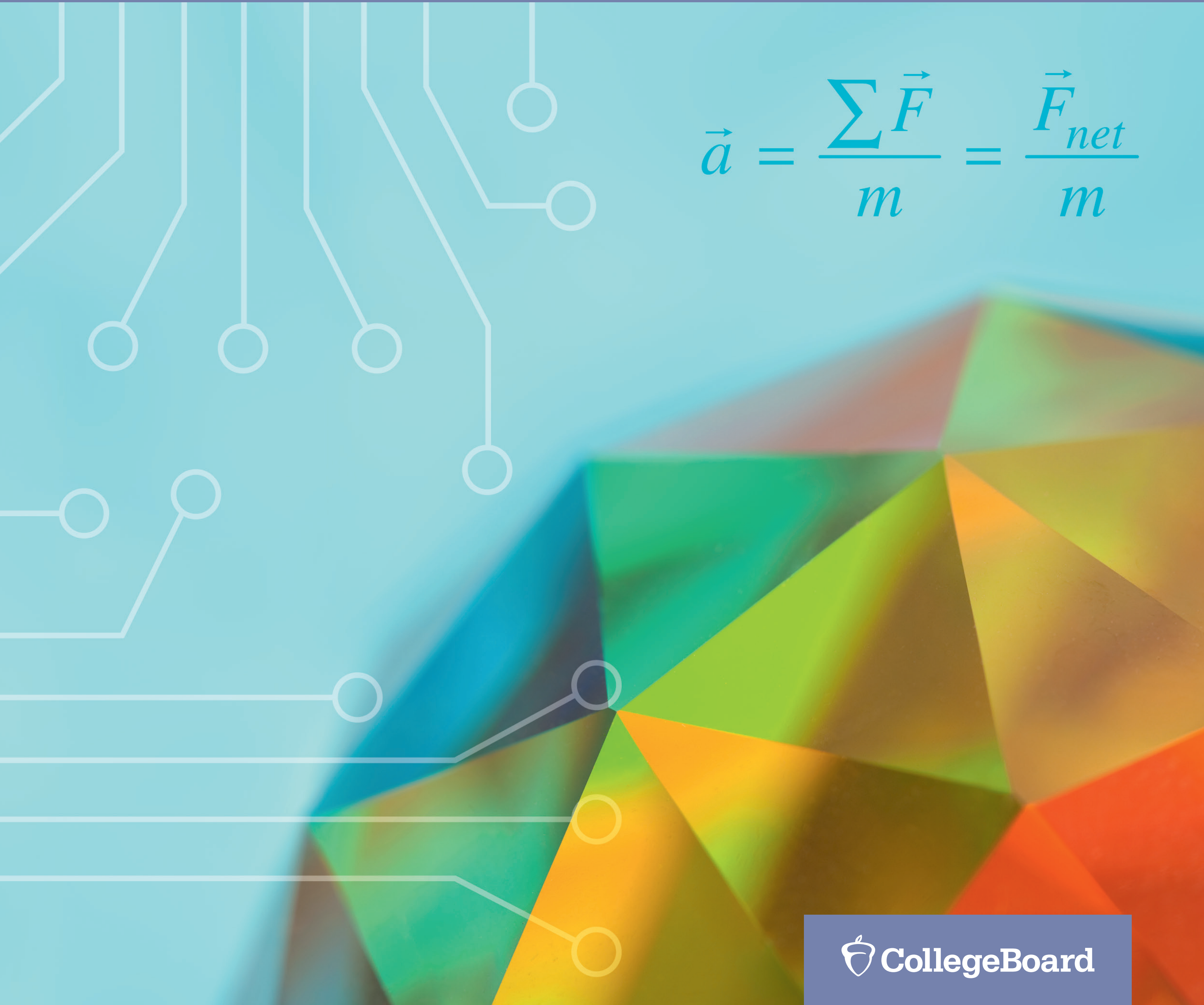
AP®

AP® Physics 1 and 2 Inquiry-Based Lab Investigations

Teacher's Manual

Effective Fall 2021



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CollegeBoard

AP® Physics 1 and 2 Inquiry-Based Lab Investigations:

A Teacher's Manual

CollegeBoard
New York, NY

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AP Science lab vision team

In 2010, the College Board convened a group of subject matter and laboratory investigation experts to provide a model of excellence for what the investigative labs should be in AP Science courses. These individuals worked diligently to create a vision for exemplary AP science labs that would serve to assist teachers in facilitating inquiry-based and student-directed investigative work. This vision also serves as the input for professional development and resource materials that will support lab investigations for the redesigned science courses.

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Chapter 1:
About This Manual

The AP Physics 1 and Physics 2 Algebra-Based courses are designed to promote student learning of essential physics content and foster development of deep conceptual understanding through an inquiry-based model of instruction. The instructional approaches utilized in this manual are informed by several decades of research on student learning and knowledge construction, especially with regard to physics principles. (Further discussion of inquiry-based instructional approaches is found in chapter 4.)

In this inquiry-based model, students learn by engaging in the seven AP Science Practices that develop their experimental and reasoning skills. By engaging in the science practices students begin to see that the study of physics is much more than just learning about our physical world; it also requires practices that are "used to establish, extend, and refine that knowledge" over time (NRC, 2012). The science practices (set out in Appendix A) enable students to make predictions of natural phenomena, develop and refine testable explanations, and use established lines of evidence and reasoning to support claims.

The laboratory investigations presented in this manual are examples of the kind of investigations that students should engage in, but they are **not to be considered mandatory;** nor are they intended to be the only investigations that students engage in during the course of study in AP Physics. It should not be assumed that any of these investigations would be specific targets for assessment on AP Exams. The investigations provided in this manual are simply models of inquiry-based labs that illustrate a variety of approaches and different levels of guidance and support that teachers can use when implementing inquiry-based laboratory work. You are also encouraged to develop your own inquiry-based investigations that meet the same cognitive objectives.

How This Manual Was Developed

To create a model of excellence for the lab component in AP science courses, the College Board, in conjunction with the Lab Vision Team and Physics Lab Development Team, worked to create an innovative vision and approach to lab investigations. Both teams of subject-matter experts consisted of master AP Physics teachers and higher-education faculty members, as well as experts in the field of inquiry-based instructional design, quantitative skill application, and lab investigations. The labs were written by physics teachers and higher education faculty members, as well as experts in the field of inquiry-based instructional design, quantitative skill application, and lab investigations. Each lab was piloted by AP teachers and students.

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Goals of Investigations in AP Physics 1 and AP Physics 2

Inquiry-based laboratory experiences support the AP Physics 1 and AP Physics 2 courses and curricular requirements by providing opportunities for students to engage in the seven science practices as they design plans for experiments, make predictions, collect and analyze data, apply mathematical routines, develop explanations, and communicate about their work. The inquiry-based investigations in this manual demonstrate a range of teacher guidance, from moderate to more fully student-directed, and support the content and science practices within the AP Physics 1 and AP Physics 2 courses.

The investigations in this manual provide examples of investigations that support recommendations by the National Science Foundation (NSF) that science teachers should include opportunities in their curricula for students to develop skills in communication, teamwork, critical thinking, and commitment to life-long learning (NSF 1996, NSF 2012, AAPT Committee on Physics in High Schools, 1992). Investigations in the style of those in this manual should engage and inspire students to investigate meaningful questions about the physical world, and they should align with the best practices described in *America's Lab Report: Investigations in High School Science* — a comprehensive synthesis of research about student learning in science laboratories from the National Research Council. Note that the investigations in this manual are neither mandatory nor all-inclusive. Feel free to use any investigations that capture the spirit of these examples.

How Inquiry-Based Investigations Support the AP Physics 1 and 2 Curriculum Framework

The AP Physics 1 and AP Physics 2 courses, equivalent to the first and second semesters of a typical introductory, algebra-based college physics course, emphasize depth of understanding over breadth of content. By delivering the content across two full-year courses, students will have more time to engage in inquiry-based learning experiences to develop conceptual understanding of content while at the same time building expertise in the science practices.

The AP Physics Exams will assess students' abilities to apply the science practices to the learning objectives in the curriculum framework. These science practices and learning objectives can be addressed by the labs in this manual and other inquiry-based labs that you may choose. This instructional approach to laboratory investigations typically takes more time than simple verification/confirmation labs; however, the reduced amount of content covered in each course will allow you to meet the curricular requirement that 25 percent of course time must be devoted to "hands-on laboratory work with an emphasis on inquiry-based investigations."

The labs in this manual are intended to serve as models, **not as required activities**; you are encouraged to develop your own teacher-guided or student-directed, inquiry-based labs that address the learning objectives in the curriculum framework. To assist and support you in this process, the College Board operates the online AP Teacher Community (https://apcommunity.collegeboard.org/), which provides opportunities for collaboration and sharing of resources and ideas. There are multiple strategies that can be applied to modify traditional confirmation investigations into guided-inquiry labs, as further discussed in chapter 4. Regardless of the approach, the goal is to engage students in the investigative process of science and allow them to discover knowledge for themselves in a self-reflective, safe, and organized manner.

How the Investigations in This Manual Connect to the AP Physics 1 and 2 Curriculum Framework

The key concepts and related content that define the AP Physics 1 and AP Physics 2 courses are organized around seven underlying principles called the big ideas, which address (1) Properties of Objects and Systems, (2) Fields and Interactions, (3) Object Interactions and Forces, (4) System Interactions and Changes, (5) Conservation Laws, (6) Waves and Wave Models, and (7) Probability, Complex Systems, and Quantum Systems. The big ideas, as described in the curriculum framework, encompass the core scientific principles, theories, and processes modeling physical interactions and systems. For each big idea, enduring understandings are identified, which incorporate the core concepts that students should retain from the learning experience.

Learning objectives for each big idea detail what students are expected to know and be able to do. Because content, inquiry, and reasoning are equally important in AP Physics 1 and AP Physics 2, each learning objective in the curriculum framework combines content with inquiry and reasoning skills as described in the science practices.

Each investigation in this manual is structured to align to one or more learning objectives from the AP Physics 1 or AP Physics 2 course and specifies the big idea(s), enduring understandings, learning objectives, and science practices most relevant to and/or addressed by the various activities in that investigation. (See A Lab at a Glance in chapter 3.)

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Chapter 2 gives an overview of the investigations, showing their connections to the curriculum framework. Although each experiment may address one primary learning objective, there is often significant overlap of the learning objectives within a given enduring understanding, and often across different big ideas. There is no particular sequence to the labs in each course, and you may choose whatever learning sequence makes sense for you and your students. It is often desirable to have students gain experience with phenomena at the beginning of a topic, to help them build a conceptual model that describes the phenomena, but it can also be desirable to have students use a model to predict the outcome of an experiment and then design the experiment to test their prediction. As suggested by *America's Lab Report: Investigations in High School Science*, the AAPT Committee on Physics in High Schools position paper, "Role of Labs in High School Physics," and a recent research summary published in *Science* (de Jong, Linn, and Zacharia, 2013), it is highly desirable to integrate laboratory work to align with work students are doing in other parts of the course. Students will gain greater conceptual understanding from a learning sequence that fully integrates laboratory work with other course content.

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Chapter 2:
Overview of the Investigations

AP Physics 1 Investigations

Lab 1: 1D and 2D Kinematics

AP Physics 1

Connection to Curriculum Framework		Material and Equipment	Time Estimated
Learning Objectives	**Science Practices**	◆ Ramps ◆ Stopwatches ◆ Metersticks ◆ Steel balls (1.5–2 cm in diameter) ◆ Carbon paper ◆ Bubble levels ◆ (Optional) Toy car that accelerates	**TOTAL: ~ 1.5–2 hours** Teacher Prep/Set-up: 10–15 minutes Student Investigation: 70–80 minutes Postlab Discussion: 15–20 minutes
3.A.1.1 3.A.1.2 3.A.1.3	1.5 2.1 2.2 4.2 4.3 5.1		

Lab 2: Newton's Second Law

AP Physics 1

Connection to Curriculum Framework		Material and Equipment	Time Estimated
Learning Objectives	**Science Practices**	◆ Dynamics tracks ◆ Carts ◆ Assorted masses ◆ Mass hangers, slotted masses ◆ Low-friction pulleys ◆ String ◆ Metersticks ◆ Stopwatches	**TOTAL: ~ 3.5 hours** Teacher Prep/Set-up: 15–20 minutes Prelab: 30 minutes Student Investigation: 110–120 minutes Postlab Discussion: 30 minutes
1.C.1.1 3.A.2.1	1.1 4.1 4.2 4.3 5.1 5.3		

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Lab 3: Circular Motion

AP Physics 1

Connection to Curriculum Framework		Material and Equipment	Time Estimated
Learning Objectives	**Science Practices**		
3.B.1.1 3.B.1.2 3.B.2.1 3.E.1.3 4.A.2.1 4.A.3.1	1.1 1.4 2.2 4.2 4.3 5.1 6.4	◆ Battery-operated flying toys with new 1.5-volt AA cells ◆ Metersticks ◆ Stopwatches ◆ (Optional) Extra sets of semidrained AA cells ◆ (Optional) Multimeter	**TOTAL: ~ 1.5–2 hours** Teacher Prep/Set-up: 15 minutes Prelab: 10 minutes Student Investigation: 45–60 minutes Postlab Discussion: 15–30 minutes

Lab 4: Conservation of Energy

AP Physics 1

Connection to Curriculum Framework		Material and Equipment	Time Estimated
Learning Objectives	**Science Practices**		
5.B.3.1	2.2 3.1 4.1 4.3 4.4 5.1 6.1 6.4 7.2	◆ Low-friction dynamics carts with spring bumper (or spring-loaded plunger carts) ◆ Ramps ◆ Metersticks ◆ Stopwatches ◆ Assorted masses ◆ Objects to prop up an inclined ramp ◆ Poster-size whiteboards	**TOTAL: ~ 3.5 hours** Teacher Prep/Set-up: 10–15 minutes Student Investigation: 90 minutes Postlab Discussion: 80 minutes

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Lab 5: Impulse and Momentum

AP Physics 1

Connection to Curriculum Framework

Learning Objectives	Science Practices
5.D.1.1	4.1
5.D.1.6	4.2
5.D.2.1	4.3
5.D.2.4	4.4
	5.1
	5.3
	6.4
	7.2

Material and Equipment

- Spring-loaded carts
- Tracks
- Bubble level
- Known and unknown calibrated masses
- Calculators
- Metersticks
- Stopwatches
- Computers with Internet access
- (Optional) Video cameras and video analysis software
- (Optional) Force sensor
- (Optional) Motion sensor with calculator or computer interface

Time Estimated

TOTAL: ~ 2 hours

Teacher Prep/Set-up:
15 minutes

Student Investigation:
70 minutes

Postlab Discussion:
25 minutes

Lab 6: Harmonic Motion

AP Physics 1

Connection to Curriculum Framework

Learning Objectives	Science Practices
3.B.3.1	2.2
3.B.3.2	4.2
3.B.3.3	4.3
	5.1
	6.4

Material and Equipment

- String
- Set of calibrated masses (20–500 g)
- Stopwatches or timers
- Metersticks
- Protractors
- Support rods
- Paper and tape (to create scrolls)
- Leaking bobs
- (Optional) Pendulum
- (Optional) Constant speed buggies
- (Optional) Motion detectors, software, computers
- (Optional) Video cameras and video analysis software
- (Optional) Spring

Time Estimated

TOTAL: ~ 3.5 hours

Teacher Prep/Set-up:
10–15 minutes

Student Investigation:
170 minutes

Postlab Discussion:
15 minutes

Lab 7: Rotational Motion

AP Physics 1

Connection to Curriculum Framework		Material and Equipment	Time Estimated
Learning Objectives	**Science Practices**	- Objects of different shapes, masses, and diameters (that can roll down an incline) - Inclined plane or grooved track - Objects to prop up an inclined plane (books, bricks, etc) - Rulers - Metersticks - Stopwatches - Mass scales - (Optional) Motion sensor or video analysis tools	**TOTAL:** **~ 3.5–4.5 hours** Teacher Prep/Set-up: 5–10 minutes Student Investigation: 180–265 minutes
3.A.1.1	1.4		
3.A.1.2	1.5		
3.A.1.3	2.1		
4.C.1.1	2.2		
5.E.2.1	4.2		
	4.3		
	5.1		

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AP Physics 2 Investigations

Lab 1: Boyle's Law

AP Physics 2

Connection to Curriculum Framework		Material and Equipment	Time Estimated
Learning Objectives	**Science Practices**		
5.B.5.4	1.1	Boyle's law apparatus Mass scales Rulers Graph paper or graphing calculators Objects with mass large enough to compress air in a syringe (Optional) Pressure sensors	**TOTAL: ~ 1 hour** Teacher Prep/Set-up: 5–10 minutes Student Investigation: 35–50 minutes Postlab Discussion: 10–15 minutes
5.B.5.5	1.4		
5.B.5.6	2.2		
5.B.7.1	4.2		
5.B.7.2	4.3		
5.B.7.3	5.1		
	6.4		

Lab 2: Fluid Dynamics

AP Physics 2

Connection to Curriculum Framework		Material and Equipment	Time Estimated
Learning Objectives	**Science Practices**		
5.B.10.1	2.1	Plastic transparent bottles or discharge containers with largely vertical walls Masking tape Metersticks Stopwatches Sinks or large containers to catch discharging water Graphical analysis program on computers or calculators (Optional) Video cameras and video analysis software	**TOTAL: ~ 1.5–2.5 hours** Teacher Prep/Set-up: 20–30 minutes Prelab: 10–20 minutes Student Investigation: 30–45 minutes Postlab Discussion: 40–50 minutes
5.B.10.2	2.2		
5.B.10.3	4.3		
5.B.10.4	6.2		
5.F.1.1	7.1		

Lab 3: Resistor Circuits

AP Physics 2

Connection to Curriculum Framework

Learning Objectives	Science Practices
5.B.9.1	1.1
5.B.9.2	1.4
5.B.9.3	2.2
5.C.3.1	4.1
5.C.3.2	4.2
5.C.3.3	4.3
	5.1
	6.4

Material and Equipment

- Four-cell battery holders
- D-cell batteries
- #14 bulbs, #48 bulbs, corresponding holders
- Connecting wire
- Basic multimeters or student single-value meters (voltmeter, ammeter)
- Extra fuses for the ammeter
- (Optional) Basic single pole throw switch

Time Estimated

TOTAL: ~ 3–3.5 hours

Teacher Prep/Set-up:
20–25 minutes

Prelab:
20 minutes

Student Investigation:
90 minutes

Postlab Discussion:
30–60 minutes

Lab 4: RC Circuits

AP Physics 2

Connection to Curriculum Framework

Learning Objectives	Science Practices
4.E.5.1	1.4
4.E.5.2	2.2
4.E.5.3	4.2
5.B.9.5	4.3
5.C.3.6	5.1
	6.1
	6.4

Material and Equipment

- D-cell batteries and battery holders
- Connecting wires
- Minature screw lamps (#40 or #50 bulbs with holders)
- Nonpolar microfarad capacitors
- Resistors with variable resistance
- Stopwatches
- Multimeters, voltmeters, and/or ammeters
- Single pole switches

Time Estimated

TOTAL: ~ 4 hours

Teacher Prep/Set-up:
15–20 minutes

Prelab:
60 minutes

Student Investigation:
120 minutes

Postlab Discussion:
45 minutes

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Lab 5: Magnetism

AP Physics 2

Connection to Curriculum Framework		**Material and Equipment**	**Time Estimated**
Learning Objectives	**Science Practices**		
2.D.2.1 2.D.3.1 2.D.4.1	1.1 1.2 1.4 2.2 4.2 4.3 5.1 7.1	◆ Small compasses ◆ Iron and ceramic magnets ◆ Containers filled with iron filings ◆ Sheets of paper, plastic bags, or sheet protectors ◆ Pith balls or paper clips hung on insulated strings ◆ Rubber rods or PVC pipes ◆ Glass or acrylic rods ◆ Rabbit fur or similar material ◆ Silk or equivalent material ◆ Styrofoam cups ◆ Aluminum foil ◆ Battery holders ◆ Copper wire (16–22 gauge) ◆ Switches ◆ Magnaprobes ◆ Magnetic field probes ◆ Flat pieces of wood or cardboard ◆ Clamps ◆ Rod stands ◆ 1.5-volt D-cell batteries ◆ (Optional) 3 × 3-inch sheets of magnetically senstive film, linear variable resistors, ammeters	**TOTAL: ~ 3–3.5 hours** Teacher Prep/Set-up: 15–20 minutes Student Investigation: 130–160 minutes Postlab Discussion: 30 minutes

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Lab 6: Electromagnetic Induction

AP Physics 2

Connection to Curriculum Framework		Material and Equipment	Time Estimated
Learning Objectives	**Science Practices**		
2.D.3.1 3.A.1.3 4.E.2.1	1.2 4.3 5.1 6.4	• Enameled magnet wire • Plastic/cardboard tubes • Neodymium axially polarized nickel-plated disc magnets • Digital multimeter with a setting that will indicate to the tenths of a millivolt • Connecting wires, preferably with alligator clips • Electrical tape • Sandpaper • String • Masking tape • Compasses • (Optional) eightpenny or tenpenny nails • (Optional) Two coils, one that fits inside the other • (Optional) Demonstration transformer • (Optional) Old AC to DC wall transformer • (Optional) Digital multimeter with AC voltmeter capability, or dedicated AC voltmeter	**TOTAL: ~ 1.5–2 hours** Teacher Prep/Set-up: 5–10 minutes Prelab: 10–20 minutes Student Investigation: 60–70 minutes Postlab Discussion: 15–20 minutes

Lab 7: Geometric Optics

AP Physics 2

Connection to Curriculum Framework		Material and Equipment	Time Estimated
Learning Objectives	**Science Practices**		
6.E.5.1 6.E.5.2	1.4 2.2 4.1 4.3 5.1 5.2	• Light sources • Converging lenses, focal length 15–25 cm • Lens holders • Metersticks • Index cards (5 × 7 inches or larger) • (Optional) Diverging lenses	**TOTAL: 3.5 hours** Teacher Prep/Set-up: 15 minutes Student Investigation: 90 minutes Extension: 60 minutes Postlab Discussion: 45 minutes

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Lab 8: The Particle Model of Light

AP Physics 2

Connection to Curriculum Framework		Material and Equipment	Time Estimated
Learning Objectives	**Science Practices**		
5.B.8.1 6.F.3.1 6.F.4.1	1.2 1.4 3.2 4.2 4.3 5.1 6.2 6.4 7.1	• Electroscope • Plastic cylinder (e.g., piece of PVC pipe) • Metal plates (zinc, copper, steel) • Steel wool • Mercury vapor lamp or ultraviolet light source • Emery cloth • Fur, felt, or wool cloth • Silk or equivalent material • Power supply with variable potential difference • Small incandescent bulb with base • Light-emitting diodes (red, green, blue) • 2–6 volt variable DC power supplies • Alligator clips and jumper wires • Potentiometers or trimpots • Multimeters • (Optional) Breadboards • (Optional) Cardboard tubes from paper towel rolls • (Optional) Small resistors (330–660 ohms) • (Optional) Photovoltaic measurement kit	**TOTAL: ~ 2.5 hours** Teacher Prep/Set-up: 15–20 minutes Prelab: 45–60 minutes Student Investigation: 45–60 minutes Postlab Discussion: 30 minutes

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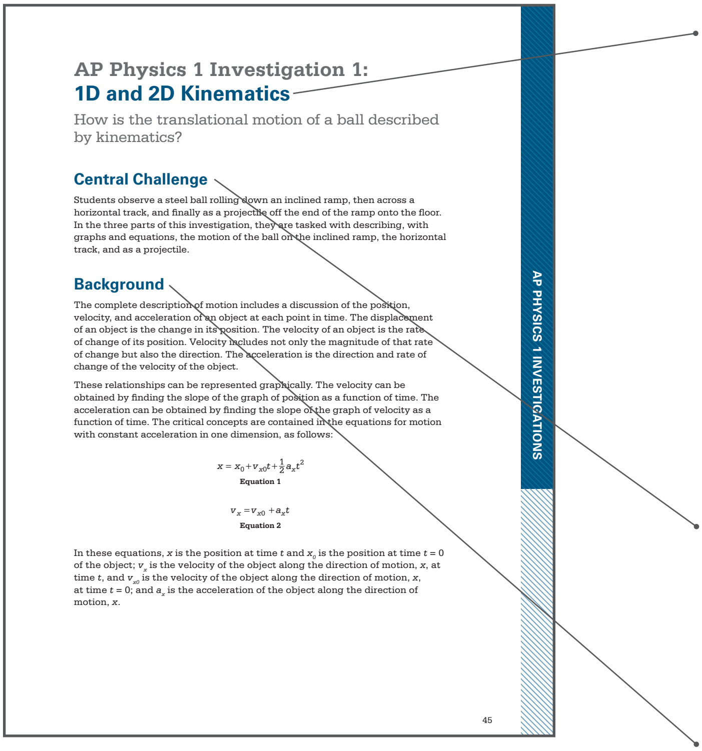
Chapter 3:
A Lab at a Glance

Although each lab investigation in this manual is unique and focuses on specified learning objectives and science practices, the overall formats are similar. As shown below, each investigation includes the following:



AP Physics 1 Investigation 1:
1D and 2D Kinematics

How is the translational motion of a ball described by kinematics?

