Student Performance Q&A:

2012 AP® Calculus AB and Calculus BC Free-Response Questions

The following comments on the 2012 free-response questions for AP® Calculus AB and Calculus BC were written by the Chief Reader, Stephen Kokoska of Bloomsburg University in Bloomsburg, Pa. They give an overview of each free-response question and of how students performed on the question, including typical student errors. General comments regarding the skills and content that students frequently have the most problems with are included. Some suggestions for improving student performance in these areas are also provided. Teachers are encouraged to attend a College Board workshop to learn strategies for improving student performance in specific areas.

Question AB1/BC1

What was the intent of this question?

This problem involved a function W that models the temperature, in degrees Fahrenheit, of water in a tub. Values of $W(t)$ at selected times between $t = 0$ and $t = 20$ minutes are given in a table. Part (a) asked students for an approximation to the derivative of the function W at time $t = 12$ and for an interpretation of the answer. Students should have recognized this derivative as the rate at which the temperature of the water in the tub is increasing at time $t = 12,$ in degrees Fahrenheit per minute. Because $t = 12$ falls between the values presented in the table, students should have constructed a difference quotient using the temperature values across the smallest time interval containing $t = 12$ that is supported by the table. Part (b) asked students to evaluate the definite integral $\int_0^{20} W'(t)\,dt$ and to interpret the meaning of this definite integral. Students should have applied the Fundamental Theorem of Calculus and used values from the table to compute $W(20) - W(0).$ Students should have recognized this as the total change in the temperature of the water, in degrees Fahrenheit, over the 20-minute time interval. In part (c) students were given the expression for computing the average temperature of the water over the 20-minute time period and were asked to use a left Riemann sum with the four intervals given by the table to obtain a numerical approximation for this value. Students were asked whether this approximation overestimates or underestimates the actual average temperature. Students should have recognized that for a strictly increasing function, the left Riemann sum will underestimate the true value of a definite integral. In part (d) students were given the symbolic first derivative $W'(t)$ of the function W that models the temperature of the water over the interval $20 \le t \le 25,$ and were asked to use this expression to determine the temperature of the water at time $t = 25.$ This temperature is computed using the expression $W(25) = W(20) + \int_{20}^{25} W'(t)\,dt,$ where $W(20) = 71$ is given in the table.

How well did students perform on this question?

The mean score was 3.96 for AB students and 5.88 for BC students out of a possible 9 points.

In general, students performed well on this question. Students had the opportunity to earn points in each part. In part (a) most students were able to set up a difference quotient, but many had difficulty with the interpretation and the units. In part (b) most students were able to correctly use the Fundamental Theorem of Calculus. However, the interpretation required three parts: an appeal to "change," units, and the interval. In part (c) most students were able to correctly set up a left Riemann sum. Many explanations were inadequate or incorrect. Most students did fairly well in part (d).

What were common student errors or omissions?

In part (a) some students had difficulty interpreting their answers in the context of the question. Students needed to provide correct units either attached to their estimate or embedded in the interpretation. In part (b) students were able to use the Fundamental Theorem of Calculus, but many were not able to interpret the meaning of the definite integral. Many students omitted one or more of the three important parts of the interpretation. In part (c) some students incorrectly assumed that the lengths of the subintervals were the same. The most common error involved the explanation associated with an underestimate. Many students were able to correctly answer "underestimate" but gave insufficient or incorrect explanations. In part (d) some students used 0 as the lower bound on the definite integral.

Based on your experience of student responses at the AP Reading, what message would you like to send to teachers that might help them to improve the performance of their students on the exam?

In general, students were able to apply the appropriate concepts in this problem and compute correct numerical answers. However, students need practice in interpretation and communication of results. In addition, students should clearly indicate the mathematical steps to a final solution.

Question AB2

What was the intent of this question?

