sphere

$V = \frac{4}{3}\pi r^3$

$S = 4\pi r^2$

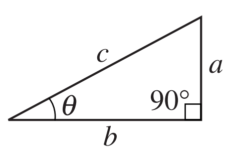
Right Triangle

$a^2 + b^2 = c^2$

$\sin\theta = \frac{a}{c}$

$\cos\theta = \frac{b}{c}$

$\tan\theta = \frac{a}{b}$



A = area
C = circumference
V = volume
S = surface area
b = base
h = height
ℓ = length
w = width
r = radius

CALCULUS

$$\frac{df}{dx} = \frac{df}{du}\frac{du}{dx}$$

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$$

$$\frac{d}{dx}\left(e^x\right) = e^x$$

$$\frac{d}{dx}(\ln x) = \frac{1}{x}$$

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1},\ n \neq -1$$

$$\int e^x\,dx = e^x$$

$$\int \frac{dx}{x} = \ln|x|$$

$$\int \cos x\,dx = \sin x$$

$$\int \sin x\,dx = -\cos x$$

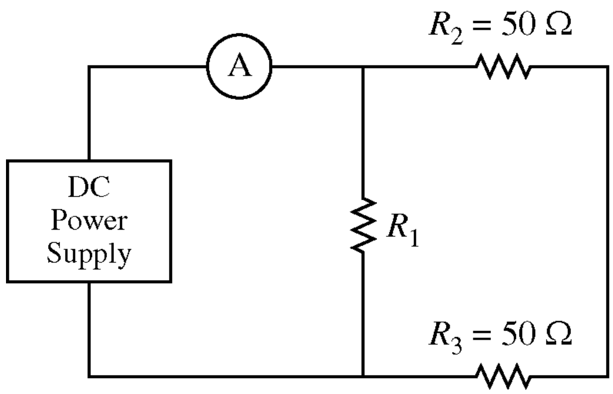
PHYSICS C: ELECTRICITY AND MAGNETISM

SECTION II

Time—45 minutes

3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.

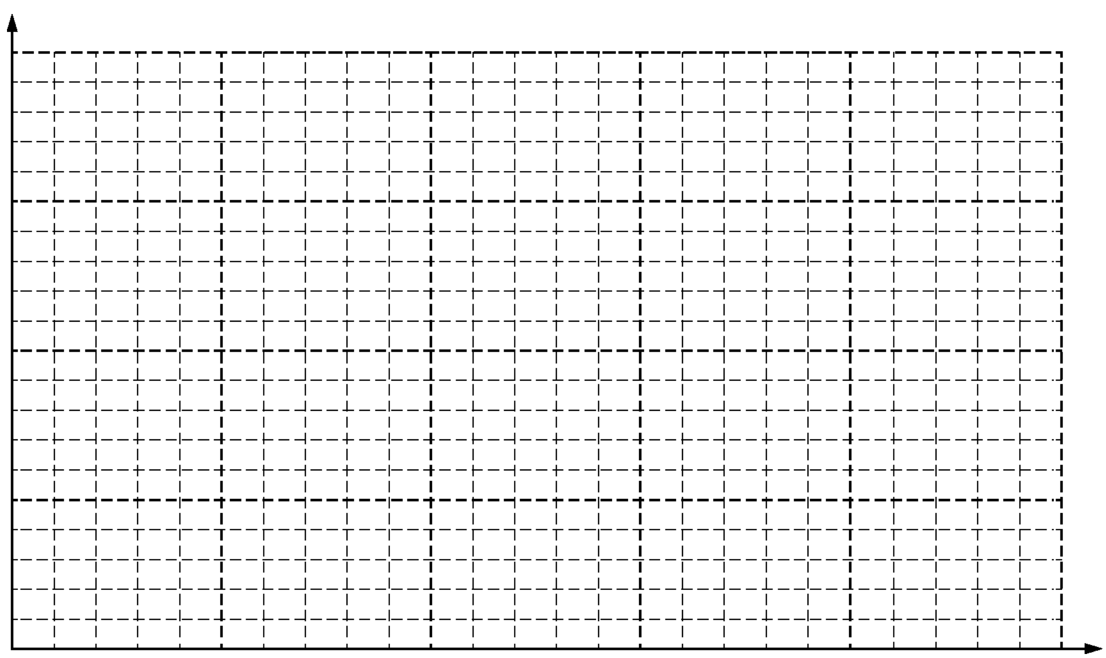


E&M. 1.