Central Challenge

Students observe a steel ball rolling down an inclined ramp, then across a horizontal track, and finally as a projectile off the end of the ramp onto the floor. In the three parts of this investigation, they are tasked with describing, with graphs and equations, the motion of the ball on the inclined ramp, the horizontal track, and as a projectile.

Background

The complete description of motion includes a discussion of the position, velocity, and acceleration of an object at each point in time. The displacement of an object is the change in its position. The velocity of an object is the rate of change of its position. Velocity includes not only the magnitude of that rate of change but also the direction. The acceleration is the direction and rate of change of the velocity of the object.

These relationships can be represented graphically. The velocity can be obtained by finding the slope of the graph of position as a function of time. The acceleration can be obtained by finding the slope of the graph of velocity as a function of time. The critical concepts are contained in the equations for motion with constant acceleration in one dimension, as follows:

$$x = x_0 + v_{x0}t + \frac{1}{2}a_x t^2$$

Equation 1

$$v_x = v_{x0} + a_x t$$

Equation 2

In these equations, x is the position at time t and x_0 is the position at time $t = 0$ of the object; v_x is the velocity of the object along the direction of motion, x, at time t, and v_{x0} is the velocity of the object along the direction of motion, x, at time $t = 0$; and a_x is the acceleration of the object along the direction of motion, x.

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The title introduces the topical focus of the lab, and the primary question that follows frames the question for students to investigate. These questions are generated from the enduring understandings, key concepts, and principles articulated in the curriculum framework.

Describes the challenge or problem students will solve and helps orient you to the investigation.

Derived from the curriculum framework, this information provides the theory or principles students should understand in order to conduct the investigation.

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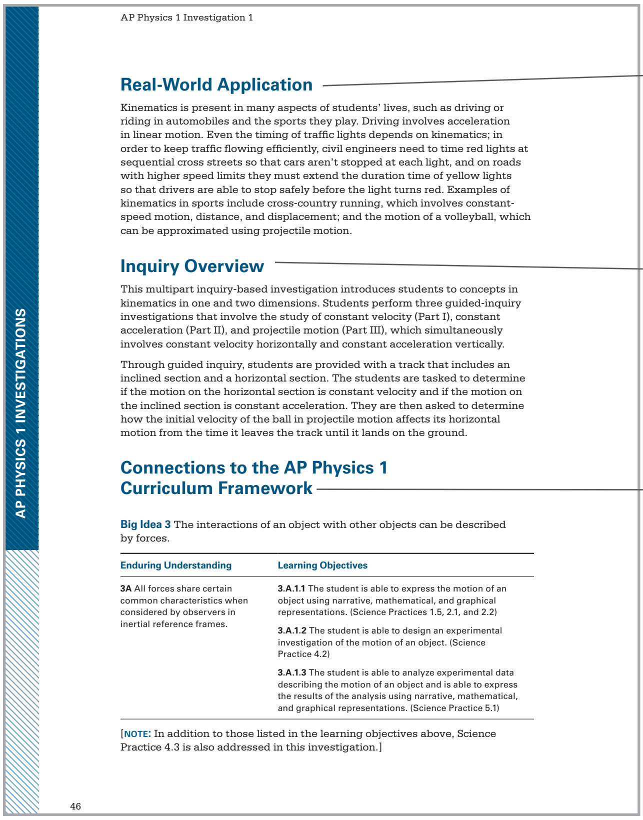

AP Physics 1 Investigation 1

AP PHYSICS 1 INVESTIGATIONS

Real-World Application

Kinematics is present in many aspects of students' lives, such as driving or riding in automobiles and the sports they play. Driving involves acceleration in linear motion. Even the timing of traffic lights depends on kinematics; in order to keep traffic flowing efficiently, civil engineers need to time red lights at sequential cross streets so that cars aren't stopped at each light, and on roads with higher speed limits they must extend the duration time of yellow lights so that drivers are able to stop safely before the light turns red. Examples of kinematics in sports include cross-country running, which involves constant-speed motion, distance, and displacement; and the motion of a volleyball, which can be approximated using projectile motion.

Inquiry Overview

This multipart inquiry-based investigation introduces students to concepts in kinematics in one and two dimensions. Students perform three guided-inquiry investigations that involve the study of constant velocity (Part I), constant acceleration (Part II), and projectile motion (Part III), which simultaneously involves constant velocity horizontally and constant acceleration vertically.

Through guided inquiry, students are provided with a track that includes an inclined section and a horizontal section. The students are tasked to determine if the motion on the horizontal section is constant velocity and if the motion on the inclined section is constant acceleration. They are then asked to determine how the initial velocity of the ball in projectile motion affects its horizontal motion from the time it leaves the track until it lands on the ground.

Connections to the AP Physics 1 Curriculum Framework

Big Idea 3 The interactions of an object with other objects can be described by forces.

Enduring Understanding	Learning Objectives
3A All forces share certain common characteristics when considered by observers in inertial reference frames.	**3.A.1.1** The student is able to express the motion of an object using narrative, mathematical, and graphical representations. (Science Practices 1.5, 2.1, and 2.2) **3.A.1.2** The student is able to design an experimental investigation of the motion of an object. (Science Practice 4.2) **3.A.1.3** The student is able to analyze experimental data describing the motion of an object and is able to express the results of the analysis using narrative, mathematical, and graphical representations. (Science Practice 5.1)

[**NOTE:** In addition to those listed in the learning objectives above, Science Practice 4.3 is also addressed in this investigation.]

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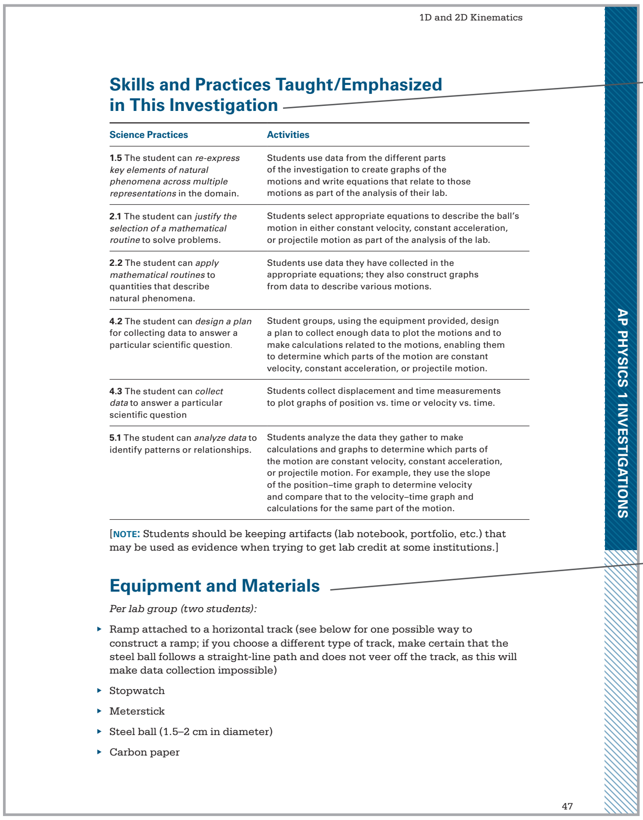
Suggestions for helping students make connections to the real-world application of the physics principles they are investigating.

Describes the scenario of the investigation and the levels of inquiry (e.g., guided versus open) present in each part of the investigation.

Connections to big ideas, enduring understandings, learning objectives, and science practices demonstrate the alignment to the curriculum framework as well as what students should *know* and be able to *do* after completing the investigation.



1D and 2D Kinematics

AP PHYSICS 1 INVESTIGATIONS

Skills and Practices Taught/Emphasized in This Investigation

Science Practices	Activities
1.5 The student can *re-express key elements of natural phenomena across multiple representations* in the domain.	Students use data from the different parts of the investigation to create graphs of the motions and write equations that relate to those motions as part of the analysis of their lab.
2.1 The student can *justify the selection of a mathematical routine* to solve problems.	Students select appropriate equations to describe the ball's motion in either constant velocity, constant acceleration, or projectile motion as part of the analysis of the lab.
2.2 The student can *apply mathematical routines* to quantities that describe natural phenomena.	Students use data they have collected in the appropriate equations; they also construct graphs from data to describe various motions.
4.2 The student can *design a plan* for collecting data to answer a particular scientific question.	Student groups, using the equipment provided, design a plan to collect enough data to plot the motions and to make calculations related to the motions, enabling them to determine which parts of the motion are constant velocity, constant acceleration, or projectile motion.
4.3 The student can *collect data* to answer a particular scientific question	Students collect displacement and time measurements to plot graphs of position vs. time or velocity vs. time.
5.1 The student can *analyze data* to identify patterns or relationships.	Students analyze the data they gather to make calculations and graphs to determine which parts of the motion are constant velocity, constant acceleration, or projectile motion. For example, they use the slope of the position–time graph to determine velocity and compare that to the velocity–time graph and calculations for the same part of the motion.

[**NOTE:** Students should be keeping artifacts (lab notebook, portfolio, etc.) that may be used as evidence when trying to get lab credit at some institutions.]

Equipment and Materials

Per lab group (two students):

- Ramp attached to a horizontal track (see below for one possible way to construct a ramp; if you choose a different type of track, make certain that the steel ball follows a straight-line path and does not veer off the track, as this will make data collection impossible)
- Stopwatch
- Meterstick
- Steel ball (1.5–2 cm in diameter)
- Carbon paper

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This chart explains how each activity in the investigation relates to the science practices in the curriculum framework.

Details the recommended equipment and materials for the investigation, as well as suggestions for purchasing, construction, and set-up where helpful.

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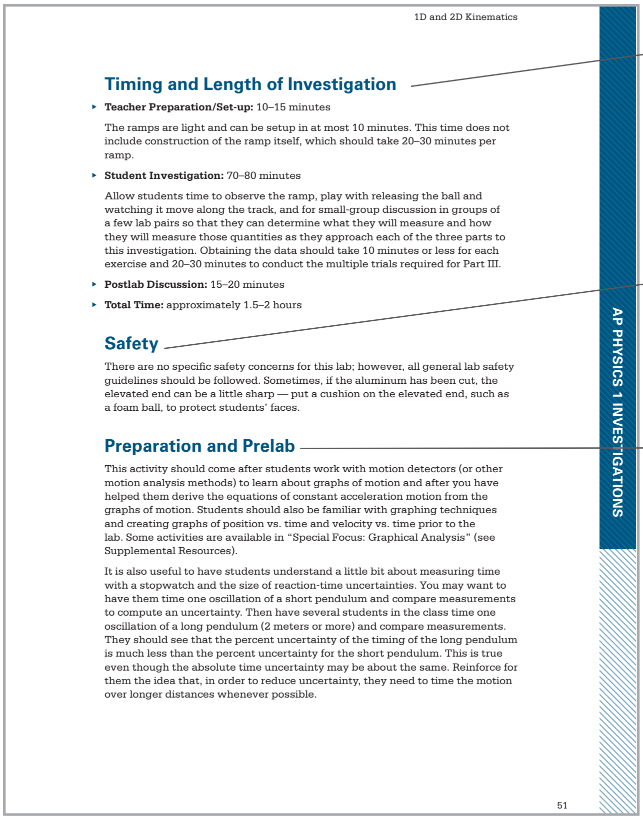

1D and 2D Kinematics

Timing and Length of Investigation

- **Teacher Preparation/Set-up:** 10–15 minutes

  The ramps are light and can be setup in at most 10 minutes. This time does not include construction of the ramp itself, which should take 20–30 minutes per ramp.

- **Student Investigation:** 70–80 minutes

  Allow students time to observe the ramp, play with releasing the ball and watching it move along the track, and for small-group discussion in groups of a few lab pairs so that they can determine what they will measure and how they will measure those quantities as they approach each of the three parts to this investigation. Obtaining the data should take 10 minutes or less for each exercise and 20–30 minutes to conduct the multiple trials required for Part III.

- **Postlab Discussion:** 15–20 minutes
- **Total Time:** approximately 1.5–2 hours

Safety

There are no specific safety concerns for this lab; however, all general lab safety guidelines should be followed. Sometimes, if the aluminum has been cut, the elevated end can be a little sharp — put a cushion on the elevated end, such as a foam ball, to protect students' faces.

Preparation and Prelab

This activity should come after students work with motion detectors (or other motion analysis methods) to learn about graphs of motion and after you have helped them derive the equations of constant acceleration motion from the graphs of motion. Students should also be familiar with graphing techniques and creating graphs of position vs. time and velocity vs. time prior to the lab. Some activities are available in "Special Focus: Graphical Analysis" (see Supplemental Resources).

It is also useful to have students understand a little bit about measuring time with a stopwatch and the size of reaction-time uncertainties. You may want to have them time one oscillation of a short pendulum and compare measurements to compute an uncertainty. Then have several students in the class time one oscillation of a long pendulum (2 meters or more) and compare measurements. They should see that the percent uncertainty of the timing of the long pendulum is much less than the percent uncertainty for the short pendulum. This is true even though the absolute time uncertainty may be about the same. Reinforce for them the idea that, in order to reduce uncertainty, they need to time the motion over longer distances whenever possible.

AP PHYSICS 1 INVESTIGATIONS

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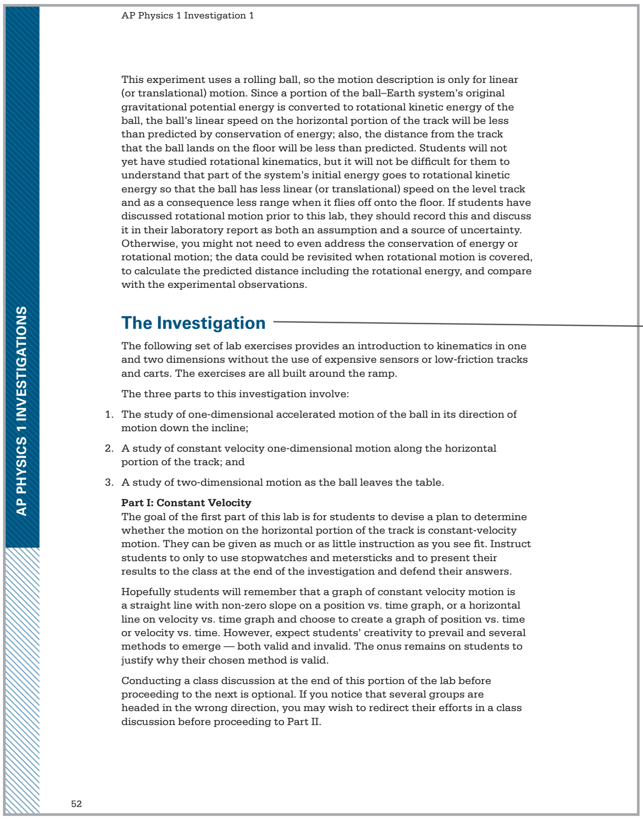
Gives guidance on the amount of time necessary for teacher preparation, student investigation, and discussion.

Suggestions for avoiding potential safety concerns or misuse of equipment during the investigation.

Suggestions for determining students' knowledge and skill levels and for preparing them for the laboratory investigation.



AP Physics 1 Investigation 1

This experiment uses a rolling ball, so the motion description is only for linear (or translational) motion. Since a portion of the ball–Earth system's original gravitational potential energy is converted to rotational kinetic energy of the ball, the ball's linear speed on the horizontal portion of the track will be less than predicted by conservation of energy; also, the distance from the track that the ball lands on the floor will be less than predicted. Students will not yet have studied rotational kinematics, but it will not be difficult for them to understand that part of the system's initial energy goes to rotational kinetic energy so that the ball has less linear (or translational) speed on the level track and as a consequence less range when it flies off onto the floor. If students have discussed rotational motion prior to this lab, they should record this and discuss it in their laboratory report as both an assumption and a source of uncertainty. Otherwise, you might not need to even address the conservation of energy or rotational motion; the data could be revisited when rotational motion is covered, to calculate the predicted distance including the rotational energy, and compare with the experimental observations.

The Investigation

The following set of lab exercises provides an introduction to kinematics in one and two dimensions without the use of expensive sensors or low-friction tracks and carts. The exercises are all built around the ramp.

The three parts to this investigation involve:

1. The study of one-dimensional accelerated motion of the ball in its direction of motion down the incline;
2. A study of constant velocity one-dimensional motion along the horizontal portion of the track; and
3. A study of two-dimensional motion as the ball leaves the table.

Part I: Constant Velocity
The goal of the first part of this lab is for students to devise a plan to determine whether the motion on the horizontal portion of the track is constant-velocity motion. They can be given as much or as little instruction as you see fit. Instruct students to only to use stopwatches and metersticks and to present their results to the class at the end of the investigation and defend their answers.

Hopefully students will remember that a graph of constant velocity motion is a straight line with non-zero slope on a position vs. time graph, or a horizontal line on velocity vs. time graph and choose to create a graph of position vs. time or velocity vs. time. However, expect students' creativity to prevail and several methods to emerge — both valid and invalid. The onus remains on students to justify why their chosen method is valid.

Conducting a class discussion at the end of this portion of the lab before proceeding to the next is optional. If you notice that several groups are headed in the wrong direction, you may wish to redirect their efforts in a class discussion before proceeding to Part II.

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This section provides an overview of the investigation and guidance on how to engage students in developing and carrying out the different components of the lab, as well as their own procedures to collect data and evidence.

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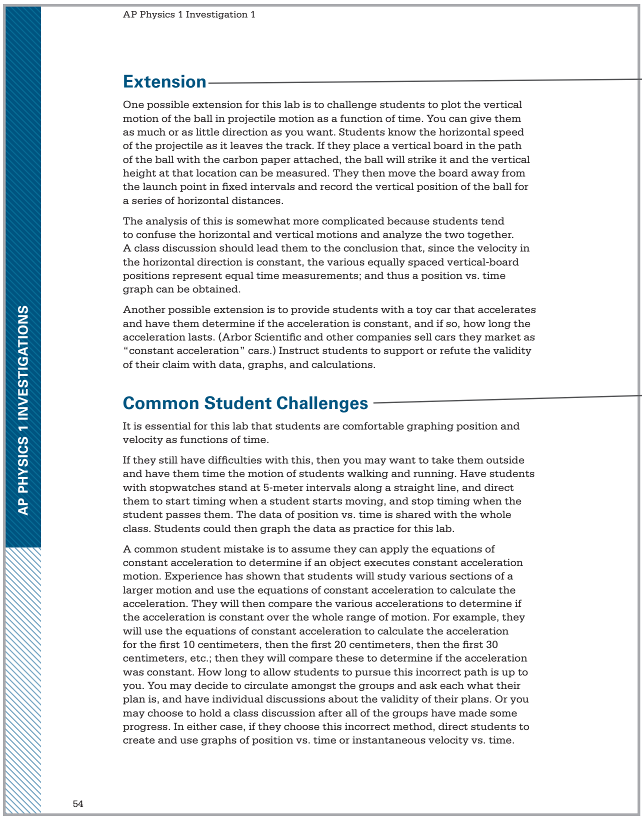


Extension

One possible extension for this lab is to challenge students to plot the vertical motion of the ball in projectile motion as a function of time. You can give them as much or as little direction as you want. Students know the horizontal speed of the projectile as it leaves the track. If they place a vertical board in the path of the ball with the carbon paper attached, the ball will strike it and the vertical height at that location can be measured. They then move the board away from the launch point in fixed intervals and record the vertical position of the ball for a series of horizontal distances.

The analysis of this is somewhat more complicated because students tend to confuse the horizontal and vertical motions and analyze the two together. A class discussion should lead them to the conclusion that, since the velocity in the horizontal direction is constant, the various equally spaced vertical-board positions represent equal time measurements; and thus a position vs. time graph can be obtained.

Another possible extension is to provide students with a toy car that accelerates and have them determine if the acceleration is constant, and if so, how long the acceleration lasts. (Arbor Scientific and other companies sell cars they market as "constant acceleration" cars.) Instruct students to support or refute the validity of their claim with data, graphs, and calculations.