Students were given the graph of a region R bounded below by the x-axis, on the left by the graph of $y = \ln x$, and on the right by the graph of the line $y = 5 - x$. In part (a) students were asked to find the area of R. This required an appropriate integral setup and evaluation. Students first needed to determine the intersection point, (A, B), of the two curves. The area could then be computed by solving each expression for x in terms of y and evaluating a single integral with respect to y. Alternatively, the area could be computed by evaluating a sum of two integrals with respect to x. In the first case, students would evaluate $\int_0^B \left(5 - y - e^y\right) dy$ and in the second case, $\int_1^A \ln x \, dx + \int_A^5 (5 - x) \, dx$. Part (b) asked for an expression involving one or more integrals that gives the volume of a solid whose base is the region R and whose cross sections perpendicular to the x-axis are squares. Students should have found the cross-sectional area function in terms of x, which is $(\ln x)^2$ on the interval $1 \le x \le A$ and $(5 - x)^2$ on the interval $A \le x \le 5$. These expressions are used as the integrands for two definite integrals with the corresponding endpoints, whose sum provides the desired expression. Part (c) asked for an equation involving one or more integrals whose solution gives the value k for which the line $y = k$ divides the region R into two smaller regions of equal area. Students should have first rewritten the equations for the curves as functions of x in terms of y. Two common solutions were setting the definite integral $\int_0^k \left(5 - y - e^y\right) dy$ equal to half the value of the area computed in part (a), and $\int_0^k \left(5 - y - e^y\right) dy = \int_k^B \left(5 - y - e^y\right) dy.$

How well did students perform on this question?

The mean score was 3.09 out of a possible 9 points.

This question was difficult for many students who attempted to work in terms of the variable x, rather than y. Using this approach, with respect to x, students had to consider two separate regions. Those students working in x performed well in parts (a) and (b). However, part (c) was very challenging.

In part (a) the most common solution was in terms of x. For those students who worked in y, many had difficulty solving for x in terms of y. Students performed well in part (b), constructing two distinct, separate integrals to find the total volume. In part (c) those working in terms of y were more successful. There were some complicated yet correct solutions in terms of x.

What were common student errors or omissions?

In general, students attempted all parts of this problem. However, part (c) was left blank more often than the other two parts. Some students did not use their calculators to determine the point of intersection of the two graphs and incorrectly concluded that the point of intersection was $(4, 1)$. Many students had difficulty solving each equation for x in terms of y. For example, many students did not know the inverse function of the natural logarithm function. For the linear function, there were several sign errors when finding the inverse function.

In part (a) the most common error was the use of $\int_1^5 (5 - x - \ln x)\, dx$ for the area of the region R. Other errors included incorrect limits. Some students incorrectly used 0 as a lower bound in $\int_0^A \ln x\, dx,$ or 4 as an upper bound in $\int_A^4 (5 - x)\, dx.$

In part (b) the most common error was the use of $\int_1^5 (5 - x - \ln x)^2\, dx$ as an expression for the volume of the solid. Many students incorrectly used the constant π in their integral expressions.

In part (c) many students tried to write an equation involving one or more integrals in the variable x. Although this is certainly an acceptable method to answer part (c), most students could not construct a valid equation. There were many varied errors by students who attempted to solve the problem in this manner.

Based on your experience of student responses at the AP Reading, what message would you like to send to teachers that might help them to improve the performance of their students on the exam?

To find the area of certain regions, it is sometimes more convenient to consider x as a function of y. Students need practice solving for x in terms of y and computing areas in which there are distinct right and left boundaries. In general, students need practice determining the best or most efficient method for computing the area of a region. Students also need practice computing the volume of a solid with known cross-sectional area. The responses suggest students have difficulty computing the area of certain known geometric regions, for example, a square, at an arbitrary point x.

Question AB3/BC3

What was the intent of this question?

This problem described a function f that is defined and continuous on the interval $[-4, 3]$. The graph of f on $[-4, 3]$ is given and consists of three line segments and a semicircle. The function g is defined by $g(x) = \int_1^x f(t)\,dt$. Part (a) asked for the values of $g(2)$ and $g(-2)$. These values are given by $\int_1^2 f(t)\,dt$ and $\int_1^{-2} f(t)\,dt$, respectively, and are computed using geometry and a property of definite integrals. Part (b) asked for the values of $g'(-3)$ and $g''(-3)$, provided they exist. Students should have applied the Fundamental Theorem of Calculus to determine that $g'(-3) = f(-3)$ and $g''(-3) = f'(-3)$. Students should have used the graph provided to determine the value of f and the slope of f at the point where $x = -3$. Part (c) asked for the x-coordinate of each point where the graph of g has a horizontal tangent line. Students were then asked to classify each of these points as the location of a relative minimum, relative maximum, or neither, with justification. Students should have recognized that horizontal tangent lines for g occur where the derivative of g takes on the value 0. These values can be read from the graph. Students should have applied a sign analysis to f in order to classify these critical points. Part (d) asked for the x-coordinates of points of inflection for the graph of g on the interval $-4 < x < 3$. Students should have reasoned graphically that these occur where f changes from increasing to decreasing, or vice versa.