Physics students are analyzing the circuit above. A variable DC power supply is connected to an ammeter and three resistors. The resistances of two of the resistors are known to be $R_2 = R_3 = 50\ \Omega$, but the resistance of the third resistor is unknown. The students collect data on the potential difference across the power supply and the current measured by the ammeter, as follows.

Potential Difference (V)	2	4	6	8	10
Current (mA)	40	55	97	138	155

(a) On the grid below, plot the data points for the current as a function of the potential difference. Clearly scale and label all axes, including units if appropriate. Draw a straight line that best represents the data.



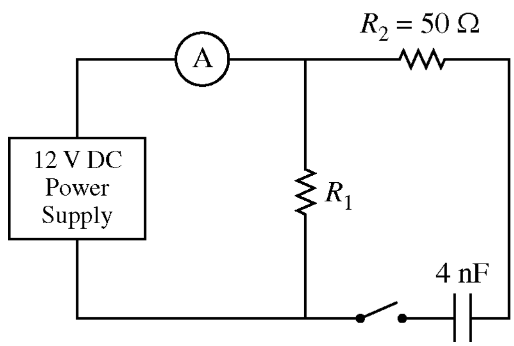
(b) Using the straight line from part (a), calculate the total resistance of the three-resistor combination.

(c) Calculate the value of R_1.

The power supply is now fixed at 12 V.

(d) Calculate the current through R_2.

(e) Resistor 3 is now removed and replaced by an open switch in series with an uncharged 4 nF capacitor, as shown below. The power supply is still fixed at 12 V.



i. Calculate the current in R_2 immediately after the switch is closed.

ii. A long time after the switch is closed, will the magnitude of the current in R_2 be greater than, less than, or equal to the current through R_2 found in part (d) ?

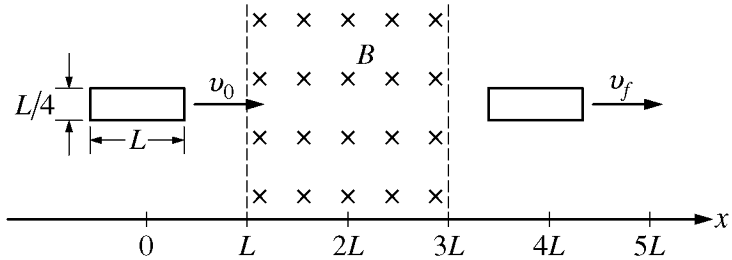
__ Greater than __ Less than __ Equal to

Justify your answer.

(f) The 4 nF capacitor is replaced with an uncharged 10 nF capacitor. Will the magnitude of the current in R_2 immediately after the switch is closed be greater than, less than, or equal to the current in part (e)i?

__ Greater than __ Less than __ Equal to

Justify your answer.



E&M. 2.

The rectangular loop of wire shown on the left in the figure above has mass M, length L, width $L/4$, and resistance R. It is initially moving to the right at constant speed v_0, with no net force acting on it. At time $t = 0$ the loop enters a region of length $2L$ that contains a uniform magnetic field of magnitude B directed into the page. The loop emerges from the field at time t_f with final speed v_f. Express all algebraic answers to the following in terms of M, L, R, B, v_0, and fundamental constants, as appropriate.

(a) Let x represent the position of the right end of the loop. Place a check mark in the appropriate box in each column in the table below to indicate whether the speed of the loop increases, decreases, or stays the same as the loop moves to the right.