Common Student Challenges

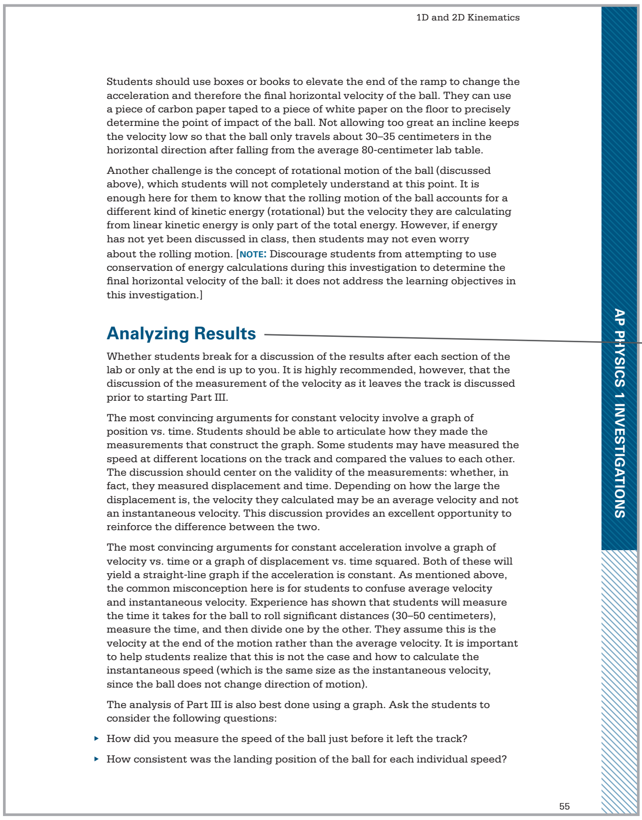
It is essential for this lab that students are comfortable graphing position and velocity as functions of time.

If they still have difficulties with this, then you may want to take them outside and have them time the motion of students walking and running. Have students with stopwatches stand at 5-meter intervals along a straight line, and direct them to start timing when a student starts moving, and stop timing when the student passes them. The data of position vs. time is shared with the whole class. Students could then graph the data as practice for this lab.

A common student mistake is to assume they can apply the equations of constant acceleration to determine if an object executes constant acceleration motion. Experience has shown that students will study various sections of a larger motion and use the equations of constant acceleration to calculate the acceleration. They will then compare the various accelerations to determine if the acceleration is constant over the whole range of motion. For example, they will use the equations of constant acceleration to calculate the acceleration for the first 10 centimeters, then the first 20 centimeters, then the first 30 centimeters, etc.; then they will compare these to determine if the acceleration was constant. How long to allow students to pursue this incorrect path is up to you. You may decide to circulate amongst the groups and ask each what their plan is, and have individual discussions about the validity of their plans. Or you may choose to hold a class discussion after all of the groups have made some progress. In either case, if they choose this incorrect method, direct students to create and use graphs of position vs. time or instantaneous velocity vs. time.

Optional classroom or laboratory activities to further guide students to achieve related learning objectives and develop the science practices.

Gives potential student challenges and misconceptions based on educators' experiences and physics education research.



Students should use boxes or books to elevate the end of the ramp to change the acceleration and therefore the final horizontal velocity of the ball. They can use a piece of carbon paper taped to a piece of white paper on the floor to precisely determine the point of impact of the ball. Not allowing too great an incline keeps the velocity low so that the ball only travels about 30–35 centimeters in the horizontal direction after falling from the average 80-centimeter lab table.

Another challenge is the concept of rotational motion of the ball (discussed above), which students will not completely understand at this point. It is enough here for them to know that the rolling motion of the ball accounts for a different kind of kinetic energy (rotational) but the velocity they are calculating from linear kinetic energy is only part of the total energy. However, if energy has not yet been discussed in class, then students may not even worry about the rolling motion. [**NOTE:** Discourage students from attempting to use conservation of energy calculations during this investigation to determine the final horizontal velocity of the ball: it does not address the learning objectives in this investigation.]

Analyzing Results

Whether students break for a discussion of the results after each section of the lab or only at the end is up to you. It is highly recommended, however, that the discussion of the measurement of the velocity as it leaves the track is discussed prior to starting Part III.

The most convincing arguments for constant velocity involve a graph of position vs. time. Students should be able to articulate how they made the measurements that construct the graph. Some students may have measured the speed at different locations on the track and compared the values to each other. The discussion should center on the validity of the measurements: whether, in fact, they measured displacement and time. Depending on how the large the displacement is, the velocity they calculated may be an average velocity and not an instantaneous velocity. This discussion provides an excellent opportunity to reinforce the difference between the two.

The most convincing arguments for constant acceleration involve a graph of velocity vs. time or a graph of displacement vs. time squared. Both of these will yield a straight-line graph if the acceleration is constant. As mentioned above, the common misconception here is for students to confuse average velocity and instantaneous velocity. Experience has shown that students will measure the time it takes for the ball to roll significant distances (30–50 centimeters), measure the time, and then divide one by the other. They assume this is the velocity at the end of the motion rather than the average velocity. It is important to help students realize that this is not the case and how to calculate the instantaneous speed (which is the same size as the instantaneous velocity, since the ball does not change direction of motion).

The analysis of Part III is also best done using a graph. Ask the students to consider the following questions:

- How did you measure the speed of the ball just before it left the track?
- How consistent was the landing position of the ball for each individual speed?

Guidance on the qualitative and quantitative analysis students may utilize to draw conclusions from their data.

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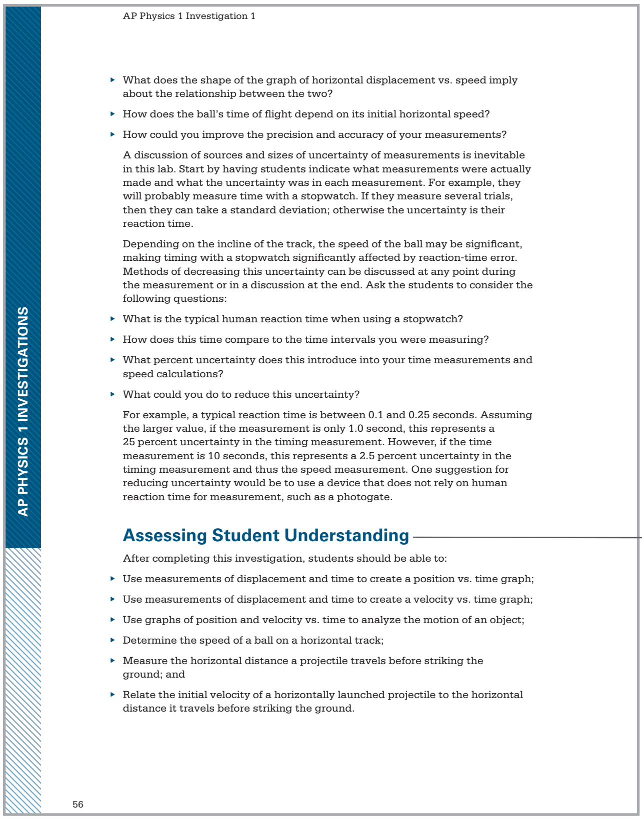

AP Physics 1 Investigation 1

AP PHYSICS 1 INVESTIGATIONS

- What does the shape of the graph of horizontal displacement vs. speed imply about the relationship between the two?
- How does the ball's time of flight depend on its initial horizontal speed?
- How could you improve the precision and accuracy of your measurements?

A discussion of sources and sizes of uncertainty of measurements is inevitable in this lab. Start by having students indicate what measurements were actually made and what the uncertainty was in each measurement. For example, they will probably measure time with a stopwatch. If they measure several trials, then they can take a standard deviation; otherwise the uncertainty is their reaction time.

Depending on the incline of the track, the speed of the ball may be significant, making timing with a stopwatch significantly affected by reaction-time error. Methods of decreasing this uncertainty can be discussed at any point during the measurement or in a discussion at the end. Ask the students to consider the following questions:

- What is the typical human reaction time when using a stopwatch?
- How does this time compare to the time intervals you were measuring?
- What percent uncertainty does this introduce into your time measurements and speed calculations?
- What could you do to reduce this uncertainty?

For example, a typical reaction time is between 0.1 and 0.25 seconds. Assuming the larger value, if the measurement is only 1.0 second, this represents a 25 percent uncertainty in the timing measurement. However, if the time measurement is 10 seconds, this represents a 2.5 percent uncertainty in the timing measurement and thus the speed measurement. One suggestion for reducing uncertainty would be to use a device that does not rely on human reaction time for measurement, such as a photogate.

Assessing Student Understanding

After completing this investigation, students should be able to:

- Use measurements of displacement and time to create a position vs. time graph;
- Use measurements of displacement and time to create a velocity vs. time graph;
- Use graphs of position and velocity vs. time to analyze the motion of an object;
- Determine the speed of a ball on a horizontal track;
- Measure the horizontal distance a projectile travels before striking the ground; and
- Relate the initial velocity of a horizontally launched projectile to the horizontal distance it travels before striking the ground.

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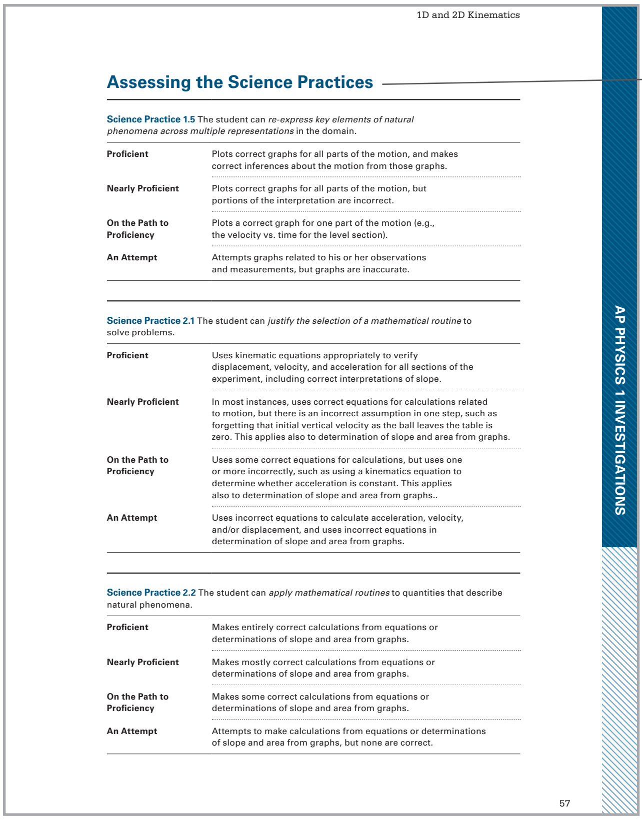
Describes what claims and connections students should be able to justify based on the evidence provided in the data they analyze.



1D and 2D Kinematics

Assessing the Science Practices

Science Practice 1.5 The student can *re-express key elements of natural phenomena across multiple representations* in the domain.

Proficient	Plots correct graphs for all parts of the motion, and makes correct inferences about the motion from those graphs.
Nearly Proficient	Plots correct graphs for all parts of the motion, but portions of the interpretation are incorrect.
On the Path to Proficiency	Plots a correct graph for one part of the motion (e.g., the velocity vs. time for the level section).
An Attempt	Attempts graphs related to his or her observations and measurements, but graphs are inaccurate.

Science Practice 2.1 The student can *justify the selection of a mathematical routine* to solve problems.

Proficient	Uses kinematic equations appropriately to verify displacement, velocity, and acceleration for all sections of the experiment, including correct interpretations of slope.
Nearly Proficient	In most instances, uses correct equations for calculations related to motion, but there is an incorrect assumption in one step, such as forgetting that initial vertical velocity as the ball leaves the table is zero. This applies also to determination of slope and area from graphs.
On the Path to Proficiency	Uses some correct equations for calculations, but uses one or more incorrectly, such as using a kinematics equation to determine whether acceleration is constant. This applies also to determination of slope and area from graphs..
An Attempt	Uses incorrect equations to calculate acceleration, velocity, and/or displacement, and uses incorrect equations in determination of slope and area from graphs.

Science Practice 2.2 The student can *apply mathematical routines* to quantities that describe natural phenomena.

Proficient	Makes entirely correct calculations from equations or determinations of slope and area from graphs.
Nearly Proficient	Makes mostly correct calculations from equations or determinations of slope and area from graphs.
On the Path to Proficiency	Makes some correct calculations from equations or determinations of slope and area from graphs.
An Attempt	Attempts to make calculations from equations or determinations of slope and area from graphs, but none are correct.

AP PHYSICS 1 INVESTIGATIONS

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Provides rubrics for evaluating the application of the science practices aligned to the learning objectives addressed in the investigation.

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Supplemental Resources

Drake, Stillman. *Galileo: Two New Sciences.* Madison, Wisconsin: University of Wisconsin Press, 1974.

"Mechanics: 1-Dimensional Kinematics." The Physics Classroom. Accessed September 1, 2014. http://www.physicsclassroom.com/calcpad/1dkin/problems.cfm. [*This website allows students to explore extra practice problems on kinematics.*]

"The Moving Man." PhET. University of Colorado Boulder. Accessed September 1, 2014. http://phet.colorado.edu/en/simulation/moving-man. [*This simulation provides an interactive way to learn about position, velocity, and acceleration graphs.*]

The Physlet Resource. Davidson College. Accessed September 1, 2014. http://webphysics.davidson.edu/physlet_resources. [*This resource provides sample "physlet" illustrations, explorations, and problems in 1-dimensional kinematics.*]

"Projectile Motion." PhET. University of Colorado Boulder. Accessed September 1, 2014. http://phet.colorado.edu/en/simulation/projectile-motion. [*Provides multiple visual representations of kinematics in one and two dimensions.*]

"Special Focus: Graphical Analysis." AP Physics 2006–2007 Professional Development Workshop Materials. College Board. Accessed September 1, 2014. http://apcentral.collegeboard.com/apc/public/repository/AP_Physics_Graphical_Analysis.pdf.

Supplemental resources for the teacher, including online simulations and activities that are useful as prelab, postlab, or extensions to the investigation.

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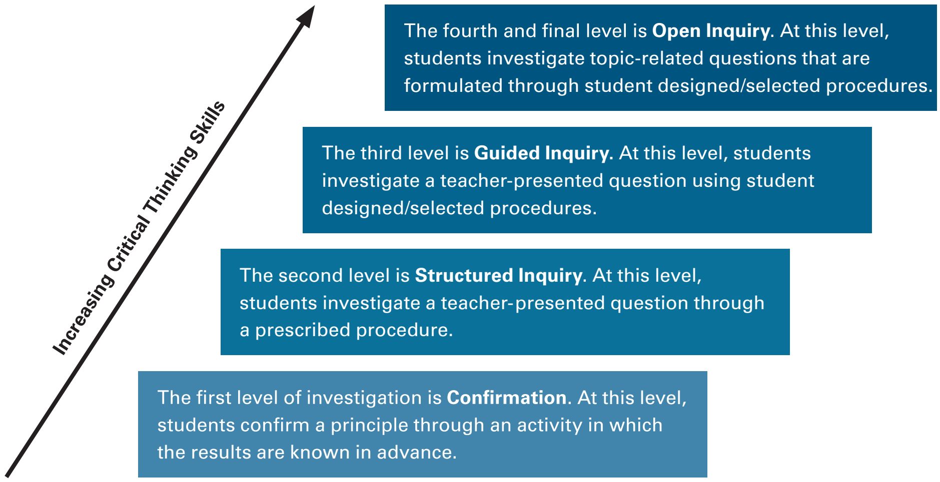
Chapter 4:
Creating an Inquiry-Based Learning Environment

"The problem is to provide students with enough Socratic guidance to lead them into the thinking and the forming of insights but not so much as to give everything away and thus destroy the attendant intellectual experience (I deliberately use the word 'guidance' to imply a distinction between this mode and the more conventional modes of providing instructions and answers)." (Arons, 1993)

Integrating Inquiry-Based Learning

Although laboratory work has often been separated from classroom work, research shows that experience and experiment are often more instructionally effective when flexibly integrated into the development of concepts. When students build their own conceptual understanding of the principles of physics, their familiarity with the concrete evidence for their ideas leads to deeper understanding and gives them a sense of ownership of the knowledge they have constructed.

Scientific inquiry experiences in AP courses should be designed and implemented with increasing student involvement to help enhance inquiry learning and the development of critical-thinking and problem-solving skills and abilities. Adaptations of Herron's approach (1971) and that of Rezba, Auldridge, and Rhea (1999) define inquiry instruction for investigations in four incremental ways:



The fourth and final level is **Open Inquiry**. At this level, students investigate topic-related questions that are formulated through student designed/selected procedures.

The third level is **Guided Inquiry**. At this level, students investigate a teacher-presented question using student designed/selected procedures.

The second level is **Structured Inquiry**. At this level, students investigate a teacher-presented question through a prescribed procedure.

The first level of investigation is **Confirmation**. At this level, students confirm a principle through an activity in which the results are known in advance.

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Typically, the level of investigations in an AP classroom should focus primarily on the continuum between guided inquiry and open inquiry. However, depending on students' familiarity with a topic, a given laboratory experience might incorporate a sequence involving all four levels or a subset of them. For instance, students might first carry out a simple confirmation investigation that also familiarizes them with equipment, and then proceed to a structured inquiry that probes more deeply into the topic and gives more practice with equipment. They would then be presented with a question and asked to design/select their own procedure. A class discussion of results could then lead to student-formulated questions that could be explored differently by different groups in open inquiry.

The idea of asking questions and inquiry is actually natural to students. However, in the classroom setting it may not seem natural to them as they may have developed more teacher-directed procedural habits and expectations in previous lab courses. As students experience more opportunities for self-directed investigations with less teacher guidance, they will become more sophisticated in their reasoning and approach to inquiry. You can promote inquiry habits in students throughout the course — during class and in the laboratory — by handing over to them more of the planning of experiments and manipulation of equipment.

Getting Students Started with Their Investigations

There are no prescriptive "steps" to the iterative process of inquiry-based investigations. However, there are some common characteristics of inquiry that will support students in designing their investigations. Often, this simply begins with using the learning objectives to craft a question for students to investigate. You may choose to give students a list of materials they are allowed to utilize in their design or require that students request the equipment they feel they need to investigate the question.

Working with learning objectives to craft questions may include:

- Selecting learning objectives from the curriculum framework that relate to the subject under study, and which may set forth specific tasks, in the form of "Design an experiment to…".

  For Example:

  > **Learning Objective 3.B.3.2:** The student is able to design a plan and collect data in order to ascertain the characteristics of the motion of a system undergoing oscillatory motion caused by a restoring force.
  >
  > Students are asked to: Design a plan and collect data in order to ascertain the characteristics of the motion of a system undergoing oscillatory motion caused by a restoring force.

- Rephrasing or refining the learning objectives that align to the unit of study to create an inquiry-based investigation for students.

For Example:

> **Learning Objective 3.B.3.1:** The student is able to predict which properties determine the motion of a simple harmonic oscillator and what the dependence of the motion is on those properties.
>
> Students are asked to: Make predictions about what properties determine the motion of a simple harmonic oscillator and then design an experiment that tests your predictions and allows for analysis of the dependence of the motion on those properties.

After students are given a question for investigation, they may:

- Refine the question you posed so the question or purpose best fits the experimental design.
- In small groups, prior to lab day, determine how they will manipulate the equipment to accomplish the goal and how they will process the data. (This may involve some initial play with the equipment to inform their plan.) Students should record their predictions and assumptions prior to collecting data.
- Conduct the experiment and then develop and record their analysis. The analysis should include a discussion of their prior predictions and assumptions as well as possible sources of uncertainty.
- Present their findings either in a written or oral report to you and the class for feedback and critique on their final design and results. Students should be encouraged to critique and challenge one another's claims based on the evidence collected during the investigation. (See chapter 5 for further information on scientific argumentation strategies).

Students should be given latitude to make design modifications or ask for additional equipment appropriate for their design. It is also helpful for individual groups to report out to the class on their basic design to elicit feedback on feasibility. With you as a guide, student groups proceed through the experiment, while you allow them the freedom to make mistakes — as long as those mistakes don't endanger students or equipment or lead the groups too far off task. Students should also have many opportunities for postlab reporting so that groups can hear of the successes and challenges of individual lab designs.

Demonstrations:

Can demonstrations occasionally count as inquiry laboratory experience if necessary? In the high school classroom, where equipment can be limited, teachers can compensate by implementing "Interactive Lecture Demonstrations" (Sokoloff and Thornton, 1997). Such demonstrations can be effectively used as low-tech alternatives by having students make and record their predictions, assist in carrying out the demonstration, record and evaluate the data, and present their conclusions. With the right guiding questions, even larger classes can be effectively engaged in such demonstration experiments, and demonstration experiments may be designed to involve students in several different levels of inquiry.

Simulations:

There are now a large variety of well-designed simulations available for physics (e.g., those on the PhET website) that can be used to allow students to investigate areas such as solar-system dynamics in a virtual laboratory where they can modify masses and orbital parameters in the simulation to explore and analyze relationships among variables. In a recent review, several studies have shown that a course sequence that includes both real and virtual laboratory experiments may be more effective than either alone (de Jong, Linn, and Zacharia, 2013). Simulations in addition to hands-on lab investigations can greatly benefit students, but simulations alone should not be considered a substitute for labs and should not represent a significant amount of authentic lab investigation time.

Creating a Safe Environment for Investigation

Giving students the responsibility for the design of their own laboratory experience involves special responsibilities for teachers. To ensure a safe working environment, you must, up front, provide the limitations and safety precautions necessary for potential procedures and equipment students may use during their investigation. You should also provide specific guidelines prior to students' discussion on investigation designs for each experiment, so that those precautions can be incorporated into final student-selected lab design and included in the background or design plan in a laboratory record. It may also be helpful to print the precautions that apply to that specific lab as Safety Notes to place on the desk or wall near student workstations. Additionally, a general set of safety guidelines should be set forth for students at the beginning of the course. The following is a list of possible general guidelines you may post:

- Before every lab, make sure you know and record the potential hazards involved in the investigation and the precautions you will take to stay safe.
- Before using equipment, make sure you know the proper use to acquire good data and avoid damage to equipment.
- Know where safety equipment is located in the lab, such as the fire extinguisher, safety goggles, and the first aid kit.
- Follow the teacher's special safety guidelines as set forth prior to each experiment. (Students should record these as part of their design plan for a lab.)
- When in doubt about the safety or advisability of a procedure, check with the teacher before proceeding.

You should interact constantly with students as they work to observe safety practices and anticipate and discuss with them any problems that may arise. Walking among student groups, asking questions, and showing interest in students' work allows you to keep the pulse of what students are doing as well as maintain a watchful eye for potential safety issues.