How well did students perform on this question?

The mean score was 2.67 for AB students and 4.29 for BC students out of a possible 9 points.

In part (a) students had difficulty associating and evaluating a definite integral with area. In part (b) many students made the correct connections between the functions f and g, but some made errors associated with the Fundamental Theorem of Calculus.

In part (c) most students were able to correctly identify the locations of the horizontal tangent lines. Most students were also able to classify these points correctly. However, the justifications were lacking in many cases. In part (d) many students were unable to determine when the graph of a function has a point of inflection.

What were common student errors or omissions?

In part (a) many students were unable to correctly evaluate either of the definite integrals. To find $g(2)$, many students used 3 as an upper bound, perhaps because this was a domain endpoint of f. Some students also had difficulty computing the area of a semicircle. When students were able to compute the correct areas, many still did not add or subtract appropriately. Several students simply evaluated $f(b) - f(a)$, which suggests a basic misunderstanding of the definition of the definite integral.

In part (b) many students made sign errors in subtracting $-2 - (-4)$ to find $g''(-3) = f'(-3)$. Several students did not correctly apply the Fundamental Theorem of Calculus, writing $g'(x) = f(x) - f(1)$. Many of these same students also wrote incorrectly that $g''(x) = f'(x) - f'(1)$. Some students confused -3 with 3 in answering this question.

In part (c) many students did not sufficiently communicate that they were solving the equation $g'(x) = 0$ to find their reported x-coordinates. However, most students were able to report the relevant points, $x = -1$ and $x = 1$. Many students described these points with inappropriate notation. In addition, many students incorrectly identified $x = -1$ as the location of a relative minimum. Several students also reported ordered pairs corresponding to points on the graph of f when discussing points on the graph of g.

Some students incorrectly included all the points where f' was undefined (including $x = -2$ in part (c) and $x = -1$ in part (d)), where $f'(x) = 0$ (including $x = 0$ in part (c)) or the endpoints ($x = -4$ and $x = 3$ in parts (c) and (d)). Many students were ambiguous in their justifications, using phrases like "the derivative" or "the slope" without reference to the proper function, f, g, f', or g'.

Based on your experience of student responses at the AP Reading, what message would you like to send to teachers that might help them to improve the performance of their students on the exam?

Teachers should continue to emphasize the interpretation of first and second derivatives and the ability to read this information from graphs. Students need practice evaluating definite integrals and sketching the graph of g given the graph of g'. In addition, students should practice evaluating derivatives, determining and classifying critical points, finding points of inflection, and sketching a graph of g' given the graph of g. These skills are particularly important for functions defined graphically. Students should have a solid understanding of the Fundamental Theorem of Calculus and practice communicating their results. The precise use of mathematical notation and absence of ambiguity are necessary to convey solution steps and justifications.

Question AB4

What was the intent of this question?

This problem presented a function f defined by $f(x) = \sqrt{25 - x^2}$ on the interval $-5 \le x \le 5$. In part (a) students were asked to find the derivative $f'(x)$. This involved correctly applying the chain rule to determine the symbolic derivative of f. Part (b) asked for an equation of the line tangent to the graph of f at the point where $x = -3$. Students needed to find the derivative at this point to determine the slope of the tangent line, the y-coordinate of the graph of f at this point, and then combine this information to provide an equation for the line. Part (c) presented a piecewise-defined function g that is equal to f on the interval $-5 \le x \le -3$ and to $x + 7$ on the interval $-3 < x \le 5$. Students were asked to use the definition of continuity to determine whether g is continuous at $x = -3$. Students should have evaluated the left-hand and right-hand limits as x approaches -3, and observed that these are the same and equal to the function value at that point. Part (d) asked students to evaluate the definite integral $\int_0^5 x\sqrt{25 - x^2}\,dx$, which can be done using the substitution $u = 25 - x^2$.