	Position of Right End of Loop			
Speed of Loop	$L < x < 2L$	$2L < x < 3L$	$3L < x < 4L$	$4L < x < 5L$
Increases				
Decreases				
Stays the same				

(b) Derive an expression for the magnitude of the current induced in the loop as its right edge enters the field.

(c) What is the direction of the induced current determined in part (b) ?

____ Clockwise ____ Counterclockwise

Justify your answer.

(d) Write, but do not solve, a differential equation for the speed v as a function of time as the loop enters the field.

(e) What is the direction of the acceleration of the loop just before its left edge leaves the field?

____ Left ___ Right ____ Up ____ Down

Justify your answer.

E&M.3.

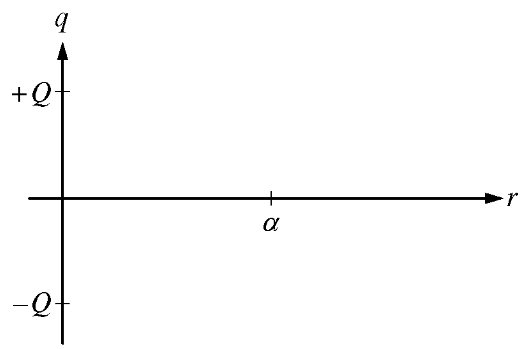
A scientist describes an electrically neutral atom with a model that consists of a nucleus that is a point particle with positive charge $+Q$ at the center of the atom and an electron volume charge density of the form

$$\rho(r) = \begin{cases} -\dfrac{\beta}{r^2} e^{-r/\alpha} & r < \alpha \\ 0 & r > \alpha \end{cases}$$

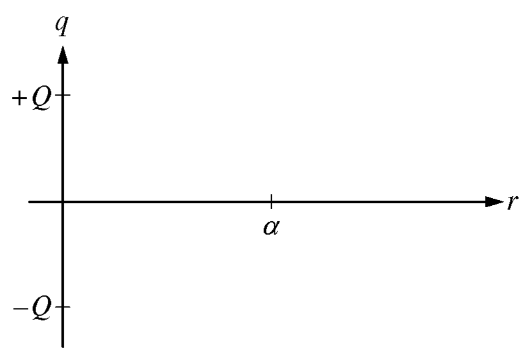
where α and β are positive constants and r is the distance from the center of the atom.

(a) On the axes below, let r stand for the radius of a Gaussian sphere. Sketch the graph for each of the following charges enclosed by the Gaussian sphere as a function of r. Explicitly label any intercepts, asymptotes, maxima, or minima with numerical values or algebraic expressions, as appropriate.

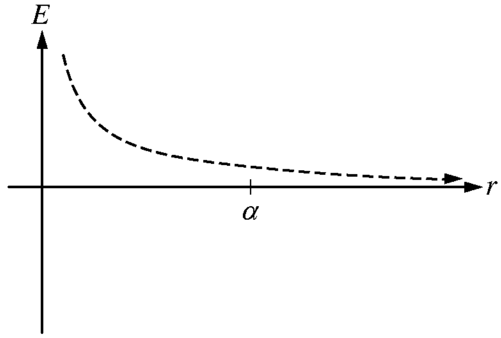
i. The nuclear charge only



ii. The electron charge only



(b) The dashed curve on the graph below represents the electric field as a function of distance r due to the positive nucleus of the atom without any electrons. The nucleus is modeled as a point particle of charge $+Q$. On the same graph, sketch the electric field as a function of distance r for the neutral atom as defined by the scientist's model, which includes the nucleus and the negative electrons surrounding it.



(c) Use Gauss's law to derive an expression for the electric field strength due to the neutral atom for the following positions in terms of $Q, \alpha, \beta, r,$ and fundamental constants, as appropriate.

i. $r > \alpha$

ii. $r < \alpha$

(d) Based on the model proposed by the scientist, what is the physical meaning of the constant α ?

STOP

END OF EXAM