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Material and Equipment Use

A wide range of equipment may be used in the physics laboratory, from generic lab items, such as metersticks, rubber balls, springs, string, metal spheres, calibrated mass sets, beakers, glass and cardboard tubes, electronic balances, stopwatches, clamps, and ring stands to items more specific to physics, such as tracks, carts, light bulbs, resistors, magnets, and batteries. Successful guided-inquiry student work can be accomplished both with simple, inexpensive materials and with more sophisticated physics equipment, such air tracks, force sensors, and oscilloscopes. Use the inquiry-based labs in this manual, with equipment listed for each experiment, to get an idea of what equipment is necessary for the lab. However, remembering that the AP lab should provide experience for students equivalent to that of a college laboratory, you should make every effort to provide a range of experiences – from those experiments students contrive from plumbing pipe, string, and duct tape to experiments in which students gather and analyze data using calculators or computer-interfaced equipment.

Chapter 5:

The Role of the Science Practices

Students begin their study of physics with some prior knowledge based on their experiences in the physical world — knowledge that serves them well in particular contexts. This knowledge is often piecemeal, may be unarticulated, and is often not able to be explicitly organized into any broad coherent scheme. Some of their ideas are scientifically accurate, while others may be partially correct or incorrect according to our present collective understanding of natural phenomena. Research in physics education has shown that merely telling students about concepts in physics has little effect on their conceptual understanding.

The role of inquiry-based physics laboratory investigations is to provide students with the opportunity to design and carry out organized investigations of the physical world, to analyze their observations in an attempt to find coherent patterns that can serve as a basis for developing conceptual and mathematical models of phenomena, and, ultimately, to organize and consolidate their understanding of these models within the theories of physics. Laboratory investigations provide students with opportunities to experience and observe phenomena by engaging in science practices, whereby they design and carry out organized investigations of phenomena in order to build models and test predictions stemming from the models they have constructed. Through this process, students begin to value the reasoning skills associated with the construction of knowledge within the scientific community.

Throughout the study of the history and philosophy of science, there are 10 key points that have emerged about the development of scientific knowledge over time. In total, these points lead to one key conclusion: science is not a body of theories and laws but rather an approach to understanding observations that allows us to make sense of the world around us. If we think about these key points, they can help us understand the reasons for using inquiry in the physics laboratory.

These key points about the nature of science (as modified from McComas, 2004) are:

- Scientific knowledge is tentative but durable.
- Laws and theories serve different roles in science and are not hierarchal relative to one another.
- There is no universal step-by-step scientific method.
- Science is a highly creative endeavor, grounded by theory.
- Scientific knowledge relies heavily, but not entirely, on observation, experimental evidence, rational arguments, creativity, and skepticism.
- Scientific progress is characterized by competition between rival theories.
- Scientists can interpret the same experimental data differently.

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- Development of scientific theories at times is based on inconsistent foundations.
- There are historical, cultural, and social influences on science.
- Science and technology impact each other, but they are not the same.

The science practices that align to the concept outline of the curriculum framework capture important aspects of the work that scientists engage in, at the level of competence expected of AP Physics students. AP Physics teachers will see how these practices are articulated within each learning objective and therefore allow laboratory investigations and instruction to emphasize both content and scientific practice.

The seven science practices for physics are set out in Appendix A.

Assessing the Science Practices

Each investigation in this manual includes sample rubrics specific to that particular lab that may be used to assess students' understanding of the science practices. Each rubric provides you with guidance on the range of student understanding and proficiency for each science practice. The rubrics provided align directly to the investigation's learning objectives. However, the very nature of inquiry-based investigation often elicits numerous other science practices that you may also want to assess.

Accordingly, a sample rubric that serves all of the science practices is set out in Appendix B, which can be used to assess student understanding in any lab you use.

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Chapter 6:
Overview of Quantitative Analysis

Experimental physics relies heavily on quantitative skills and analysis. Successful data collection and analysis in the AP Physics laboratory requires many skills, including making accurate and precise measurements of quantities using a range of instruments, converting units, making estimates, carrying out algebraic and statistical calculations, and constructing and interpreting graphs. The information obtained using these skills must then be integrated with reasoning and higher-order thinking skills in order for students to successfully analyze and interpret data and formulate and communicate conclusions. This chapter describes how the investigations in this manual can support the development and successful application of the quantitative skills needed in the AP Physics laboratory.

How Quantitative Skills Are Addressed in This Manual

Most students come to AP Physics with some quantitative skills from earlier course work. The 16 investigations in this manual provide opportunities for students to practice and improve their existing skills and develop new skills. Since each investigation by its nature demands a different combination of skills, the particular skills needed for each lab activity are described in the section "Skills and Practices Taught/Emphasized in This Investigation."

For example, in *AP Physics 1 Investigation #6: Harmonic Motion*, students first use Science Practice 4.2 (design a plan for collecting data) as they make a plan to determine the factors that affect the period of a simple pendulum. They are then asked to graph the period as a function of mass, angle, and length, and to derive an equation relating period and length, involving Science Practice 1.4 (use representations and models) and Science Practice 2.2 (apply mathematical routines).

In choosing lab investigations, you should assess your students' existing skills in order to plan appropriate instruction prior to the lab, anticipate student challenges and questions, and provide extra monitoring during the lab. You should also consider what new skills you want your students to develop while carrying out the investigation.

Most of the labs in this manual contain a prelab or preparation component in order to introduce new techniques and equipment and reinforce fundamental skills before students undertake the investigation and apply their quantitative and other skills to address the central challenge of the lab.

Key Quantitative Skills in AP Physics Labs

The most important quantitative skills in the AP Physics laboratory can be roughly classified into four types: measuring that includes estimation of uncertainties, calculating, creating representations such as tables and graphs, and analyzing results. In practice and as seen in the examples below, these skills can overlap, since in the laboratory setting they are often used in conjunction with each other to accomplish experimental goals.

Measuring

Measurement skills encompass choosing the appropriate measuring tools for different tasks, calibrating the tools that require calibrating, using the tools to make accurate and precise measurements, and making and recording appropriate estimates of uncertainty. Tools needed for the investigations in this manual range from metersticks to motion sensors to video analysis software. Students might not have gained prior experience with some of these tools prior to their first AP Physics course, so you might need to provide instruction on how to use a particular tool the first time they encounter it. You can do this by demonstrating a technique yourself or showing them a short video or simulation that demonstrates the device (several of the labs in this manual provide references to such materials).

During a lab, you should monitor students' measuring technique, estimation of uncertainty, and use of significant figures, providing feedback and coaching as needed. Emphasize the importance of carefully taking multiple measurements in order to help students realize that sufficient and accurate data is essential to solve problems and answer questions. No amount of analysis or calculation can make up for insufficient data.

Calculating

Most labs require several types of calculations. These include converting units, solving for unknowns in algebraic calculations, and making statistical calculations such as percent error and standard deviation. Many calculations involve using logarithms and scientific notation. Some of these calculations mirror those required in other parts of the AP Physics course, so you will find it helpful to discuss connections between the content students are studying and the calculations they are doing as part of other tasks, such as homework problems. For example, in *AP Physics 2 Investigation #1: Boyle's Law*, students need to use mathematical equations to calculate the area of the piston and volume of gas for each value of height. Students also need to calculate the area under the PV graph that represents the work done on the system by the external force. These types of calculations should be familiar to students. If an experiment uses mathematical routines that students have not yet studied in other parts of the course, you might need to model calculations related to the content or provide resources for student reference.

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Most students will have had experience with unit conversions, simple algebraic calculations, and the use of significant figures and scientific notation; but they may not have performed calculations of uncertainty, and may require instruction in such calculations as mean, standard deviation, and percent error, which are central to data analysis and interpretation. Labs also provide an opportunity for students to use spreadsheet programs, such as Microsoft Excel®, and graphing programs, such as Vernier Graphical Analysis™, to perform calculations and share data. Facility with spreadsheets is a valuable skill both within and beyond AP Physics, and it is well worth the instructional time for you to demonstrate their use and for students to practice with them (see Resources at the end of this chapter for links to online tutorials on Excel).

The following labs in this manual provide opportunities for students to use spreadsheets and/or graphing programs for data analysis:

- *AP Physics 1 Investigation #1: 1D and 2D Kinematics*. Students tabulate their data and graph the suggested position vs. time and velocity vs. time relationships.
- *AP Physics 1 Investigation #5: Impulse and Momentum*. Students tabulate data and graph position vs. time to calculate the velocity of the carts before and after the collision, and/or use a data table and equation definition to multiply mass times velocity to calculate momentum for each trial.
- *AP Physics 1 Investigation #6: Harmonic Motion*. Students can use average and standard deviation spreadsheet functions to analyze data from multiple measurements of the pendulum period.
- *AP Physics 1 Investigation #7: Rotational Motion*. In the quantitative part of this lab, students determine the speed of objects arriving at the bottom of a ramp. They can use a spreadsheet to plot a position vs. time graph and take a slope, and for multiple trials they can create a data table and use spreadsheet functions to calculate averages and standard deviations.
- *AP Physics 2 Investigation #1: Boyle's Law*. Students design a plan to find the area under the curve. Solutions include "counting squares," making a best-fit line that includes and excludes approximately equal areas, using a graphing program with an integration function to plot the data and calculate the area, or using a spreadsheet and the method of trapezoids to find the area.
- *AP Physics 2 Investigation #2: Fluid Dynamics*. Students can use a graphical analysis program on a computer or calculator to go from an h vs. t graph to a v vs. t graph.
- *AP Physics 2 Investigation #7: Geometric Optics*. Students can use a spreadsheet to create graphs to determine the focal length of a lens.
- *AP Physics 2 Investigation #8: The Particle Model of Light*. Students can use a spreadsheet or graphing program to calculate the slope using a large number of data points.

Using Web tools such as Google Sheets® allows students to share data and collaborate online as well. For example, in *AP Physics 1 Investigation #6: Harmonic Motion*, two groups can investigate the effect of length on period, two groups can investigate the effect of mass on period, and another two groups can investigate the effect of angle on period. You can then create a Google spreadsheet that the groups can use to input and share their data, with one sheet for each of the variables: length, mass, and angle. All students would then make three graphs: period as a function of length, period as a function of mass, and period as a function of angle.

Tabulating data, generating graphs by hand, and interpreting graphs is an essential skill in AP Physics. Using software tools to generate and interpret graphs is not essential, but it is highly desirable because they allow for relatively fast analysis, are generally more accurate and precise, and are more professional in appearance than hand-drawn graphs. Use of spreadsheets for graphing and analysis is also a transferable skill that is useful in many other courses and scientific contexts.

Creating tables and graphs

Creating and using representations and models (Science Practice 1) are skills that are essential to organizing data in the laboratory for analysis. Learning to construct data tables requires students to:

a. Identify dependent and independent variables and controlled quantities;

b. Choose appropriate quantities and units for measurement; and

c. Use word processing or spreadsheet programs (if possible) as needed.

Presentation and analysis of data often calls for the use of graphs of different types. Generating and using graphs requires the skills of plotting coordinates, determining independent and dependent variables, and choosing appropriate scales. The laboratory section of the AP Physics 1 and 2 Exams requires graphing skills with questions relating to analyzing data, including displaying data in graphical or tabular form; fitting curves (which may be lines) to data points in graphs; determining slopes; and performing calculations with data or making extrapolations and interpolations from data.

These skills can be applied differently in different contexts. For example, in *AP Physics 1 Investigation #6: Harmonic Motion*, students use their data to make predictions about the period of a pendulum with a given mass, angle, and length by interpolating or extrapolating from their graphical data or by using the equation they obtained for period versus length to calculate a predicted value.

Different types of graphs may be appropriate for different types of analysis. In studying one-dimensional kinematics, graphing and graphical analysis of position, velocity, and acceleration versus time can be used to determine functional relationships. In energy experiments, energy bar graphs constructed for different configurations of a system are useful for tracking how energy is distributed in a system. PV diagrams are useful for representing thermodynamic processes as in the Boyle’s law experiment. Energy-level diagrams are used to represent atomic transitions in studying topics in modern physics.

Students should be able to recognize whether their laboratory data implies a linear, quadratic, inverse, inverse-square, exponential, logarithmic, sinusoidal, or power-law relationship. They should be familiar with graphing techniques that allow them to alter the graphical axes to show a linear relationship.

As with the measurement and calculation skills discussed earlier, students are likely to have prior experience with some graphing skills, while other uses of graphs may be new and will require you to provide instruction and extra guidance in order to support successful skill development.

Analyzing results

Data analysis and evaluation of evidence (Science Practice 5) helps to derive meaning out of quantitative information generated through measurement, calculations, and graphing. Often the most difficult part of the quantitative work in the laboratory, this process is essential in order to answer experimental questions successfully. Students can develop and strengthen these skills through practice over time in a variety of laboratory experiences. Three quantitative analysis tasks in laboratory work are discussed below: interpreting data, assessing accuracy and precision, and analysis of uncertainty (error analysis).

Interpreting data

Interpreting measured and calculated values in terms of the physical principles under study is something many students find challenging, especially as experiments and calculations become more sophisticated. A relatively simple example is in *AP Physics 1 Investigation #1: Kinematics*, where students measure position and velocity as functions of time. Once they make these measurements, they need to analyze the data in order to answer the central experimental question — how the initial velocity of the ball in projectile motion affects its horizontal motion from the time it leaves the track until it lands on the ground.

Assessing accuracy and precision

Quantitative analysis also includes assessing the accuracy and precision of results. In *AP Physics 1 Investigation #3: Circular Motion* for example, students make various measurements needed to make predictions about the period of motion of a conical pendulum. They need to decide how many trials are needed; determine if their measurements are sufficiently close together; decide if any of their calculations are outliers that should be discarded; compare the various groups' methods; and decide which methods were more precise than others. All of these decisions require analysis of precision and accuracy. The concept of uncertainty should also be part of the lab investigation that can be addressed prelab and/or postlab. For example, in *AP Physics 2 Investigation #4: Resistor Circuits* some students may automatically conclude that two potential differences are the same across branches in parallel if they are within 5–10 percent of each other. Other students may conclude they are different. This can lead to a discussion of the uncertainty in the meter measurements and the possible effects of the resistance of connecting wires. For lab problems on exams, discussion of accuracy, precision, and error analysis will primarily be

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qualitative. However, quantitative treatment of these is expected for college course work, so you should also ensure that students understand standard deviation as a measure of precision and percent error as a measure of accuracy, and that repetition makes for more robust data that enhances the physicists' ability to draw conclusions and answer questions. You could choose to delve further into statistical analysis with your students, addressing topics such as confidence intervals and *p*-values, if desired. Knowing how to use statistics to analyze the quality of laboratory results is extremely important in higher-level courses and an introduction to these concepts will thus benefit students.

Analysis of uncertainty (error analysis)

A third quantitative analysis task is analysis of uncertainty, or error analysis, which involves assessing likely sources of uncertainty and their effects on measured and calculated values. For example, in part I of *AP Physics 1 Investigation #7: Rotational Motion*, students make the necessary measurements and calculations needed for a kinematic analysis. They are then required to do an analysis of uncertainty to decide whether observed differences in the speeds of two objects are in fact different or simply due to uncertainties in the measurements.

SYSTEMATIC UNCERTAINTY includes uncertainty or errors due to the calibration of instruments and uncertainty due to faulty procedures or assumptions. Even if an instrument has been properly calibrated, it can still be used in a fashion that leads to systematically wrong (always either high or low) results. If instruments are calibrated and used correctly, then you can expect accurate results, but even the most basic measurements might include things such as parallax errors in measuring length or reading an analog meter and human reaction-time errors with a stopwatch, creating inaccuracies in results. Another common example of a systematic error in the physics lab is the assumption that air resistance is not a factor for a falling body, which makes real results inaccurate.

When the systematic uncertainties in an experiment are small, the experiment is said to be *accurate*. Accuracy is a measure of how close you are to the accepted answer.

RANDOM UNCERTAINTY include errors of judgment in reading a meter or a scale and uncertainties due to fluctuating experimental conditions. Because no instrument is infinitely precise, even if measurement conditions are not fluctuating, careful measurements of the same quantity by the same person will not yield the same result over multiple trials. For many measurements, environmental fluctuations (e.g., the temperature of the laboratory or the value of the line voltage) or small variations in starting conditions will necessarily give different results each time.

When the random uncertainties in an experiment are small, the experiment is said to be *precise*. Precision tells you how well you know the answer you have determined; it is how sure you are of your measurement or, alternatively, how unsure you are of your measurement, regardless of whether the measurement is accurate or correctly made.

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Expectations for error analysis on the AP Physics Exams

Experiment and data analysis questions on the AP Physics 1 and 2 Exams will not require students to calculate standard deviations, use formal methods to calculate the propagation of error, or carry out a calculated linear best-fit. However students should be able to:

- Discuss which measurement or variable in a procedure contributes most to overall uncertainty in the final result and on conclusions drawn from a given data set;
- Recognize that there may be no significant difference between two reported measurements if they differ by less than the smallest division on a scale;
- Reason in terms of percentage error;
- Report results of calculations to an appropriate number of significant digits;
- Construct an estimated best-fit line to data that they plot;
- Articulate the effects of error and error propagation on conclusions drawn from a given data set and how the results and conclusions would be affected by changing the number of measurements, measurement techniques, or precision of measurements; and
- Review and critique an experimental design or procedure and decide whether the conclusions can be justified based on the procedure and the evidence presented.

Expectations for error analysis in college

Some colleges and universities expect students to submit a laboratory notebook to receive credit for lab courses. Given the emphasis on time spent in the laboratory, students are expected to have been introduced to the formal methods of error analysis as presented in this chapter and to have carried out the procedures on at least some of the laboratory experiments they undertook, particularly since the use of computers and calculators have significantly reduced the computational burden of these procedures.

Resources

“AP Physics 1 and 2 Lab Investigations: Student Guide to Data Analysis.” AP Physics 1 and AP Physics 2 Essential Course Resources. College Board. Accessed February 10, 2015. https://media.collegeboard.com/digitalServices/pdf/ap/physics-1-2-data-analysis-student-guide.pdf

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"Special Focus: Graphical Analysis." AP Physics 2006–2007 Professional Development Workshop Materials. College Board. Accessed October 14, 2014. http://apcentral.collegeboard.com/apc/public/repository/AP_Physics_Graphical_Analysis.pdf.

"Training Courses for Excel 2013." Microsoft Office. Accessed October 14, 2014. http://office.microsoft.com/en-us/excel-help/training-courses-for-excel-2013-HA104032083.aspx.

"Useful Excel Commands for Lab." Department of Physics. Randolph College. Accessed July 19, 2021. http://physics.randolphcollege.edu/lab/106_116lab/default.htm.

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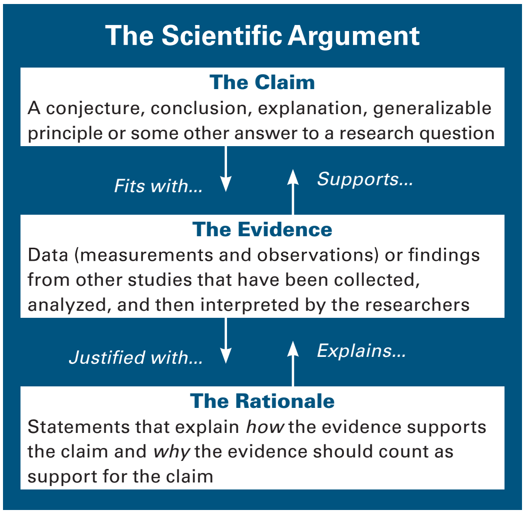
Chapter 7:
Written, Verbal, and Visual Communication

Engaging Students in Scientific Argumentation

Guide students to develop the mindset that there are no wrong answers to questions; instead, they should consider their responses to be steps toward developing the best explanation possible. Responses become valid only as they are supported by physics concepts and explanations. The physics concepts are in turn supported by evidence students collect during their investigations. To foster the inquiry process, cultivate a classroom environment in which students can be “wrong” without embarrassment, and can offer explanations with confidence that they will be taken seriously. Once explanations are proposed, guide students to final conclusions through a process of open scientific discussion and argumentation based on evidence.

The goal is to enable students to build skills in constructing arguments from evidence so that they can defend their conclusions. Laboratory experience, as distinguished from simple, everyday experience, must involve active engagement of students’ minds as well as their hands. Such experience is gained in investigations where students are required to articulate their observations, build mental models, draw conclusions from their observations, and make and test predictions based on their models. Students should then be able to construct claims based on their investigations that are supported by evidence, and to explain how that evidence supports what they claim about their observations.



The Scientific Argument

The Claim
A conjecture, conclusion, explanation, generalizable principle or some other answer to a research question

The Evidence
Data (measurements and observations) or findings from other studies that have been collected, analyzed, and then interpreted by the researchers

The Rationale
Statements that explain *how* the evidence supports the claim and *why* the evidence should count as support for the claim

Model developed by Sampson & Grooms, 2008; Sampson, Grooms, & Walker, 2009

Communicating Scientific Evidence from Investigations

Give students time to consider their explanations and the opportunity to discuss and respond, and allow the ensuing discussion to evolve as students develop their own understanding. Students should have opportunities to convey their evidence from investigations in the laboratory in several ways. Laboratory work should be recorded directly in written form (e.g., a bound journal or portfolio) in real time. You may choose to have the analysis of this work completed in the journal or developed in written form as a formal report. Reporting results can also be verbal, as individual students or student groups report their observations and conclusions to a larger group for discussion and feedback.

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Chapter 8:
Making AP Physics 1 and 2 Inclusive for All Learners

As a teacher, you should anticipate having students with special needs in your class, and plan to meet the individual needs of those students in order to support their endeavors to be successful in the course. This chapter provides guidance regarding issues that are particularly pertinent to special-needs students in the guided-inquiry physics laboratory.

Safety

The most important consideration for teachers is always the safety of students in the laboratory. You may need to make special efforts in order to ensure that students with special needs can work effectively and safely in the lab. Their inclusion can be successful when you have sufficient information about their particular needs, have the proper materials to assist them in the lab (as needed), and receive support from professionals who specialize in working with these students. In some cases, you will need to spend more time with special-needs students in the laboratory. Thus, the total number of students you can adequately supervise may be smaller, so teacher–studio ratio is particularly important. You may need to have additional professionals in the laboratory to be able to guide and manage all students safely. Some students may need specialized equipment or other aids to support their work in the lab. A team of professionals (counselors, science teachers, special-education teachers, and school administrators) should discuss class size, specialized equipment, and other issues pertinent to the requirements of the special-needs student prior to laboratory work, and you must ensure that recommendations are followed. You can help the team to identify risks that might arise in the specific context of the physics laboratory.