How well did students perform on this question?

The mean score was 4.09 out of a possible 9 points.

Most students were able to begin this problem in a variety of ways in order to earn points. In general, students did well in parts (a) and (b). In part (c) most students did not correctly use the definition of continuity. In part (d) students commonly earned either 0 or 3 points. If the student knew how to find an antiderivative in this problem, then he or she typically earned all 3 points in part (d).

What were common student errors or omissions?

In part (a) the most common errors were a chain rule error (omitting the derivative of $25 - x^2$) and a sign error in the derivative of $25 - x^2$ (many students wrote $2x$ and not $-2x$). There were many arithmetic errors in part (b), and students used $x = 3$ instead of $x = -3$. In addition, many students had difficulty evaluating $-x$ when $x = -3$.

In part (c) many students did not appeal to the definition of continuity. Many simply checked $f(x)$ and $x + 7$ at $x = -3$ and concluded that g was continuous at $x = -3$. Many students used a poor presentation or inadequate communication when appealing to the definition of continuity. Some students presented a limit symbol without an argument, for example, $\lim_{x \to -3} = 4$. Some students considered only the one-sided limits of g at $x = -3$ without examining the value of $g(-3)$. Some students appealed to the value of $f(-3)$ but never explicitly communicated that $g(-3) = f(-3)$.

The most common error in part (d) was a basic misunderstanding of how to integrate a product of functions. Many students separated the factors and attempted to integrate each independently. For those students who used substitution, some had a sign or arithmetic error in the coefficient of $\left(25 - x^2\right)^{3/2}$.

Based on your experience of student responses at the AP Reading, what message would you like to send to teachers that might help them to improve the performance of their students on the exam?

Teachers should continue to stress the use of standard mathematical notation. In particular, students should practice the use of parentheses appropriately, for example, in the use of the chain rule. Teachers should stress the importance of definitions and theorems in the study of calculus, and students need practice with problems in which they must appeal to these concepts. Teachers should remind students that it is not necessary to simplify expressions on the AP Calculus Exams. Students should realize that if they do attempt to simplify an expression, they must be careful not to make arithmetic, evaluation, or algebraic errors.

Question AB5/BC5

What was the intent of this question?

The context of this problem is weight gain of a baby bird. At time $t = 0$, when the bird is first weighed, its weight is 20 grams. A function B modeling the weight of the bird satisfies $\frac{dB}{dt} = \frac{1}{5}(100 - B)$, where t is measured in days since the bird was first weighed. Part (a) asked whether the bird is gaining weight faster when it weighs 40 grams or when it weighs 70 grams. Students had to evaluate and compare $\frac{dB}{dt}$ for these two values of B. Part (b) asked for $\frac{d^2B}{dt^2}$ in terms of B. Students should have used a sign analysis of the second derivative to explain why the graph of B cannot resemble the given graph. Part (c) asked students to use separation of variables to solve the initial value problem $\frac{dB}{dt} = \frac{1}{5}(100 - B)$ with $B(0) = 20$ to find $B(t)$.

How well did students perform on this question?

The mean score was 2.87 for AB students and 4.75 for BC students out of a possible 9 points.

In part (a) most students earned 2 points. In part (b) many students did not use the chain rule. In part (c) many students could not separate the variables appropriately in order to begin to solve the differential equation.

What were common student errors or omissions?

In part (a) some students did not sufficiently communicate that they had used the given differential equation. This emphasizes the need for students to use proper mathematical notation in solution steps.

In part (b) many students did not use the chain rule. Several students found the derivative of $\frac{dB}{dt}$ with respect to B rather than t. Some students left their answers in the form $-\frac{1}{5}\frac{dB}{dt}$. These students did not earn the first point but were eligible for the second point.

In part (c) many students did not know how to separate the variables in order to solve this differential equation. For those students who did successfully separate the variables, there were several, varied calculus errors (mostly in the antidifferentiation step) or algebraic errors in arriving at a final answer.

Based on your experience of student responses at the AP Reading, what message would you like to send to teachers that might help them to improve the performance of their students on the exam?

Students need more practice solving separable differential equations as well as practice with problems in context. Students were able to explain their reasoning fairly well in part (a). However, they still need more practice explaining results. Teachers should also realize that excellent algebraic skills are necessary for students to succeed in AP Calculus. Teachers should continue to emphasize the use of good mathematical notation.

Question AB6

What was the intent of this question?

This problem presented students with a particle in rectilinear motion during the time interval $0 \le t \le 12.$ The particle's position at time $t = 0$ is given, and the velocity $v(t)$ is provided. Part (a) asked students to determine the times when the particle is moving to the left, which they should have done by considering the sign of the given velocity function. In part (b) students were asked to provide an integral expression for the total distance traveled by the particle from time $t = 0$ to time $t = 6,$ which they should have recognized as given by the definite integral of $|v(t)|$ over the given time interval. Part (c) asked for the acceleration at time t. Students should have recognized that the acceleration $a(t)$ is the derivative of the velocity function. Students should have provided a symbolic derivative for the given velocity function, correctly using the chain rule. Students were then asked whether the speed of the particle is increasing, decreasing, or neither at time $t = 4.$ Students should have evaluated both the velocity and the acceleration functions at time $t = 4.$ Because $v(4) < 0$ and $a(4) < 0,$ the speed of the particle is increasing. Part (d) asked students to find the position of the particle at time $t = 4.$ This is calculated using the expression $x(4) = x(0) + \int_0^4 v(t)\, dt.$

How well did students perform on this question?

The mean score was 3.59 out of a possible 9 points.

Students did very well in parts (a) and (b). In part (c) most students found $a(t)$ correctly. In part (d) few students earned all 3 points, but many earned at least 1 point for the antiderivative or use of the initial condition.

What were common student errors or omissions?

In part (a) many students provided incorrect intervals with a left endpoint of 4. Some students integrated $v(t)$ instead of using $v(t)$ to determine when the particle was moving to the left. Some students presented a sign chart as a solution rather than a written communication. In part (b) some students presented an integral for displacement, $\int_0^6 v(t)\,dt,$ rather than total distance traveled. Some students incorrectly added an initial condition to the expression for total distance.

In part (c) some students did not use the chain rule correctly to find $a(t)$. Many students did not consider $v(4)$ in addition to $a(4)$ in order to determine whether the speed of the particle is increasing, decreasing, or neither. These students often concluded that the speed is decreasing based on the sign of the acceleration alone. There were also several sign errors in computing $v(4)$. In general, the most common errors in part (c) were incorrect values for the two trigonometric functions at $t = 4$. In part (d) many students did not include the initial condition. Several students did not find the correct antiderivative of $\cos\left(\frac{\pi}{6}t\right)$.

Based on your experience of student responses at the AP Reading, what message would you like to send to teachers that might help them to improve the performance of their students on the exam?

It is still very important for students to learn unit-circle values for trigonometric functions and to be able to use them without the use of a calculator. Students need to practice communication skills and the use of proper mathematical notation. Teachers should provide more practice with particle motion problems. Students need to understand the difference between displacement and total distance, and between speed and velocity. Students also need additional practice with initial value problems, especially in cases in which the initial value is not 0.

Question BC2

What was the intent of this question?

This problem described the path of a particle whose position at time t is given by $(x(t), y(t)),$ where $\frac{dx}{dt} = \frac{\sqrt{t+2}}{e^t}$ and $\frac{dy}{dt} = \sin^2 t.$ Part (a) asked whether the particle's horizontal direction of motion is toward the left or toward the right at time $t = 2.$ Students should have determined the sign of $\frac{dx}{dt}$ at this time to establish the direction of motion. Students were asked to find the slope of the particle's path at that time. The slope can be found by evaluating $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$ at $t = 2.$ Part (b) asked students to find the x-coordinate of the particle's position at time $t = 4.$ This is calculated using the expression $x(4) = x(0) + \int_0^4 x'(t)\,dt.$ Part (c) asked for the speed of the particle at time $t = 4$ seconds. This value is found by evaluating $\sqrt{(x'(t))^2 + (y'(t))^2}$ at time $t = 4.$ Students were then asked for the acceleration vector at this time, which is given by $\langle x''(4), y''(4)\rangle.$ Part (d) asked for the distance traveled by the particle over the interval $2 \le t \le 4$ seconds. This is found by integrating $\sqrt{(x'(t))^2 + (y'(t))^2}$ over the interval $2 \le t \le 4.$

How well did students perform on this question?