Accommodations

Both physical and nonphysical accommodations that enhance learning might be needed. The most common special needs relate to (1) vision, (2) mobility, (3) autism spectrum, (4) learning and attention, (5) hearing, and (6) health. Consultation with educational professionals who specialize in the particular special needs of a student is important. Awareness of organizations such as DO-IT (Disabilities, Opportunities, Internetworking, and Technology) can provide teachers with information about working in the laboratory/classroom with students with special needs. Many students with learning issues have individualized education programs (IEPs) that can guide the accommodations.

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Consider the following suggestions:

- **Students with vision impairments** might benefit greatly from enhanced verbal descriptions and demonstrations. Lab equipment can be purchased with Braille instructions, promoting independent participation for visually impaired students. Students with visual challenges might also benefit from preferential seating that allows them to see demonstrations more easily. If possible, provide students with raised-line drawings and tactile models for illustrations. You might also consider using technology to increase accessibility to the lab experience. For example, video cameras attached to a computer or television monitor can be used to enlarge images.
- **Students who have mobility challenges** may need a wheelchair-accessible laboratory. Keep the lab uncluttered and make sure that aisles are wide enough for wheelchair movement. Students can often see a demonstration better if a mirror is placed above the instructor. Lab adaptations are available for students with mobility problems to assist them in most lab activities. You will need to know a student's limitations before planning a successful lab experience.
- **Students with autism spectrum disorders** (including Asperger's syndrome and pervasive developmental disorder) may have a range of communication and impulsive behavior challenges requiring accommodations and close monitoring in the laboratory setting to ensure a safe and supportive learning environment. These students' particular challenges and needs are highly individualized. Guidance and support from appropriate professionals is particularly important in preparing you to meet such a student's needs. An educational aide or support staff member working with the student in the lab is sometimes helpful, as a lower student–educator ratio is often beneficial and may, in some cases, be called for in the student's IEP.
- **Students with hearing difficulty** might benefit from preferential seating near you when demonstrations are given. It is also helpful to provide hearing-impaired students with written instructions prior to the lab and use instructor captioning when showing videos and DVDs.
- **Students who have learning and attention special needs** may require a combination of oral, written, and pictorial instruction. Scaffolding instruction increases learning, and safety issues and procedural instructions may need to be repeated. Having audiotaped instructions may be helpful to allow students to hear them as often as needed for comprehension. Some students who have special needs related to attention need frequent breaks to allow them to move around and refocus. Providing a student with preferential seating to avoid distraction is also helpful. Students with reading and writing challenges often require more time to prepare for lessons and to complete the follow-up activities. Students with learning and attention challenges sometimes benefit greatly from the use of technology, such as scanning and speaking pens that help with reading. Other such students might benefit from using laptops to take notes during class.

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- **Students with health issues,** such as asthma, allergies, or insulin-dependent diabetes, may benefit from certain accommodations. Care should be taken to avoid risking a student's health because of exposure to chemicals or allergens such as noxious gases or vapors, latex gloves, or food components (e.g., milk or egg proteins, peanuts) while conducting laboratory investigations. Students with asthma or allergies may benefit from wearing a mask designed for physical laboratory use. You should be aware of any student requiring epinephrine administration (e.g., an EpiPen) in the case of an allergic reaction.

Universal Design

A laboratory environment that is universal in design is one that is accessible to students both with and without special needs. By creating such an environment, you will address most concerns and accommodations for students with special needs and, at the same time, improve learning opportunities for *all* students in the lab. Be proactive whenever possible in implementing accommodations, including the following:

- Provide both written and oral directions.
- Give students adequate time to prepare for labs and to complete follow-up activities.
- Make the aisles wide enough for wheelchairs.
- Install a mirror above the area where demonstrations are performed.
- Use tables that can be adjusted for height.

Supporting English Language Learner Students

All AP teachers should be prepared to accommodate English language learner (ELL) students in their courses. You can employ a number of strategies to support such students; many of these strategies will benefit all students, not just ELL students. Examples include:

- Using printed pictures and graphics (e.g., pictures of lab glassware) to support English text in curricular materials and lab handouts;
- Teacher demonstrations of basic procedures and techniques;
- Video clips showing laboratory techniques; and
- Multimedia simulations of chemical phenomena.

Another idea to consider is pairing students with less developed English language skills with another student who speaks their first language and has more developed English language skills; though of course this is not a substitute for teacher supervision and support. Close teacher monitoring and prompting in the lab will further help students who appear confused or on the wrong track during inquiry activities, and will prevent any potential safety hazards from arising.

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Developing a Community of Learners

Teachers must foster the creation of a learning environment that includes and respects *all* students. For example, creating cooperative learning groups provides students with the opportunity to share their knowledge and skills and to learn from each other. This is particularly advantageous for special-needs students. You may find it helpful to talk with students to discover firsthand what accommodations you need to implement in order to make students' lab experiences successful. By modeling attitudes and behaviors expected from students, you can develop activities that help *all* students build meaningful academic and personal relationships.

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AP Physics 1 Investigation 1:
1D and 2D Kinematics

How is the translational motion of a ball described by kinematics?

Central Challenge

Students observe a steel ball rolling down an inclined ramp, then across a horizontal track, and finally as a projectile off the end of the ramp onto the floor. In the three parts of this investigation, they are tasked with describing, with graphs and equations, the motion of the ball on the inclined ramp, the horizontal track, and as a projectile.

Background

The complete description of motion includes a discussion of the position, velocity, and acceleration of an object at each point in time. The displacement of an object is the change in its position. The velocity of an object is the rate of change of its position. Velocity includes not only the magnitude of that rate of change but also the direction. The acceleration is the direction and rate of change of the velocity of the object.

These relationships can be represented graphically. The velocity can be obtained by finding the slope of the graph of position as a function of time. The acceleration can be obtained by finding the slope of the graph of velocity as a function of time. The critical concepts are contained in the equations for motion with constant acceleration in one dimension, as follows:

$$x = x_0 + v_{x0}t + \frac{1}{2}a_x t^2$$

Equation 1

$$v_x = v_{x0} + a_x t$$

Equation 2

In these equations, x is the position at time t and x_0 is the position at time $t = 0$ of the object; v_x is the velocity of the object along the direction of motion, x, at time t, and v_{x0} is the velocity of the object along the direction of motion, x, at time $t = 0$; and a_x is the acceleration of the object along the direction of motion, x.

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Real-World Application

Kinematics is present in many aspects of students' lives, such as driving or riding in automobiles and the sports they play. Driving involves acceleration in linear motion. Even the timing of traffic lights depends on kinematics; in order to keep traffic flowing efficiently, civil engineers need to time red lights at sequential cross streets so that cars aren't stopped at each light, and on roads with higher speed limits they must extend the duration time of yellow lights so that drivers are able to stop safely before the light turns red. Examples of kinematics in sports include cross-country running, which involves constant-speed motion, distance, and displacement; and the motion of a volleyball, which can be approximated using projectile motion.

Inquiry Overview

This multipart inquiry-based investigation introduces students to concepts in kinematics in one and two dimensions. Students perform three guided-inquiry investigations that involve the study of constant velocity (Part I), constant acceleration (Part II), and projectile motion (Part III), which simultaneously involves constant velocity horizontally and constant acceleration vertically.

Through guided inquiry, students are provided with a track that includes an inclined section and a horizontal section. The students are tasked to determine if the motion on the horizontal section is constant velocity and if the motion on the inclined section is constant acceleration. They are then asked to determine how the initial velocity of the ball in projectile motion affects its horizontal motion from the time it leaves the track until it lands on the ground.

Connections to the AP Physics 1 Curriculum Framework

Big Idea 3 The interactions of an object with other objects can be described by forces.

Enduring Understanding	Learning Objectives
3A All forces share certain common characteristics when considered by observers in inertial reference frames.	**3.A.1.1** The student is able to express the motion of an object using narrative, mathematical, and graphical representations. (Science Practices 1.5, 2.1, and 2.2) **3.A.1.2** The student is able to design an experimental investigation of the motion of an object. (Science Practice 4.2) **3.A.1.3** The student is able to analyze experimental data describing the motion of an object and is able to express the results of the analysis using narrative, mathematical, and graphical representations. (Science Practice 5.1)

[**NOTE:** In addition to those listed in the learning objectives above, Science Practice 4.3 is also addressed in this investigation.]

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Skills and Practices Taught/Emphasized in This Investigation

Science Practices	Activities
1.5 The student can *re-express key elements of natural phenomena across multiple representations* in the domain.	Students use data from the different parts of the investigation to create graphs of the motions and write equations that relate to those motions as part of the analysis of their lab.
2.1 The student can *justify the selection of a mathematical routine* to solve problems.	Students select appropriate equations to describe the ball's motion in either constant velocity, constant acceleration, or projectile motion as part of the analysis of the lab.
2.2 The student can *apply mathematical routines* to quantities that describe natural phenomena.	Students use data they have collected in the appropriate equations; they also construct graphs from data to describe various motions.
4.2 The student can *design a plan* for collecting data to answer a particular scientific question.	Student groups, using the equipment provided, design a plan to collect enough data to plot the motions and to make calculations related to the motions, enabling them to determine which parts of the motion are constant velocity, constant acceleration, or projectile motion.
4.3 The student can *collect data* to answer a particular scientific question	Students collect displacement and time measurements to plot graphs of position vs. time or velocity vs. time.
5.1 The student can *analyze data* to identify patterns or relationships.	Students analyze the data they gather to make calculations and graphs to determine which parts of the motion are constant velocity, constant acceleration, or projectile motion. For example, they use the slope of the position–time graph to determine velocity and compare that to the velocity–time graph and calculations for the same part of the motion.

[**NOTE:** Students should be keeping artifacts (lab notebook, portfolio, etc.) that may be used as evidence when trying to get lab credit at some institutions.]

Equipment and Materials

Per lab group (two students):

- Ramp attached to a horizontal track (see below for one possible way to construct a ramp; if you choose a different type of track, make certain that the steel ball follows a straight-line path and does not veer off the track, as this will make data collection impossible)
- Stopwatch
- Meterstick
- Steel ball (1.5–2 cm in diameter)
- Carbon paper

- Bubble level
- (Optional) Toy car that accelerates

The ramps are constructed from aluminum sliding door C-channel, and they can be built for approximately $10 per lab station from materials that are readily available at local home-improvement stores.

Per ramp:

- One 2-foot piece of 1/2-inch aluminum C-channel
- One 2-foot piece of 3/8-inch aluminum C-channel
- Two 6-inch pieces of aluminum C-channel (preferably 1 inch wide, but scraps will do)
- Two #6-32 × 1/2-inch machine screws
- Two nuts to fit the machine screws

To construct four ramps:

Get two 8-foot lengths of C-channel, one 1/2-inch wide to form the horizontal tracks at the base of the ramps and one 3/8-inch wide to form the inclined sections of the ramps. The bottom end of the 3/8-inch piece used for the upper, angled part of each ramp fits snugly into the upper end of the 1/2-inch horizontal track piece. Also purchase one piece of wider C-channel to cut into short sections to attach for "feet."

Cut the ½-inch C-channel into four 2-foot lengths with a hacksaw or band saw to make the four horizontal sections. Cut the smaller 3/8-inch C-channel into four 2-foot lengths to make the four upper track pieces that will be angled.

Two feet are needed for each ramp. The feet can be made from larger or leftover C-channel turned upside down under the track piece so the nuts on the bottom fit inside the channel and attach to the ramp pieces with machine screws and nuts. Drill two 3/16-inch holes in each section of the C-channel, 6–8 inches from the ends. Attach the feet to the wider C-channel with the machine screws (wing nuts are preferable, but any #6-32 nut will do). It is very important that the screws be set so that they in no way interfere with the path of the ball. To make each foot, turn the short piece of 1-inch (or scrap) C-channel upside down under the track and attach the two together with the screws and nuts.

Duct tape or a C-clamp can be used to fasten the ramp and track to the table so that repeated trials are consistent and not affected by changing the elevation of the upper track. With this design, the inclined piece of C-channel is movable (necessary to perform the exercise in Part III of this investigation) since one end can be elevated to different heights with small wooden blocks.

Another option is to construct the tracks to be twice as long (i.e., with a 4-foot lower section and 4-foot upper section); these are harder to store, but they provide more length on which students can take measurements. Just double the cut lengths in the directions above to accomplish this.

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Figure 1 is a good picture of what the C-channel looks like, how the feet are attached, and how it should be supported.

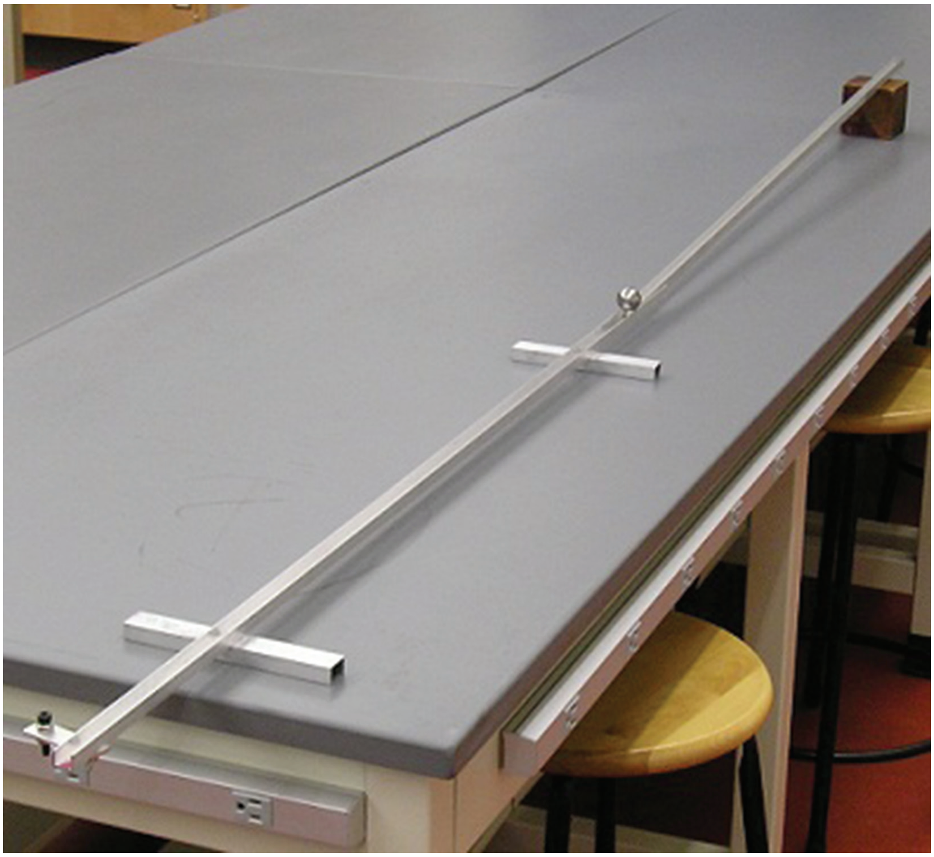

Figure 1

Figure 2 shows how the narrower piece of channel fits into the wider piece of channel to provide a smooth transition from the angled ramp part of the track to the horizontal section.

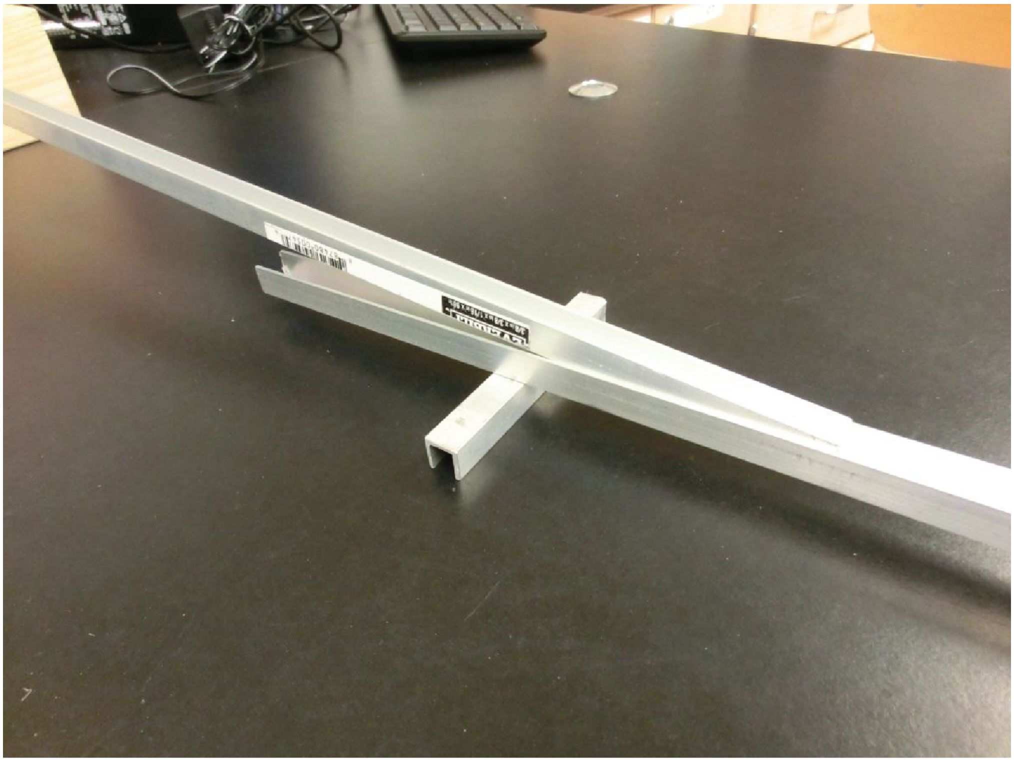

Figure 2

Alternate equipment ideas:

- Use 6-foot lengths of flexible vinyl threshold, which is also available from local home-improvement stores. These provide an ideal track for tennis balls and are very inexpensive. The inclined ramp portion would need to be supported by a board, as it is flexible and will move if unsupported as the tennis ball rolls along it. The tennis balls will not make a mark on the carbon paper so other methods would need to be used to determine the landing point of the projectile. [**NOTE:** It is important that ramps are grooved so that the ball moves in a straight motion down the ramp without veering or falling off.]
- Commercially made ramps are also available from popular scientific equipment companies. These are, however, significantly more expensive, and in some of them the flat, horizontal section and the inclined section are all one piece, so the angle of incline is fixed. These do not offer students the flexibility of changing the incline.
- If the technology is available, give students photogates and the computer interfaces necessary to operate them. Avoid giving students motion detectors, however. They should be required to take simple displacement and time measurements to make their conclusions in this activity.

Timing and Length of Investigation

- **Teacher Preparation/Set-up:** 10–15 minutes

  The ramps are light and can be setup in at most 10 minutes. This time does not include construction of the ramp itself, which should take 20–30 minutes per ramp.

- **Student Investigation:** 70–80 minutes

  Allow students time to observe the ramp, play with releasing the ball and watching it move along the track, and for small-group discussion in groups of a few lab pairs so that they can determine what they will measure and how they will measure those quantities as they approach each of the three parts to this investigation. Obtaining the data should take 10 minutes or less for each exercise and 20–30 minutes to conduct the multiple trials required for Part III.

- **Postlab Discussion:** 15–20 minutes
- **Total Time:** approximately 1.5–2 hours

Safety

There are no specific safety concerns for this lab; however, all general lab safety guidelines should be followed. Sometimes, if the aluminum has been cut, the elevated end can be a little sharp — put a cushion on the elevated end, such as a foam ball, to protect students' faces.

Preparation and Prelab

This activity should come after students work with motion detectors (or other motion analysis methods) to learn about graphs of motion and after you have helped them derive the equations of constant acceleration motion from the graphs of motion. Students should also be familiar with graphing techniques and creating graphs of position vs. time and velocity vs. time prior to the lab. Some activities are available in "Special Focus: Graphical Analysis" (see Supplemental Resources).

It is also useful to have students understand a little bit about measuring time with a stopwatch and the size of reaction-time uncertainties. You may want to have them time one oscillation of a short pendulum and compare measurements to compute an uncertainty. Then have several students in the class time one oscillation of a long pendulum (2 meters or more) and compare measurements. They should see that the percent uncertainty of the timing of the long pendulum is much less than the percent uncertainty for the short pendulum. This is true even though the absolute time uncertainty may be about the same. Reinforce for them the idea that, in order to reduce uncertainty, they need to time the motion over longer distances whenever possible.

This experiment uses a rolling ball, so the motion description is only for linear (or translational) motion. Since a portion of the ball–Earth system's original gravitational potential energy is converted to rotational kinetic energy of the ball, the ball's linear speed on the horizontal portion of the track will be less than predicted by conservation of energy; also, the distance from the track that the ball lands on the floor will be less than predicted. Students will not yet have studied rotational kinematics, but it will not be difficult for them to understand that part of the system's initial energy goes to rotational kinetic energy so that the ball has less linear (or translational) speed on the level track and as a consequence less range when it flies off onto the floor. If students have discussed rotational motion prior to this lab, they should record this and discuss it in their laboratory report as both an assumption and a source of uncertainty. Otherwise, you might not need to even address the conservation of energy or rotational motion; the data could be revisited when rotational motion is covered, to calculate the predicted distance including the rotational energy, and compare with the experimental observations.

The Investigation

The following set of lab exercises provides an introduction to kinematics in one and two dimensions without the use of expensive sensors or low-friction tracks and carts. The exercises are all built around the ramp.

The three parts to this investigation involve:

1. The study of one-dimensional accelerated motion of the ball in its direction of motion down the incline;
2. A study of constant velocity one-dimensional motion along the horizontal portion of the track; and
3. A study of two-dimensional motion as the ball leaves the table.

Part I: Constant Velocity

The goal of the first part of this lab is for students to devise a plan to determine whether the motion on the horizontal portion of the track is constant-velocity motion. They can be given as much or as little instruction as you see fit. Instruct students to only to use stopwatches and metersticks and to present their results to the class at the end of the investigation and defend their answers.

Hopefully students will remember that a graph of constant velocity motion is a straight line with non-zero slope on a position vs. time graph, or a horizontal line on velocity vs. time graph and choose to create a graph of position vs. time or velocity vs. time. However, expect students' creativity to prevail and several methods to emerge — both valid and invalid. The onus remains on students to justify why their chosen method is valid.

Conducting a class discussion at the end of this portion of the lab before proceeding to the next is optional. If you notice that several groups are headed in the wrong direction, you may wish to redirect their efforts in a class discussion before proceeding to Part II.

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Part II: Constant Acceleration

The goal of the second exercise is for students to design an experiment to determine if the motion of the ball down the ramp is one of constant acceleration. This is more challenging for students. Since you are not directly telling students what to measure, they may need several chances to fail before they find the right measurements that will yield a valid claim about the motion of the ball.

Challenge students to present an analysis of the motion that justifies their claim that it is constant acceleration. Some students will recall that the graph of position vs. time for a constant acceleration motion is a parabola. However, it will be difficult for students to prove that the graph is a parabola unless they are familiar with curve-fitting programs on their calculator or a computer. In this case, you may choose to guide students to the realization that a plot of displacement vs. the square of time should yield a straight line with a slope of $\frac{1}{2}a_x$ for the motion on the inclined ramp, and therefore justifies their claim about the motion.