The mean score was 5.07 out of a possible 9 points.

In general, student performance on this problem was very good. Most students demonstrated a solid understanding of parametrically defined curves. However, there were several arithmetic, algebraic, and decimal presentation errors in this calculator-active question. Many students were able to present correct mathematical statements and equations but produced incorrect results from their calculators. Some students did not correctly use the initial condition.

What were common student errors or omissions?

In part (a) the most common error was an attempt to determine the horizontal movement of the particle from either $\frac{dy}{dx}$ or the value of $x(2)$. Many students made arithmetic or algebraic mistakes in computing the value of the slope after their presentation of $\frac{dy/dt}{dx/dt}$.

In part (b) many students showed no evidence that they were using $x(2)$ in finding the particle's position at time $t = 4$. Some students tried to compute a symbolic antiderivative of $\frac{dx}{dt}$. However, most students abandoned this attempt and used their calculators to obtain a final answer.

In part (c) many students used an incorrect formula for speed. For those students who attempted to find the derivatives of $\frac{dx}{dt}$ and $\frac{dy}{dt}$ analytically, many made quotient rule errors or algebraic errors. This led to an error in the final acceleration vector.

In part (d) there were two common errors. Many students used $\left|\frac{dy}{dx}\right|$ as the integrand in computing the distance. In addition, several students incorrectly used the formula for arc length assuming $y = f(x)$.

Based on your experience of student responses at the AP Reading, what message would you like to send to teachers that might help them to improve the performance of their students on the exam?

Students need practice communicating their mathematical steps to arrive at a solution. There were some decimal presentation errors in this problem. Students need to be reminded of the instructions for decimal presentation as well as how to report and use intermediate values used to find final answers.

Students need more practice using calculus to justify direction of motion. Teachers should provide more opportunities for students to work with the Fundamental Theorem of Calculus in the form $x(b) = x(a) + \int_a^b x'(t)\, dt$. Students preparing for the AP Calculus BC Exam need practice in applications of calculus to parametrically defined curves. Many students seemed unfamiliar with the concepts of the speed of a particle in the plane or the distance traveled by a particle.

Students need more practice writing complex mathematical expressions with proper notation and syntax. This will help them avoid presentation errors and allow better communication of results. Students also need more practice entering and using mathematically complex expressions on their calculators.

Question BC4

What was the intent of this question?

Students were presented with a table of values for f' at selected values of x given that f is a twice-differentiable function. The values for $f(1)$ and $f''(1)$ are also given. Part (a) asked students to write an equation for the line tangent to the graph of f at $x = 1$ and then use this line to approximate $f(1.4)$. Students should have used the given values for $f(1)$ and $f'(1)$ to construct an equation equivalent to $y = f(1) + f'(1)(x - 1)$. Students could then substitute $x = 1.4$ to obtain the desired approximation. Part (b) asked students to use a midpoint Riemann sum with two subintervals of equal length, based on values in the table, to approximate $\int_1^{1.4} f'(x)\,dx$. They were then asked to use this approximation to estimate $f(1.4)$. This estimate is obtained by using the midpoint Riemann sum in the expression $f(1) + \int_1^{1.4} f'(x)\,dx$. Part (c) asked students to use Euler's method, starting at $x = 1$ with two steps of equal size, to approximate $f(1.4)$. Part (d) asked for the second-degree Taylor polynomial for f about $x = 1$, which was then used to obtain yet another approximation for $f(1.4)$. Students should have used the given values for $f(1)$, $f'(1)$, and $f''(1)$ to write the Taylor polynomial.

How well did students perform on this question?

The mean score was 5.43 out of a possible 9 points.

In general, students did very well in solving this unique approximation problem. The midpoint Riemann sum and the Taylor polynomial were the keys to success on this question. Many students presented minimal supporting work and did not earn points owing to inadequate communication. Some students used incorrect values from the table or statement of the problem.

What were common student errors or omissions?

The most common errors were algebraic and arithmetic mistakes. In part (a) many students used incorrect values from the table or statement of the problem. For example, commonly used incorrect values were $f(1) = 8$ and $f'(1) = 20$.

In part (b) many students calculated either a trapezoidal sum or used four subintervals instead of two subintervals. Some students made arithmetic errors in computing the final value for the Riemann sum. Many wrote incorrect statements involving the Fundamental Theorem of Calculus.

In part (c) many students made arithmetic errors in calculating the product $f'(x)\cdot\Delta x$ and/or $f(x) + f'(x)\cdot\Delta x$. Several students presented unlabeled or poorly communicated work and did not clearly indicate the final answer.

In part (d) many students used incorrect values or no values from the table or from the statement of the problem or did not center the polynomial at $x = 1$. For those students who were able to obtain the Taylor polynomial, some made algebraic and arithmetic errors in computing the approximation.

Based on your experience of student responses at the AP Reading, what message would you like to send to teachers that might help them to improve the performance of their students on the exam?

Students should be reminded that on the AP Calculus Exams, simplification of expressions is not required. If a student simplifies an expression, the student must carefully check the work for algebraic and arithmetic errors. Students need more practice with problems involving tabular data and the use of appropriate values from a table. Teachers should encourage students to clearly label values and answers. Students need practice communicating intermediate and final results. Communication was very important in this problem, especially in parts (b) and (c) with Euler's method and the Fundamental Theorem of Calculus.

Teachers should encourage students to read each question carefully, to parse important information, and to answer each part of the question explicitly. If a specific method or more than one answer is required, students need to demonstrate use of the stated method and must present distinct answers to each part of the question.

Question BC6

What was the intent of this question?

This problem presented the Maclaurin series for an infinitely differentiable function g. Part (a) asked students to use the ratio test to determine the interval of convergence for the given Maclaurin series. Students should have observed that for $x = -1$ and $x = 1,$ the resulting series is alternating with terms decreasing in absolute value to 0. Therefore, the series converges for $x = -1$ and $x = 1.$ Part (b) asked students to show that the approximation for $g\left(\frac{1}{2}\right)$ obtained by using the first two nonzero terms of the series differs from the actual value by less than $\frac{1}{200}.$ Because this is an alternating series with terms decreasing in absolute value to 0, students should have observed that the absolute value of the third term bounds the error and is strictly less than $\frac{1}{200}.$ Part (c) asked the students to find the first three nonzero terms and the general term of the Maclaurin series for $g'(x).$ Students should have computed the symbolic derivative of the first three nonzero terms and the general term of the series for $g(x).$

How well did students perform on this question?

The mean score was 4.23 out of a possible 9 points.

Given that this was the last problem on the AP Calculus BC Exam, and it involved an infinite series, students did very well. Many students were very successful in part (a), earning 4 of the 5 possible points. Many students had difficulty completing the analysis and presenting a proper appeal to the Alternating Series Test. In part (b) most students knew that the third term in the series was associated with the error bound but were unsure of the correct formula. In part (c) most students were able to differentiate the series for g term by term.

What were common student errors or omissions?

In part (a) many students made algebraic errors in replacing n with $n + 1.$ Some students also incorrectly solved the inequality $\left|x^2\right| < 1,$ which, for many, led to equations with nonreal solutions. Some students worked with the limit incorrectly. The most common error in part (a) involved the analysis at the endpoints of the interval of convergence. Many students did not cite the Alternating Series Test. These students either gave no justification for the conclusion or gave incorrect reasons involving the harmonic series.

In part (b) some students incorrectly considered this a Lagrange error bound problem. Some students made arithmetic errors in simplifying $\frac{\left(\frac{1}{2}\right)^5}{7}$. In part (c) the most common errors involved incorrect differentiation of the terms of g or the general term. Some students incorrectly linked unequal terms and incorrectly included the summation expression instead of the general term in the indicated sum.

Based on your experience of student responses at the AP Reading, what message would you like to send to teachers that might help them to improve the performance of their students on the exam?

Many of the student errors on this question were linked to process rather than conceptual understanding. In applying the ratio test, teachers should stress the role of each step, the importance of proper notation, and the need for exact algebraic skills. Series is an important part of the AP Calculus BC course, and teachers should spend sufficient time covering the topics associated with series. Students still need practice solving inequalities, especially those involving a quadratic expression.