Students may choose to plot a graph of velocity vs. time. Experience has shown that students tend to think they can calculate the velocity at any point by dividing the distance traveled by the time. Remind students that this is the average velocity over that interval and not the instantaneous velocity at the end of the interval.

Also remind them that they are not to assume that the acceleration is constant. You might need to stop the entire class to have them debrief and share measurement techniques if they head off in the wrong direction. They are to use data to demonstrate that acceleration is constant without necessarily finding its value. Students should not be allowed to use the equations of constant acceleration to prove the acceleration is constant. They must use a position vs. time graph or velocity vs. time graph.

Part III: Projectile Motion

The goal of the last part of the investigation is to provide students with an introduction to projectile motion. Ask the students to determine how the initial velocity of a projectile launched horizontally affects the distance it travels before it strikes the ground. Their experiments in Part I will prepare them to measure several different velocities for the ball as it leaves the track. The ball rolls off the end of the track and strikes the ground a distance from where it left the track. Give students as much direction as you want on how to reliably measure the x component of the displacement (the horizontal distance it travels). They likely have not had experience with carbon paper, so you may need to explain to them how it works: a steel ball landing on the paper will cause a dot to appear on a piece of paper placed under the carbon side of the paper.

Once students have displacement data for several different values of launch velocity, they use a graph to determine the relationship between the two variables. Once you have discussed the equations of constant acceleration applied to projectile motion, students refer back to their graph and how it supports the mathematical derivations.

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Extension

One possible extension for this lab is to challenge students to plot the vertical motion of the ball in projectile motion as a function of time. You can give them as much or as little direction as you want. Students know the horizontal speed of the projectile as it leaves the track. If they place a vertical board in the path of the ball with the carbon paper attached, the ball will strike it and the vertical height at that location can be measured. They then move the board away from the launch point in fixed intervals and record the vertical position of the ball for a series of horizontal distances.

The analysis of this is somewhat more complicated because students tend to confuse the horizontal and vertical motions and analyze the two together. A class discussion should lead them to the conclusion that, since the velocity in the horizontal direction is constant, the various equally spaced vertical-board positions represent equal time measurements; and thus a position vs. time graph can be obtained.

Another possible extension is to provide students with a toy car that accelerates and have them determine if the acceleration is constant, and if so, how long the acceleration lasts. (Arbor Scientific and other companies sell cars they market as “constant acceleration” cars.) Instruct students to support or refute the validity of their claim with data, graphs, and calculations.

Common Student Challenges

It is essential for this lab that students are comfortable graphing position and velocity as functions of time.

If they still have difficulties with this, then you may want to take them outside and have them time the motion of students walking and running. Have students with stopwatches stand at 5-meter intervals along a straight line, and direct them to start timing when a student starts moving, and stop timing when the student passes them. The data of position vs. time is shared with the whole class. Students could then graph the data as practice for this lab.

A common student mistake is to assume they can apply the equations of constant acceleration to determine if an object executes constant acceleration motion. Experience has shown that students will study various sections of a larger motion and use the equations of constant acceleration to calculate the acceleration. They will then compare the various accelerations to determine if the acceleration is constant over the whole range of motion. For example, they will use the equations of constant acceleration to calculate the acceleration for the first 10 centimeters, then the first 20 centimeters, then the first 30 centimeters, etc.; then they will compare these to determine if the acceleration was constant. How long to allow students to pursue this incorrect path is up to you. You may decide to circulate amongst the groups and ask each what their plan is, and have individual discussions about the validity of their plans. Or you may choose to hold a class discussion after all of the groups have made some progress. In either case, if they choose this incorrect method, direct students to create and use graphs of position vs. time or instantaneous velocity vs. time.

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Students should use boxes or books to elevate the end of the ramp to change the acceleration and therefore the final horizontal velocity of the ball. They can use a piece of carbon paper taped to a piece of white paper on the floor to precisely determine the point of impact of the ball. Not allowing too great an incline keeps the velocity low so that the ball only travels about 30–35 centimeters in the horizontal direction after falling from the average 80-centimeter lab table.

Another challenge is the concept of rotational motion of the ball (discussed above), which students will not completely understand at this point. It is enough here for them to know that the rolling motion of the ball accounts for a different kind of kinetic energy (rotational) but the velocity they are calculating from linear kinetic energy is only part of the total energy. However, if energy has not yet been discussed in class, then students may not even worry about the rolling motion. [**NOTE:** Discourage students from attempting to use conservation of energy calculations during this investigation to determine the final horizontal velocity of the ball: it does not address the learning objectives in this investigation.]

Analyzing Results

Whether students break for a discussion of the results after each section of the lab or only at the end is up to you. It is highly recommended, however, that the discussion of the measurement of the velocity as it leaves the track is discussed prior to starting Part III.

The most convincing arguments for constant velocity involve a graph of position vs. time. Students should be able to articulate how they made the measurements that construct the graph. Some students may have measured the speed at different locations on the track and compared the values to each other. The discussion should center on the validity of the measurements: whether, in fact, they measured displacement and time. Depending on how the large the displacement is, the velocity they calculated may be an average velocity and not an instantaneous velocity. This discussion provides an excellent opportunity to reinforce the difference between the two.

The most convincing arguments for constant acceleration involve a graph of velocity vs. time or a graph of displacement vs. time squared. Both of these will yield a straight-line graph if the acceleration is constant. As mentioned above, the common misconception here is for students to confuse average velocity and instantaneous velocity. Experience has shown that students will measure the time it takes for the ball to roll significant distances (30–50 centimeters), measure the time, and then divide one by the other. They assume this is the velocity at the end of the motion rather than the average velocity. It is important to help students realize that this is not the case and how to calculate the instantaneous speed (which is the same size as the instantaneous velocity, since the ball does not change direction of motion).

The analysis of Part III is also best done using a graph. Ask the students to consider the following questions:

- How did you measure the speed of the ball just before it left the track?
- How consistent was the landing position of the ball for each individual speed?

- What does the shape of the graph of horizontal displacement vs. speed imply about the relationship between the two?
- How does the ball's time of flight depend on its initial horizontal speed?
- How could you improve the precision and accuracy of your measurements?

A discussion of sources and sizes of uncertainty of measurements is inevitable in this lab. Start by having students indicate what measurements were actually made and what the uncertainty was in each measurement. For example, they will probably measure time with a stopwatch. If they measure several trials, then they can take a standard deviation; otherwise the uncertainty is their reaction time.

Depending on the incline of the track, the speed of the ball may be significant, making timing with a stopwatch significantly affected by reaction-time error. Methods of decreasing this uncertainty can be discussed at any point during the measurement or in a discussion at the end. Ask the students to consider the following questions:

- What is the typical human reaction time when using a stopwatch?
- How does this time compare to the time intervals you were measuring?
- What percent uncertainty does this introduce into your time measurements and speed calculations?
- What could you do to reduce this uncertainty?

For example, a typical reaction time is between 0.1 and 0.25 seconds. Assuming the larger value, if the measurement is only 1.0 second, this represents a 25 percent uncertainty in the timing measurement. However, if the time measurement is 10 seconds, this represents a 2.5 percent uncertainty in the timing measurement and thus the speed measurement. One suggestion for reducing uncertainty would be to use a device that does not rely on human reaction time for measurement, such as a photogate.

Assessing Student Understanding

After completing this investigation, students should be able to:

- Use measurements of displacement and time to create a position vs. time graph;
- Use measurements of displacement and time to create a velocity vs. time graph;
- Use graphs of position and velocity vs. time to analyze the motion of an object;
- Determine the speed of a ball on a horizontal track;
- Measure the horizontal distance a projectile travels before striking the ground; and
- Relate the initial velocity of a horizontally launched projectile to the horizontal distance it travels before striking the ground.

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Assessing the Science Practices

Science Practice 1.5 The student can *re-express key elements of natural phenomena across multiple representations* in the domain.

Proficient	Plots correct graphs for all parts of the motion, and makes correct inferences about the motion from those graphs.
Nearly Proficient	Plots correct graphs for all parts of the motion, but portions of the interpretation are incorrect.
On the Path to Proficiency	Plots a correct graph for one part of the motion (e.g., the velocity vs. time for the level section).
An Attempt	Attempts graphs related to his or her observations and measurements, but graphs are inaccurate.

Science Practice 2.1 The student can *justify the selection of a mathematical routine* to solve problems.

Proficient	Uses kinematic equations appropriately to verify displacement, velocity, and acceleration for all sections of the experiment, including correct interpretations of slope.
Nearly Proficient	In most instances, uses correct equations for calculations related to motion, but there is an incorrect assumption in one step, such as forgetting that initial vertical velocity as the ball leaves the table is zero. This applies also to determination of slope and area from graphs.
On the Path to Proficiency	Uses some correct equations for calculations, but uses one or more incorrectly, such as using a kinematics equation to determine whether acceleration is constant. This applies also to determination of slope and area from graphs..
An Attempt	Uses incorrect equations to calculate acceleration, velocity, and/or displacement, and uses incorrect equations in determination of slope and area from graphs.

Science Practice 2.2 The student can *apply mathematical routines* to quantities that describe natural phenomena.

Proficient	Makes entirely correct calculations from equations or determinations of slope and area from graphs.
Nearly Proficient	Makes mostly correct calculations from equations or determinations of slope and area from graphs.
On the Path to Proficiency	Makes some correct calculations from equations or determinations of slope and area from graphs.
An Attempt	Attempts to make calculations from equations or determinations of slope and area from graphs, but none are correct.

Science Practice 4.2 The student *can design a plan* for collecting data to answer a particular scientific question.

Proficient	Follows directions and adds a thorough description of a design plan (with clearly labeled diagrams), including predictions and assumptions.
Nearly Proficient	Follows directions and adds a design plan that is mostly complete (with diagrams), and including assumptions.
On the Path to Proficiency	Follows directions but does not clearly indicate a plan for experimental design and procedure.
An Attempt	Misinterprets directions or does not indicate a viable plan for experimental design and procedure.

Science Practice 4.3 The student can *collect data* to answer a particular scientific question.

Proficient	Collects accurate data in a methodical way and presents the data in an organized fashion.
Nearly Proficient	Collects mostly but not entirely accurate and complete data or the presentation of the data is somewhat disorganized.
On the Path to Proficiency	Collects somewhat inaccurate or incomplete data and the presentation of the data lacks organization.
An Attempt	Collects inaccurate or incomplete data and doesn't provide any organization for this data.

Science Practice 5.1 The student can *analyze data* to identify patterns or relationships.

Proficient	Appropriately uses a velocity–time graph to determine the acceleration of the ball and position–time graphs to determine the speed of the ball on the track. Accurately graphs horizontal displacement vs. speed and interprets the results.
Nearly Proficient	Makes conclusions and calculations from data (perhaps graphs) but indicates no clear correlations.
On the Path to Proficiency	Requires significant assistance in analyzing velocity–time graphs or relating horizontal distance traveled for a projectile launched horizontally to the initial speed of the projectile.
An Attempt	Attempts to use incorrect features of a velocity–time graph to determine the acceleration of an object.

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Supplemental Resources

Drake, Stillman. *Galileo: Two New Sciences.* Madison, Wisconsin: University of Wisconsin Press, 1974.

“Mechanics: 1-Dimensional Kinematics.” The Physics Classroom. Accessed September 1, 2014. http://www.physicsclassroom.com/calcpad/1dkin/problems.cfm. [*This website allows students to explore extra practice problems on kinematics*.]

“The Moving Man.” PhET. University of Colorado Boulder. Accessed September 1, 2014. http://phet.colorado.edu/en/simulation/moving-man. [*This simulation provides an interactive way to learn about position, velocity, and acceleration graphs.*]

The Physlet Resource. Davidson College. Accessed September 1, 2014. http://webphysics.davidson.edu/physlet_resources. [*This resource provides sample “physlet” illustrations, explorations, and problems in 1-dimensional kinematics.*]

“Projectile Motion.” PhET. University of Colorado Boulder. Accessed September 1, 2014. http://phet.colorado.edu/en/simulation/projectile-motion. [*Provides multiple visual representations of kinematics in one and two dimensions.*]

“Special Focus: Graphical Analysis.” AP Physics 2006–2007 Professional Development Workshop Materials. College Board. Accessed September 1, 2014. http://apcentral.collegeboard.com/apc/public/repository/AP_Physics_Graphical_Analysis.pdf.

AP Physics 1 Investigation 2: Newton’s Second Law

What factors affect the acceleration of a system?

Central Challenge

In this lab students investigate how the acceleration of an object is related to its mass and the force exerted on the object, and use their experimental results to derive the mathematical form of Newton’s second law.

Students should have already completed the study of kinematics and Newton’s first law.

Background

Newton’s laws are the basis of classical mechanics and enable us to make quantitative predictions of the dynamics of large-scale (macroscopic) objects. These laws, clearly stated in Isaac Newton’s *Principia* over 300 years ago, explain how forces arising from the interaction of two objects affect the motion of objects.

Newton’s first law states that an object at rest remains at rest, and an object moves in a straight line at constant speed unless the object has a net external force exerted on it.

Newton’s second law states that when a next external force is exerted on an object of mass *m,* the acceleration that results is directly proportional to the net force and has a magnitude that is inversely proportional to the mass. The direction of the acceleration is the same as the direction of the net force.

The mass of an object in Newton’s second law is determined by finding the ratio of a known net force exerted on an object to the acceleration of the object. The mass is a measure of the inertia of an object. Because of this relationship, the mass in Newton’s second law is called inertial mass, which indicates how the mass is measured.

Newton’s laws of motion are only true in frames of reference that are not accelerating, known as inertial frames.

Real-World Application

There are numerous real-world applications of Newton’s second law that can spark student interest. Students can research their favorite sport and apply the concepts learned in this investigation to understand how the magnitude of the acceleration varies when a force is exerted on objects of different mass, such as golf balls, tennis balls, and baseballs.

Another application could be the physics involved when a car encounters ice. Students think the engine makes the car move, but why doesn't it work on ice? It doesn't work because an external force must be exerted on an object by another object to cause acceleration; the tires push back on the ground, the ground pushes forward on the tires, and the car goes forward. Ice interferes with this interaction of external forces on the tires and the ground, and so the wheels just spin.

In this investigation, students use a modified Atwood's machine. Atwood's machines are systems with two masses connected by a cable and pulley, providing for a constant acceleration of any value required (see Figure 1). Some students might be interested in a real-life application of this technology, such as an elevator and its counterweight.

Inquiry Overview

This investigation is structured as a guided inquiry. Students are presented with the question, "What factors affect the acceleration of a system?"

After observing the demonstrations suggested in Part I of the investigation, the students will be guided to discover the factors to be investigated. The students will also design the procedure of the investigation and the data collection strategy.

Students might need some guidance with the analysis of data and the construction of graphs. More specifically, they might be confused about how to merge the results of the two parts of the investigation to answer the overall lab question.

In the Investigation section, specific guiding questions are offered to support students in the design and interpretation of their experiments. Part II of the investigation is divided into two separate activities. The first is limited to the relation of acceleration to force, and the second is limited to the relation of acceleration to mass.

Connections to the AP Physics 1 Curriculum Framework

Big Idea 1 Objects and systems have properties such as mass and charge. Systems may have internal structure.

Enduring Understanding	Learning Objectives
1.A The internal structure of a system determines many properties of the system.	**1.C.1.1** The student is able to design an experiment for collecting data to determine the relationship between the net force exerted on an object, its inertial mass, and its acceleration. (Science Practice 4.2)

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Big Idea 3 The interactions of an object with other objects can be described by forces.

Enduring Understanding	Learning Objectives
3.A The internal structure of a system determines many properties of the system.	**3.A.2.1** The student is able to represent forces in diagrams or mathematically using appropriately labeled vectors with magnitude, direction, and units during the analysis of a situation. (Science Practice 1.1)

[**NOTE:** In addition to those listed in the learning objectives above, the following science practices are also addressed in the various lab activities: 4.1, 4.3, 5.1, and 5.3.]

Skills and Practices Taught/Emphasized in This Investigation

Science Practices	Activities
1.1 The student can *create representations and models* of natural or man-made phenomena and systems in the domain.	Students produce multiple representations of the data in the form of graphs and diagrams as follows: ◆ Graphs of the data: › acceleration vs. force › acceleration vs. mass ◆ Force diagrams that represent the forces exerted on the objects
4.1 The student can *justify the selection of the kind of data* needed to answer a particular scientific question.	Students identify the quantities that need to be measured in order to determine the acceleration of the system.
4.2 The student can *design a plan* for collecting data to answer a particular scientific question.	Students design a procedure to investigate the relationships among the net force exerted on an object, its inertial mass, and its acceleration.
4.3 The student can *collect data* to answer a particular scientific question.	Students gather the following data: ◆ net force and acceleration when the total mass is kept constant ◆ total mass and acceleration when the net force is kept constant
5.1 The student *can analyze data* to identify patterns or relationships.	Students analyze the graphs to identify the relationship between the variables
5.3 The student *can evaluate the evidence provided by data sets* in relation to a particular scientific question.	Students articulate an operational definition of Newton's second law based on the evidence presented by the graphs.

[**NOTE:** Students should be keeping artifacts (lab notebook, portfolio, etc.) that may be used as evidence when trying to get lab credit at some institutions.]

Equipment and Materials

Per lab group (three to four students):

- Dynamics track
- Cart
- Assorted masses
- Mass hanger and slotted masses
- Low-friction pulley
- String
- Meterstick
- Stopwatch

If you do not have a dynamics track, then any flat, smooth surface, perhaps even the lab tables themselves, will work just fine. The carts should have wheels with a small rotational-inertia and low-friction bearings.

Data acquisition using motion detectors or photogates is recommended when available, as it helps reduce experimental procedural errors. Another option is to record a video of the motion of the cart and use video analysis software to analyze the motion.

Timing and Length of Investigation

- **Teacher Preparation/Set-up:** 15–20 minutes

  This time is needed to prepare the demos and set out equipment from which students may choose for their investigation.

- **Prelab:** 30 minutes

  It is advisable to conduct the activities and prelab discussion in one class or lab period.

- **Student Investigation:** 110–120 minutes

  Design of procedure: 20–30 minutes

  Data collection: 30 minutes

  Data analysis: 60 minutes

  You may assign the design of the data collection procedures as homework. Students gather the materials and do their own setup for their investigations. At the beginning of the lab period, have volunteers present their draft procedures to the class, and solicit feedback from the various groups.

- **Postlab Discussion:** 30 minutes
- **Total Time:** approximately 3.5 hours

  [**NOTE:** This investigation is designed to enable a deeper understanding of Newton's second law and therefore it might take more time than investigations performed in the context of the previous AP Physics B course.]

Safety

There are no major safety concerns for this lab. However, pay attention to high speeds of carts, masses flying off carts, masses hitting the feet of students, and student fingers being squeezed when stopping a cart at the pulley when a high proportion of mass is on the hanger. Also, to keep students and equipment from being damaged, restrict the total slotted mass. General lab safety guidelines should always be observed.

Preparation and Prelab

Prelab Activities
The following activities are optional and could be conducted to assess students' prior knowledge, skill levels, and understanding of key concepts. Setup the modified Atwood machine and pose questions such as those suggested below in this four-part prelab session:

Part I:
What will a graph of the cart's velocity (v) vs. time (t) look like after the system is released from rest?

After making and discussing their predictions, students carry out an experiment, using a motion detector to record v vs. t, or using video capture, in which case students will have to put some thought into how to produce the velocity vs. time graph. But the main point of this part is for students to see and make sense of the conclusion that the slope of the velocity vs. time graph is constant.

Part II:
(a) If the cart's mass is increased, will the new velocity vs. time graph look the same or different from the graph in Part I?

(b) If the hanging mass is increased, will the new velocity vs. time graph look the same or different from the graph in Part I?

Again, these are qualitative questions, but students can obtain quantitative data to answer them. As usual with these kinds of qualitative questions, the lab works well if students first make and discuss their predictions before designing and carrying out the experiments.

Part III:
If both the cart's mass and the hanging mass are doubled, will the new velocity vs. time graph look the same or different from the graph in Part I?

Part IV:
What if the cart is moving initially?

What will the velocity vs. time graph look like, compared to the graph from Part I, if the cart at $t = 0$ is given a brief push away from the pulley? Will the graph be the same? If not, what will be different?

Some students may spontaneously have the idea of doing another trial where the cart is given a brief push towards the pulley — and it would be great for them to try that! They should be able to identify that the y-intercept in the velocity–time graph represents the initial velocity of the cart.

The Investigation

Part I:
In the first part of this activity, students observe a number of demonstrations that include variations of an object being accelerated.

A modified Atwood's machine with a system consisting of a cart and a hanger with slotted masses like the one shown in Figure 1 is a suitable setup.

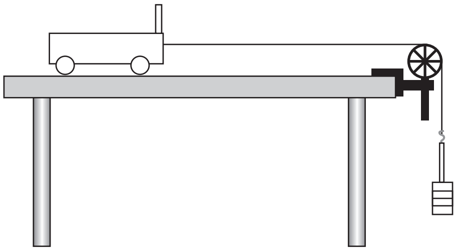

Figure 1

Examples include a demonstration where the total mass of the system is kept constant and the net force is varied, and a demonstration where the net force is kept constant and the total mass of the system is varied. Instructors could use any available lab equipment that allows for a variation of the force exerted on the object with added masses. Ask the students these three questions:

1. "What do you observe?"
2. "What can you measure?"
3. "What can you change?"

A guided discussion should yield some of the following answers to the questions:

1. The cart-mass hanger system is accelerated.
2. Quantities that can be measured include the mass of the cart, the mass of the hanger, distance traveled by the cart, distance traveled by the hanger and the slotted masses, the time of travel, etc.
3. Quantities that can be changed are the net force on the system and the total mass of the system.

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Students may have difficulty identifying the net force exerted on the system. Drawing free-body diagrams might help in determining that the net force on the system is equal to the gravitational force of Earth on the hanger and slotted masses. Some students will indicate that a force of kinetic friction is exerted on the cart.

Part II:
After the discussion, instruct students to design two data collection strategies to determine how two factors affect the acceleration of the system: the net force on the system and the total mass of the system.

Activity 1: Students design procedures that include calculation of the acceleration when the total mass of the system is kept constant and the net force is varied.

Activity 2: Students design procedures that include calculation of the acceleration when the total mass of the system is varied and the net force is kept constant.

A few tips:

- Discourage students from trying to combine the two activities into one.
- Encourage students to be careful to keep the string parallel to the track throughout the data collection.
- The length of the string connecting the cart to the mass hanger should allow the mass hanger to reach the floor just before the cart reaches the pulley.
- Make sure that the string does not rub against anything, such as the pulley mount.

Extension

An extension to this lab is to investigate the effect of friction on the acceleration of the cart. Alternative investigations that use dynamics concepts can be provided as challenges. For examples of this type of activity, see "Turning a Common Lab Exercise into a Challenging Lab Experiment: Revisiting the Cart on an Inclined Track" and "Time Trials — An AP Physics Challenge Lab" in Supplemental Resources.

Another engaging extension activity consists of having students apply the concepts learned in this investigation to their favorite sports. Students could do short presentations in the class, or they could create a poster with their findings if time for presenting is a constraint.

The Science360 Video Library, sponsored by the National Science Foundation, gathers the latest science videos by scientists, colleges and universities, and science and engineering centers. "Newton's Three Laws of Motion" and "Science of the Summer Olympics: Engineering In Sports" are recommended for students to explore (see Supplemental Resources).

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Common Student Challenges

Some of the common challenges that students have regarding Newton's first law include the idea that forces are required for motion with constant velocity. When observing the demonstrations, students need to recognize that the velocity of the object is changing as a result of the net force exerted on the object. It should be clear that the net force determines an object's acceleration, not its velocity. To counter this student misconception, you can use a motion detector and a force probe to study the motion of a cart being pulled by a mass hanging from a string that passes over a pulley (as shown in the Investigation section). Simultaneously graph the force on the cart and the motion of the cart. Direct students to notice the shape of the force graph (horizontal line) and acceleration graph are the same, but the velocity vs. time graph is a line with a positive slope. A constant forward force produces an increasing velocity and a constant acceleration.

Students might not see the connection between Newton's laws and kinematics, so it is important for them to recognize Newton's second law as "cause and effect." It is important to present Newton's second law in its operational form of $\vec{a} = \frac{\Sigma \vec{F}}{m}$, as the commonly used $\sum \vec{F} = m\vec{a}$ leads some students to believe that the product of mass and acceleration (ma) is a force.

A specific student challenge in this investigation is to recognize that both the cart and the falling mass are included in the total inertial mass of the system being affected by the gravitational force on the falling mass. During the investigation, all masses to be used as falling masses should be placed in the cart when not pulling the cart. Students will be tempted to have the cart on the table and replace the falling mass with a different falling mass that is on the lab table. This, in effect, changes the total mass being pulled. This is a good opportunity to have students discuss the meaning of *system*. The system that is being accelerated is the cart and falling mass.

Another specific student challenge is the role of friction of the cart and the pulley as well as the rotational inertia of the wheels of the cart and the pulley. These can be ignored when conducting the investigation for sufficient hanging mass, but should be discussed at some point in the analysis of results.

Analyzing Results

How students analyze their results depends on how they decided to make measurements and complete the calculations. Some students may use a stopwatch to measure the time of the acceleration over a fixed distance. These students would then use the equations of constant acceleration motion to calculate the acceleration. Other students may choose to use motion sensors to plot the velocity vs. time for the cart. In that case, they would use the slope of the graph for the acceleration.

The sources of experimental uncertainty depend on the equipment used as the precision is limited by the apparatus resolution. In this investigation, uncertainty might be related to the measurements of time, length, or mass (or combinations of each). Students can minimize the uncertainties by taking measurements in multiple trials and averaging the results. See Resources for options of support in this area.

The development of mathematical models from graphs of acceleration vs. force and acceleration vs. mass are an expectation of this investigation. In order to determine the relationship between net force and acceleration and between total mass and acceleration, students plot a graph with an independent variable on the horizontal axis and a dependent variable on the vertical axis. If students are not familiar with linearization methods, guide them as they linearize the acceleration vs. mass graph.

The use of multiple representations in this lab is highly recommended as it leads to a deeper conceptual understanding of Newton’s second law. The lab report should include verbal descriptions of their observations as well as labeled free-body diagrams of the forces exerted on the system.

Sample qualitative graphs for this lab include:

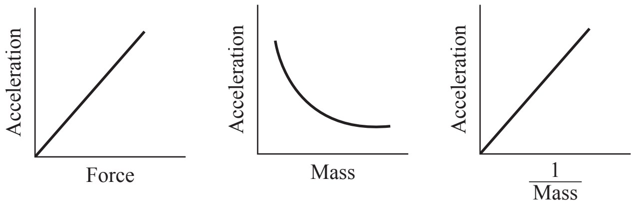

Figure 2

Following are several guiding questions that will help students interpret their graphs generated in Part II of the investigation:

Activity 1:

How does your data indicate if the acceleration was proportional to the force?

Students determine the relationship between the acceleration and the force from the graph. A straight line represents a direct variation between the acceleration and the net force.

What does the slope of the acceleration vs. force graph represent?

The slope of the acceleration vs. force graph represents the reciprocal of the mass of the system.

What is the algebraic relationship between acceleration and net force in this system?

The algebraic relationship between acceleration and net force is expressed as $a \propto \Sigma F$.

[**NOTE:** You may want to point out to students that the graph does not go through zero. This accounts for the frictional force between the cart and the surface.]

Activity 2:

How does the data indicate if the acceleration was inversely proportional to the mass?

Students determine the relationship between the acceleration and the mass from the graph. A hyperbola represents an inverse variation between the acceleration and the mass.

What does the slope of the acceleration vs. the inverse of the mass represent?

The slope of the acceleration vs. the inverse of the mass graph represents the net force of the system.

What is the algebraic relationship between acceleration and mass in this system?

The algebraic relationship between acceleration and mass is expressed as $a \propto \frac{1}{m}$.

As part of the analysis, students could find the percent difference between the theoretical value of the acceleration from one configuration of the masses using the free-body diagram of the system and the experimental value.

[**NOTE:** Percent difference is applied when comparing two experimental quantities, E1 and E2, neither of which can be considered the "correct" value. The percent difference is the absolute value of the difference over the mean times 100.]

Assessing Student Understanding

By the end of the investigation, students should be able to:

- Articulate that the acceleration of an object is directly proportional to the net force: $a \propto \Sigma F$;
- Articulate that the acceleration is inversely proportional to the mass:$a \propto \frac{1}{m}$;
- Determine a relationship between arbitrary combinations of mass, force, and acceleration using dimensional analysis;
- Calculate the proportionality constant (k) for the relationship derived from dimensional analysis: $a = k\frac{\Sigma F}{m}$;
- Obtain a proportionality constant value of 1.0; and
- Identify the sources of experimental uncertainty and ways to minimize experimental uncertainties.

Assessing the Science Practices

Science Practice 1.1 The student can *create representations and models* of natural or man-made phenomena and systems in the domain.

Proficient	Creates accurate and appropriate graphical representations of the relationship between acceleration and net force and between acceleration and mass.
Nearly Proficient	Creates mostly correct graphical representations of the relationship between acceleration and net force and between acceleration and mass. The graphs may not fully reflect all aspects of the relationships among the variables.
On the Path to Proficiency	Creates flawed or incomplete graphical representations of the relationship between acceleration and net force and/or between acceleration and mass.
An Attempt	Provides incorrect graphical representations of the relationship between acceleration and net force and/or between acceleration and mass.

Science Practice 4.1 The student can *justify the selection of the kind of data* needed to answer a particular scientific question.

Proficient	Provides accurate and detailed justification explaining the relevance of the variation of mass and net force in the system.
Nearly Proficient	Provides accurate justification for the relevance of the variation of mass and net force in the system with only an occasional or minor error.
On the Path to Proficiency	Provides justification for the relevance of the variation of mass and/or net force in the system with occasional and/or minor errors; justification may be correct but lacks completeness.
An Attempt	Provides generally weak justification for the relevance of the variation of mass and/or net force in the system justification; includes minimal reasoning and evidence.

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Science Practice 4.2 The student can *design a plan* for collecting data to answer a particular scientific question.

Proficient	Designs an effective data collection plan to answer the question via well-selected quantitative measurements of acceleration, providing rationales for all choices. Accurately evaluates uncertainty in measurements. Effectively explains equipment selection for acquiring data (balance and meterstick and stopwatch or motion detector or photogates). Accurately explains different sources of error in data. Accurately identifies and explains independent, dependent, and controlling variables, and justifies choices as follows: (1) Determination of the acceleration when the total mass of the system is kept constant and the net force is varied. (2) Determination of the acceleration when the total mass of the system is varied and the net force is kept constant.
Nearly Proficient	Designs an appropriate data collection plan to answer the question via quantitative measurements of acceleration; measurements may lack complete details. Identifies equipment (balance and meterstick and stopwatch or motion detector or photogates). Identifies appropriate data sources and estimated error. Accurately identifies and describes independent, dependent, and controlling variables as follows: (1) Determination of the acceleration when the total mass of the system is kept constant and the net force is varied. (2) Determination of the acceleration when the total mass of the system is varied and the net force is kept constant.
On the Path to Proficiency	Designs a data collection plan to answer the question via quantitative measurements of acceleration; measurements may not be clearly defined or articulated. Acknowledges need to consider estimated error. Accurately identifies independent, dependent, and controlling variables with few errors as follows: (1) Determination of the acceleration when the total mass of the system is kept constant and the net force is varied. (2) Determination of the acceleration when the total mass of the system is varied and the net force is kept constant.
An Attempt	Presents an incomplete data collection plan to answer the question. Makes errors in identifying the variables (independent, dependent, and controlling).

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Science Practice 4.3 The student can *collect data* to answer a particular scientific question.

Proficient	Collects appropriate data to fully determine the relationship among the acceleration, net force, and inertial mass of the system with precision of observations, accuracy of records, and accurate use of scientific tools and conditions. Accurately applies mathematical routines and appropriately uses measurement strategies.
Nearly Proficient	Collects appropriate and adequate data to answer some aspects of the relationship among the acceleration, net force, and inertial mass of the system with only minor errors in the precision of observation, record keeping, and use of tools and conditions. Selects appropriate mathematical routines and provides measurements with only few minor errors.
On the Path to Proficiency	Collects appropriate data to determine the relationship among the acceleration, net force, and inertial mass of the system. Provides observation logs and record keeping that contain several errors. Selects appropriate mathematical routines and provides measurements with few errors or only a single significant error.
An Attempt	Collects relevant but significantly inadequate data to determine the relationship among the acceleration, net force, and inertial mass of the system. Provides observations and/or record keeping that are incomplete and/or inadequate for answering a particular question. Selects inappropriate mathematical routines; measurements contain many errors.

Science Practice 5.1 The student can *analyze data* to identify patterns or relationships.

Proficient	Comprehensively describes the patterns and relationships within data relative to the relationship among the acceleration, net force, and inertial mass of the system. Accurately applies appropriate mathematical routines. Correctly identifies all of the sources of experimental error, and suggests ways to minimize the uncertainties.
Nearly Proficient	Identifies most patterns within data relative to the relationship among the acceleration, net force, and inertial mass of the system with only an occasional minor error. Selects appropriate mathematical routines and applies them with only minor errors. Correctly identifies most of the sources of experimental error, and suggests ways to minimize the uncertainties.
On the Path to Proficiency	Identifies the most obvious patterns within data, relative to the relationship among the acceleration, net force, and inertial mass of the system with some errors and inaccuracies. Selects appropriate mathematical routines but makes some application errors. Identifies some of the sources of experimental error, and suggests ways to minimize the uncertainties.
An Attempt	Identifies a few legitimate patterns in data, though these may be irrelevant to determine the relationship among the acceleration, net force, and inertial mass of the system. Identifies some mathematical routines that are appropriate. Identifies some of the sources of experimental error, but does not suggest ways to minimize the uncertainties.

Science Practice 5.3 The student can *evaluate the evidence provided by data sets* in relation to a particular scientific question.

Proficient	Provides a connection along with a clear justification, such as the calculation of the proportionality constant (k), for the relationship derived from dimensional analysis to determine the relationship between the acceleration and the inertial mass of the system and the relationship between the acceleration and the net force of the system.
Nearly Proficient	Provides a connection but no justification is offered, or a justification is offered but it is vague regarding the relationship between the acceleration and the inertial mass of the system and/or the relationship between the acceleration and the net force of the system. Attempts to represent the proportionalities among acceleration, net force, and inertial mass as an equation; rearranges and solves for the constant of proportionality k.
On the Path to Proficiency	Provides a connection but the generalization of the relationship between the acceleration and the inertial mass of the system and/or the relationship between the acceleration and the net force of the system is not correct.
An Attempt	Fails to recognize or provide a connection to the relationship between the acceleration and the inertial mass of the system, and the relationship between the acceleration and the net force of the system.

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AP Physics 1 Investigation 3: Circular Motion

How do you determine the period of a conical pendulum?

Central Challenge

In this investigation, students use a toy that executes motion in a conical pendulum to study circular motion. Given only a meterstick and a stopwatch, they must design a procedure and make measurements to predict the period of motion of the conical pendulum.

Background

A conical pendulum consists of an object moving in uniform circular motion at the end of a string of negligible mass (see Figure 1). A free-body diagram of the object is shown in Figure 2. F_T represents the tension in the string and the gravitational force on the object is mg where m is the object's mass and g is the acceleration due to gravity.

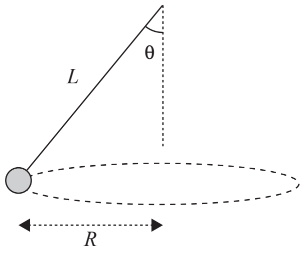

Figure 1

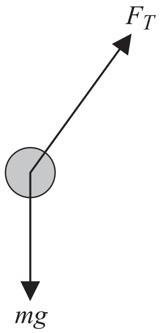

Figure 2

The circular motion of the object is in the horizontal plane, so the horizontal component of the tension is serving as the centripetal force. Since there is no vertical motion of the object, the vertical component of the tension is equal to the gravitational force on the object. In equation form:

$$F_T \sin\theta = ma_c$$

$$F_T \sin\theta = m\frac{v^2}{R}$$

and

$$F_T \cos\theta = mg$$

where R is the radius of the object's motion, v is the speed, and θ is the angle the string makes with the vertical, as shown in Figure 1.

Combining these equations we get:

$$\tan\theta = \frac{v^2}{gR}$$

The speed of an object in circular motion is given by $v = \frac{2\pi R}{T}$ where T is the period of the circular motion. Substituting this relationship into the equation above and rearranging we get $T^2 = \frac{4\pi^2 R}{g\tan\theta}$.

Thus, by measuring only lengths such as L and R (see Figure 1), and using them to calculate the angle from the vertical, students can predict the period of a conical pendulum.

[**NOTE:** L is the length of the pendulum, as measured from the point of attachment of the string to the center of mass of the object at the end of the pendulum (assuming the string has negligible mass), and R is measured from the center of the circle to the center of mass of the object.]

Real-World Application

There are many real-world applications of circular motion dealing with interchanges, intersections, and driving a car in general. You can talk about various amusement park rides as well — roller coasters deal heavily with circular motion. The swing ride is an example of a conical pendulum in which the riders sit in swings and move in circular motion around a central support structure (see Figure 3). Other rides, such as the rotor ride, Enterprise wheel, and Ferris wheel, spin the rider in circular motion either horizontally or vertically. NASA uses circular motion in a centrifuge to simulate the high g-forces on astronauts in flight. Medical equipment such as the centrifuge use circular motion principles to separate out components in test tubes.

Figure 3

Inquiry Overview

This investigation is a guided inquiry in which students make measurements with a meterstick and use them to predict the period of a self-propelled mass, such as a flying airplane (or flying pig or cow), that moves like a conical pendulum. This is a new twist on what is a familiar lab (see "Circular Motion Studies with a Toy Airplane" in Supplemental Resources).

As part of their experimental design, students should also plan to make multiple measurements to determine or verify the relationship between the length of the pendulum and the angle the string makes with the vertical as the object executes circular motion. They can vary the length and plot graphs of period vs. length, speed vs. length, and angle vs. length, and compare the graphical results to the theoretical results derived using Newton's second law.

Connections to the AP Physics 1 Curriculum Framework

Big Idea 3 The interactions of an object with other objects can be described by forces.

Enduring Understanding	Learning Objectives
3.B Classically, the acceleration of an object interacting with other objects can be predicted by using $\vec{a} = \frac{\Sigma\vec{F}}{m}$.	**3.B.1.1** The student is able to predict the motion of an object subject to forces exerted by several objects using an application of Newton's second law in a variety of physical situations with acceleration in one dimension. (Science Practice 6.4) **3.B.1.2** The student is able to design a plan to collect and analyze data for motion (static, constant, or accelerating) from force measurements and carry out an analysis to determine the relationship between the net force and the vector sum of the individual forces. (Science Practices 4.2 and 5.1) **3.B.2.1** The student is able to create and use free-body diagrams to analyze physical situations to solve problems with motion qualitatively and quantitatively. (Science Practices 1.1, 1.4, and 2.2)
3E A force exerted on an object can change the kinetic energy of the object.	**3.E.1.3** The student is able to use force and velocity vectors to determine qualitatively or quantitatively the net force exerted on an object and qualitatively whether kinetic energy of that object would increase, decrease, or remain unchanged. (Science Practices 1.4 and 2.2)

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Big Idea 4 Interactions between systems can result in changes in those systems.

Enduring Understanding	Learning Objectives
4.A The acceleration of the center of mass of a system is related to the net force exerted on the system, where $\vec{a} = \frac{\Sigma \vec{F}}{m}$.	**4.A.2.1:** The student is able to make predictions about the motion of a system based on the fact that acceleration is equal to the change in velocity per unit time, and velocity is equal to the change in position per unit time. (Science Practice 6.4) **4.A.3.1:** The student is able to apply Newton's second law to systems to calculate the change in the center-of-mass velocity when an external force is exerted on the system. (Science Practices 2.2 and 5.1)

[**NOTE:** In addition to those listed in the learning objectives above, Science Practice 4.3 is also addressed in this investigation.]

Skills and Practices Taught/Emphasized in This Investigation

Science Practices	Activities
1.1 The student can *create representations and models* of natural or man-made phenomena and systems in the domain.	Students draw free-body diagrams of the object as it executes circular motion.
1.4 The student can *use representations and models* to analyze situations or solve problems qualitatively and quantitatively.	Students use the free-body diagram and Newton's second law to write equations related to the motion of the object.
2.2 The student can *apply mathematical routines* to quantities that describe natural phenomena.	Students use equations derived from Newton's second law to analyze the motion of the object.
4.2 The student can *design a plan* for collecting data to answer a particular scientific question.	Students design a plan to use only length measurements to predict the period of a conical pendulum.
4.3 The student can *collect data* to answer a particular scientific question.	Students make measurements of various lengths associated with the motion of the object as it moves in a circle.
5.1 The student can *analyze data* to identify patterns or relationships	Students apply mathematical routines to choose data that will allow them to predict the period of the object's motion. Students analyze the uncertainty in their measurements and make adjustments to reduce these uncertainties where possible.
6.4 The student can *make claims and predictions about natural phenomena* based on scientific theories and models.	Students use Newton's second law and length measurements to predict the period of an object moving in a circle.

[**NOTE:** Students should be keeping artifacts (lab notebook, portfolio, etc.) that may be used as evidence when trying to get lab credit at some institutions.]

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Equipment and Materials

Per lab group (two to four students):

- Battery-operated toy airplane (or flying pig or cow — see Figure 4) with new 1.5-volt AA cells installed
- Meterstick
- Stopwatch (for verification only)
- (Optional) Extra sets of AA cells for the plane that have been drained so they are not at full operating potential difference. [**NOTE:** The cells in the sets should be less than 1.5 V each under load, but each cell in the set of two should be at the same potential.]
- (Optional) Multimeter to test electric potential difference of each cell

[**NOTE:** Ceiling-suspended, battery-operated airplanes (9-inch wingspan, two AA batteries required) can be obtained from The Physics Toolbox — see Supplemental Resources.]

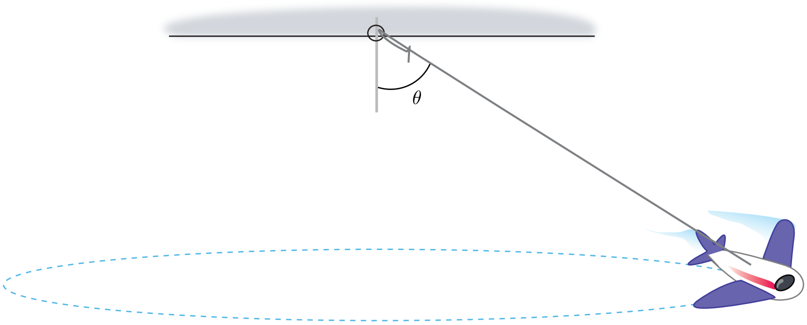

Figure 4

Timing and Length of Investigation

- **Teacher Preparation/Set-up:** 15 minutes

  The toys need to be suspended so they can execute circular motion — extend them from the ceiling or from a tall stick or pole. You should do this setup prior to the lab. [**NOTE:** Strong hooked magnets can be attached to ceiling metal cross grids to support the swivel hook that comes with the flying toy. Avoid attaching the devices to the ceiling on or at the corners of light fixtures or on sprinkler system apparatus.]

- **Prelab:** 10 minutes

  To demonstrate the conical pendulum, put students in groups and pose the problems to them.

- **Student Investigation:** 45–60 minutes

  Students design a plan to make measurements, and make the measurements and calculate the period.

- **Postlab Discussion:** 15–30 minutes

  Students present their results, and share the method they used to predict the period.

- **Total Time:** 1.5–2 hours

Safety

All general safety guidelines should be observed. In addition, some toy airplanes have small plastic propellers that rotate rapidly; students must take care to keep their fingers away from the propellers. Students should also not walk around too much to avoid getting hit in the head by a conical pendulum. Students should be wearing safety goggles on the off-chance that a string breaks.

To prevent students from climbing up on tables or chairs to change ceiling connections, it may be wise to preinstall multiple devices with new cells and with different lengths; then students can take multiple trials by simply moving to a different pendulum (assuming they all are constructed similarly).

Preparation and Prelab

This lab is best implemented at the end of the circular motion unit and used as a review. Students should already have solved many problems involving circular motion. They should be able to draw a free-body diagram and identify the radius of an object's motion.

Demonstrate for students how to start the toy airplane flying in circular motion. All that remains after that is to present them with this task: using only a meterstick, make measurements that allow for calculation of the period of the plane's motion. Students should not be shown the derivation above in the Background section; rather, they should be required to complete it themselves and decide what measurements to make.

The Investigation

Students should work in groups of two to four. The number of students per group depends upon how many toy airplanes are available or the time available for groups to rotate through using the setup. Each group should have direct access to a device.

Each group designs and executes a plan for taking measurements with a meterstick to calculate the period of a conical pendulum. They then measure the period with a stopwatch and compare the stopwatch measurement to their prediction.

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Some groups will start to measure before they have a plan. Some groups will ask if they can find the mass of the plane. They should not be allowed to use a balance to find the mass of the plane. If they can find the mass of the plane with only a meterstick (no other masses, etc.) then that's fine, but the only measurement tool they are allowed is a meterstick.

Circulate among the groups and encourage students to draw a free-body diagram of the plane and use it to write some equations. Some groups will need more assistance than others. Most groups will measure the length of the pendulum (from pivot to center of object). Some groups will measure the radius of the circular motion (from center of circle to center of object); other groups will measure how far below the support point (ceiling) the circle is. Groups need to use this measurement to calculate the vertex angle of the conical pendulum (the angle the string makes with the vertical; see Figure 1). Encourage students to only run the plane when they are making measurements so the battery doesn't run out too quickly — this will help maintain a constant speed for the plane during the experiment.

Once the students have completed their measurements and calculations, they share them with the rest of the class, perhaps using whiteboards or large sheets of paper, for a discussion related to methods of analysis.

Extension

An extension option is to provide students with AA cells that have different potential differences to power the planes, to first determine whether the potential difference affects speed. Then students can investigate how the speed affects the angle and the radius of the motion for a constant length of string supporting the plane.

Common Student Challenges

One of the biggest problems students face with circular motion is the idea of centripetal force. Many students seem to think that a "magic" centripetal force is exerted on an object when it is in circular motion, and that the direction of this force is directed outward, not inward to the center of the circle. Students think this because they are confusing centripetal force with inertia. They think that if they were in a car making a fast turn and the door opened, they would be thrown out of the car. Thus, they believe there is a force related to circular motion directed to the outside of the circle.

It is important to emphasize that a force is an interaction between two objects and help students identify the object exerting the force toward the center of another object's circular motion. Ask them to envision that to keep them in the car going around a circle, the door must exert an inward force, since their inertia would cause them to continue moving in a straight line. Trying to make an object, such as a basketball, roll in a circle by only tapping it with a meterstick will also emphasize in which direction the external force must act.

Some teachers go so far as to tell students that there is no such thing as a centripetal force, just like there is no such thing as a down force. The word *centripetal* refers to a direction. Emphasize that some external force, such as the normal force, gravity, friction, or tension must act centripetally to allow an object to execute circular motion. An activity that will help students with this concept is as follows: students draw several free-body diagrams of objects in circular motion and then select (e.g., draw a circle around) the force or forces that act centripetally. In this particular lab, the centripetal direction is horizontally toward the center of the circle in which the plane is flying, so the horizontal component of the tension force is acting centripetally.

Students also need to be reminded, when making measurements of pendulum length (L) and radius of the circle (R) that the measurements should be made to the center of the moving object. Students often mistakenly take the length of a pendulum as the length of the string or chain supporting the object; however, the pendulum length is from pivot or connection to the center of mass of the pendulum/mass system. If the supporting chain or string has negligible mass, then the pendulum length is measured from the pivot to the center of mass of the object attached.

Analyzing Results

Ask students to use a stopwatch and compare their calculated period (calculated using the length measurements) to a period measured directly with the stopwatch. They should compute a percent difference between the measured and calculated periods and describe how reasonable their results are. The “measured” period is what students measured with the stopwatch. This should be treated as the theoretical value in this case. The “calculated” period is the one derived from the distance measurements they made. This should be treated as the “experimental” value in their discussion of percent difference. Technically they are both measured values, but in calculating the percent difference, the period measured with the stopwatch has much less uncertainty, and thus can be used to approximate a true value for the period.

Encourage students to consider the uncertainties in their measurements. For example, if they measured the radius of the plane’s motion while it was moving, how precisely could they measure the radius? What is the uncertainty in each of their measurements? What do they think the total uncertainty is? Are their measured and calculated values the same within the limits of precision of their measurements? In other words, is the percent difference between the measured and calculated values less than the total uncertainty in their length measurements?

Students should also consider whether the speed of the plane was actually constant. Ask them how they might have noticed in their data that it wasn’t, and how that would affect their prediction.

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Students then use the measurements of varying lengths to determine or verify the relationship between the length of the pendulum and the angle the string makes with the vertical, in order to determine the relationship between the length of the pendulum and the angle the string makes with the vertical as the object executes circular motion. The equation derived above $\left(T^2 = \frac{4\pi^2 R}{g\tan\theta}\right)$ relates the radius to the period and the angle. The radius and length of string are related by $R = L\sin\theta$. Substitute this into the above equation and one obtains $T^2 = \frac{4\pi^2}{g}L\cos\theta$.

Once students have contemplated these questions within their own groups, then the whole class has a discussion comparing the various groups' methods and which methods were more precise than others. They also present their graphs for comparison and discussion.

Assessing Student Understanding

After completing this investigation, students should be able to:

- Draw a free-body diagram of an object moving as a conical pendulum;
- Design a plan to make measurements to analyze the motion of a conical pendulum;
- Evaluate the uncertainties in the measurements of length made for a conical pendulum;
- Use Newton's second law to analyze the motion of a conical pendulum;
- Predict the period of a conical pendulum using only length measurements;
- Calculate speed, period, and angle for various lengths; and
- Graph the relationships and compare them to Newton's second law.

Assessing the Science Practices

Science Practice 1.1 The student can *create representations and models* of natural or man-made phenomena and systems in the domain.

Proficient	Draws an accurate picture of the motion of a conical pendulum, and draws an accurate free-body diagram of the conical pendulum.
Nearly Proficient	Draws an accurate free-body diagram of the conical pendulum but adds a fictitious centripetal force to the diagram.
On the Path to Proficiency	Draws an almost accurate free-body diagram with one or more additional forces that are incorrect.
An Attempt	Draws an inaccurate free-body diagram of the conical pendulum.

Science Practice 1.4 The student can *use representations and models* to analyze situations or solve problems qualitatively and quantitatively.

Proficient	Makes no mistakes in using a free-body diagram to analyze the motion of a conical pendulum using Newton's second law.
Nearly Proficient	Makes minor mistakes in using the free-body diagram or picture to write equations to analyze the motion of a conical pendulum.
On the Path to Proficiency	Makes major mistakes in using Newton's laws and the free-body diagram to analyze the conical pendulum.
An Attempt	Unable to write equations using the free-body diagram.

Science Practice 2.2 The student can *apply mathematical routines* to quantities that describe natural phenomena.

Proficient	Uses Newton's second law to analyze the motion of a conical pendulum, using length measurements only to calculate period, speed, and angle.
Nearly Proficient	Makes mostly correct calculations from equations; may confuse the use of sine and cosine.
On the Path to Proficiency	Makes some correct calculations from equations.
An Attempt	Makes no correct calculations from equations.

Science Practice 4.2 The student can *design a plan* for collecting data to answer a particular scientific question.

Proficient	Designs an accurate and appropriate plan to make length measurements to predict the period of a pendulum.
Nearly Proficient	Follows directions and adds a design plan that is mostly complete, including diagrams and assumptions.
On the Path to Proficiency	Follows directions but does not clearly indicate a plan for experimental design and procedure.
An Attempt	Misinterprets directions or does not indicate a plan for experimental design and procedure.

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Science Practice 4.3 The student can *collect data* to answer a particular scientific question.

Proficient	Collects accurate and appropriate length data from multiple trials using different lengths to predict the period of a conical pendulum.
Nearly Proficient	Collects data that is missing a few minor pieces or is disorganized in its presentation. For example, doesn't perform trials for multiple pendulum lengths.
On the Path to Proficiency	There are major gaps in the data collected, and the presentation lacks any organization.
An Attempt	Collects inaccurate or incomplete data or doesn't provide any organization for this data.

Science Practice 5.1 The student can *analyze data* to identify patterns or relationships.

Proficient	Analyzes the uncertainties in the length measurements made and determines the uncertainty in the period calculated. Graphs the relationships from period, speed, and angle as functions of pendulum length and compares them to Newton's second law.
Nearly Proficient	Estimates uncertainties and calculates a total uncertainty without being clear on how to use this to evaluate the accuracy of the result.
On the Path to Proficiency	Estimates uncertainties in measurements but does not compute a total uncertainty or compare it to the percent difference.
An Attempt	Cannot accurately evaluate the uncertainties in measurements.

Science Practice 6.4 The student can *make claims and predictions about natural phenomena* based on scientific theories and models.

Proficient	Accurately predicts how changing the length of a conical pendulum will change the period of the pendulum and applies this prediction to changes in speed and angle.
Nearly Proficient	Makes a minor mistake in the predictions about the motion of a conical pendulum.
On the Path to Proficiency	Makes only limited predictions about the motion of a conical pendulum.
An Attempt	Cannot make accurate predictions about the motion of a conical pendulum.

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Supplemental Resources

Butcher, Frank "Circular Motion Studies with a Toy Airplane." *The Physics Teacher* 25, no. 9 (1987): 572–573.

"Circular Motion." PBS LearningMedia. Accessed September 1, 2014. http://www.teachersdomain.org/resource/lsps07.sci.phys.maf.circmotion. [*This website links to the circular motion videos available on the Rutgers PAER video website and also has background discussions, questions, and standard relations to circular motion. These can be used for a prelab demonstration to show that if an object is moving in a circle, there must be a force directed towards the center of the circle.*]

"The Forbidden F-Word." The Physics Classroom. Accessed September 1, 2014. http://www.physicsclassroom.com/class/circles/u6l1d.cfm. [*This website provides some of the basics of circular motion that can easily be related to curves on roads and highways. The website provides some basic animations as well, and can be used for extensions where students go for further knowledge.*]

"Ladybug Motion 2D." PhET. University of Colorado Boulder. Accessed September 1, 2014. http://phet.colorado.edu/en/simulation/ladybug-motion-2d. [*This simulation provides an interactive way to learn about position, velocity, and acceleration vectors.*]

The Physics Toolbox. Accessed September 1, 2014. https://www.physicstoolboxinc.com/p-180-centripetal-force-airplane.aspx.

AP Physics 1 Investigation 4: Conservation of Energy

How does the compression of a spring affect the motion of a cart?

Central Challenge

In this investigation, students experiment with the concept of the conservation of energy by qualitatively investigating the relationship between elastic potential energy and gravitational potential energy. Students take a spring-loaded cart and release it so that it travels up a ramp. In addition to making observations and measurements, they make predictions as to what would happen if the angle of the ramp changed. Then, students experiment quantitatively with the relationship between the compression of the spring and the gravitational potential energy of the Earth-cart system. They do this by repeating measurements of the cart on the ramp for different compressions of the spring.

Background

The gravitational potential energy (U_g) of an Earth-cart system can be calculated with the equation $U_g = mgy$. Total energy for a closed system is conserved and so the decrease in the spring potential energy (U_{spring}) is equal to the gain in the U_g as the cart moves up the incline.

Conservation of energy is the hallmark organizing principle in all sciences. As the total energy of a closed system remains constant, a loss of one form of energy must be equal to a gain in another form of energy. Potential energy of a system is due to the interactions and relative positions of its constituent objects. Energy transferred into or out of a system can change the kinetic, potential, and internal energies of the system. Energy transfers within a system can change the amount of kinetic energy in the system and the amount of potential and internal energy, or the amount of different types of potential energy. These transfers of energy can be seen in many instances: amusement parks, electric generators, fluid flow dynamics, and heating.

Real-World Application

In this lab, students find that the loss of spring potential energy is equal to the gain in kinetic energy of a cart. In turn, kinetic energy then decreases as the gravitational potential energy increases. Operators of trains and trucks use these principles for emergency stops. At the train station, a huge spring is compressed to bring the train to rest should the brakes fail. Similarly, a truck driver might use an uphill ramp on the side of a road to bring the truck to rest. In the case of the train, the loss in kinetic energy is equal to the gain in the spring potential energy. In the case of the truck, the loss in kinetic energy is equal to the gain in gravitational potential energy. In both cases, some energy is converted into thermal energy.

People seeking thrills jump off bridges secured by a bungee cord. In this case, the energy transformations include a loss of gravitational potential energy and a gain of kinetic energy. The kinetic energy then decreases and is accompanied by an increase in the spring potential energy. Once again, some energy is converted into thermal energy.

In designing amusement park or carnival rides, it is also necessary to apply the principle of conservation of mechanical energy. For example, to build a roller coaster one must accurately predict the speed at the top of a loop to insure that the ride is safe.

Inquiry Overview

This investigation is divided into three different parts. Each part engages the student in guided-inquiry activities.

In Part I, a spring-loaded cart is placed on an incline and the cart's motion is observed once the spring is released. Students design their own experiment to test how the angle of the ramp changes the motion of the cart for the same compression of the spring.

In Part II, students design their own experiment to determine how changes in the compression of the spring change the amount of increase of the gravitational potential energy of the Earth-cart system.

In Part III, students consider how to improve their experimental design to take into account overlooked aspects of the earlier experiments. As an extension, they can also begin a new experiment where the transfer of energy out of the Earth-cart system changes the compression of the spring.

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Connections to the AP Physics 1 Curriculum Framework

Big Idea 5 Changes that occur as a result of interactions are constrained by conservation laws.

Enduring Understanding	Learning Objectives
5.B The energy of a system is conserved.	**5.B.3.1** The student is able to describe and make qualitative and/or quantitative predictions about everyday examples of systems with internal potential energy. (Science Practices 2.2, 6.4, and 7.2)

[**NOTE:** In addition to those listed in the learning objective above, the following science practices are addressed in the various lab activities: 3.1, 4.1, 4.3, 4.4, 5.1, and 6.1.]

Skills and Practices Taught/Emphasized in This Investigation

Science Practices	Activities
2.2 The student can *apply mathematical routines* to quantities that describe natural phenomena.	Part II: Students find the mathematical relationship between the compression of the spring and the gain in gravitational potential energy. Since this is not a linear relationship, students need to find alternative means of graphing and analyzing the data to secure a linear relationship (i.e., plotting the square of the compression vs. the gain in U_g in the case of many data points). Students with four data points or more should be able to show that the relation between compression of the spring and the energy the spring can provide a cart is not linear. They should also show that a quadratic relationship is supported by the data.
3.1 The student can *pose scientific questions.*	Part I: Students make observations of a cart going up a ramp and pose a question about how the angle of the incline will change the motion. Part II: Students pose questions about the relationship between the compression of the spring and the gain in gravitational potential energy of the Earth-cart system.
4.1 The student can *justify the selection of the kind of data* needed to answer a particular scientific question.	Part II: Students decide how to measure the compression of the spring and the change in gravitational potential energy. They also decide on the number of trials required.
4.3 The student can *collect data* to answer a particular scientific question	Parts I, II, and III: Students collect data as they design their own experiments and/or engage in the different data collection activities.

Science Practices	Activities
4.4 The student can *evaluate sources of data* to answer a particular scientific question.	Part III: Students consider the role that friction played in their experimental design and data collection.
5.1 The student can *analyze data* to identify patterns or relationships	Part II: Students decide if their data better fits a linear model or a quadratic model.
6.1 The student can *justify claims with evidence.*	Part I: Students create a claim regarding the motion of the cart up different inclines (e.g., more time, more distance, more speed, more height) and then use their experimental evidence to support or refute their claim.
6.4 The student can *make claims and predictions about natural phenomena* based on scientific theories and models.	Part I: Although a cart on a steeper slope will travel at a different acceleration, a different distance, and for a different elapsed time, the Earth-cart system will gain an identical amount of U_g. This allows students to use the theory of conservation of energy to make claims and predictions about the investigation.
7.2 The student can *connect concepts* in and across domain(s) to generalize or extrapolate in and/or across enduring understandings and/or big ideas.	Part II: The relationship between the compression of the spring and the gain in height leads to an understanding of the conservation of energy where the compression of the spring is related to the spring potential energy and the gain in height corresponds to a gain in gravitational potential energy. Part III: The conservation of energy principle (an enduring understanding) does not result in constant total energy in this experiment. Students recognize that this is due to the fact that the system is not closed since there are losses of energy due to friction.

[**NOTE:** Students should be keeping artifacts (lab notebook, portfolio, etc.) that may be used as evidence when trying to get lab credit at some institutions.]

Equipment and Materials

Per lab group:

- Low-friction dynamics cart with spring bumper (or spring-loaded plunger cart)
- Ramp
- Meterstick
- Stopwatch
- Assorted masses
- Books or blocks (to create incline)
- Poster-size whiteboards for sharing group work

Timing and Length of Investigation

- **Teacher Preparation/Set-up:** 10–15 minutes
- **Part I:**

  Student Investigation: 20 minutes (this includes prelab time)

  Postlab Discussion: 20 minutes (allow 5–10 minutes per group)
- **Part II:**

  Prelab: 10–15 minutes

  Student Investigation: 40 minutes

  Postlab Discussion: 40 minutes (or allow 5–10 minutes per group)
- **Part III:**

  Prelab: 15 minutes

  Student Investigation (procedural time to repeat experiments): 30 minutes

  Postlab Discussion: 20 minutes
- **Total Time:** approximately 3.5 hours

Safety

Remind students that the carts should not be on the floor where someone could slip on one. They should also consider how the spring-loaded cart could hurt someone if the plunger released near the body, especially the eye. All general lab safety guidelines should always be observed.

Preparation and Prelab

Part I of this investigation serves to determine students' prior knowledge regarding the change in the gravitational potential energy of the Earth-cart system. This then serves as the prelab for Part II.

The Investigation

Part I: Introducing the Apparatus and Experimental Design

Introduce this part of the investigation by setting up a demonstration with a spring-loaded cart on an inclined ramp (see Figure 1). Release the cart and have the students observe the motion.

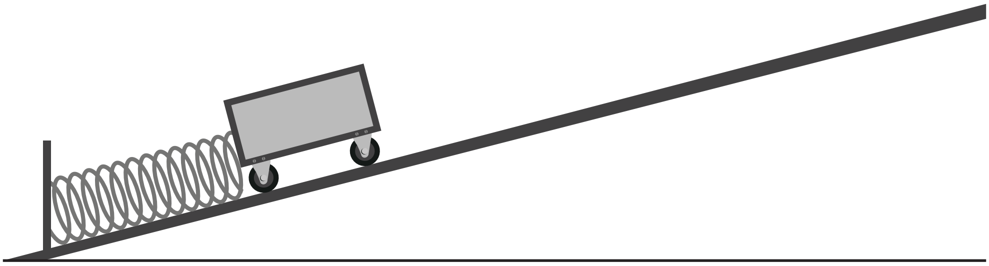

Figure 1

Prompt students: *If the cart were to be shot up a steeper vs. shallower ramp, describe how its motion will change.*

[**NOTE:**] You should not ask how the height changes, since that limits your ability to find out everything that a student is thinking about concerning the change. Expect some students to focus on greater height, greater distance, or greater time. Others may say that the cart will go a different amount up the slope or that it reaches the same height, and still others may say that it will take more or less time to reach the top. All are suitable responses, and all can be developed into experimental designs.

GUIDE STUDENTS: Instruct students to first make and justify their predictions individually, have them discuss those predictions in small groups, and then have a whole-class discussion (do NOT reveal the "right" answer). Next, have students design an experiment with the cart and ramp to investigate the question above. Each group should discuss their design and findings, and prepare to present them to the class (individual poster-size whiteboards are great for this). As a whole class, discuss the results. If there was enough friction that it affected the results, you may need to bring it into the discussion here. If there was negligible friction, the final height achieved would be the same in either case. However, since the distance travelled to reach the same height is larger on the smaller angle ramp, friction usually means it will not go as high. If not careful, students will use this observation to support the wrong conclusion.

In reviewing the experimental design, you should discuss whether multiple measurements should have been made for each angle and, if so, how many measurements would be sufficient. Ask students if one angle change was sufficient or if multiple angle changes should have been made.

If this did not come up in the class discussions, in reviewing the experimental designs and results, raise the question of the role of friction in the experiment. If there was much more friction, how would the results have changed?

Part II: Applying the Principle of Conservation of Energy

In this part of the investigation, students explore their understanding of energy and energy conservation.

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BACKGROUND: Traditionally, students have learned that the principle of conservation of energy states that energy can neither be created nor destroyed, and the total energy of a closed system remains constant. They should have also learned that the gravitational potential energy of the Earth-cart system can be calculated with the equation $U_g = mgy$. Remind them that if energy is indeed conserved, then the work on the spring from compressing it must give it some spring potential energy (U_{spring}).

Energy exists in the compression of the spring (spring potential energy [U_{spring}]), in the movement of the cart (kinetic energy [K]), and in the Earth-cart system (gravitational potential energy [U_g]).

ASK STUDENTS: As a way of testing student understanding of this principle for this part of the investigation, have students answer the following questions:

For each of the following four locations of the cart shown in Figure 2, what is the magnitude of the U_{spring}, K, and U_g at that location? Specifically, which is large, which is small, and which is zero?

Location 1: Cart is next to fully compressed spring

Location 2: Spring is no longer compressed; cart is slightly in front of spring

Location 3: Cart is halfway up the ramp

Location 4: Cart is at peak distance along the ramp

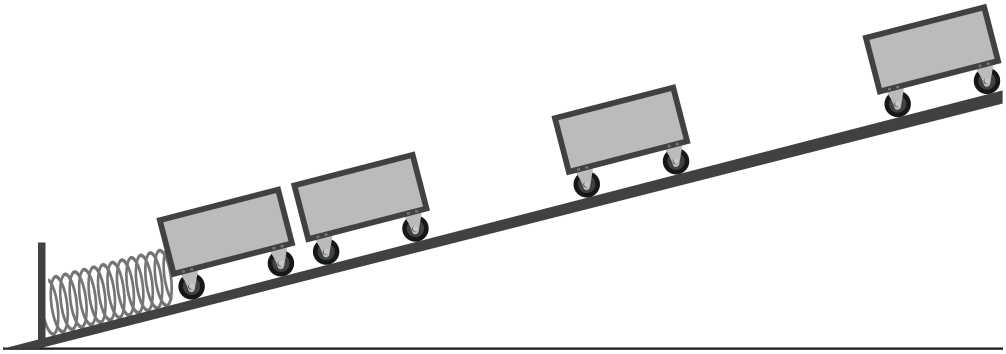

Figure 2

The students should recognize that the U_{spring} must then be equal to the U_g of the Earth-cart system after the cart gets to its peak position and no longer has any kinetic energy (K). Ask them if these statements are consistent with what they found in Part I of the investigation and to explain how they are or are not. If friction were eliminated, would the new expected experimental results be consistent with this energy explanation?

GUIDE STUDENTS: Introduce this part of the investigation by repeating the demonstration with a spring-loaded dynamics cart on an inclined ramp. Release the cart and have students observe the motion. Describe to the students that we can change the U_{spring} by compressing the spring different amounts. Some apparatus allow two possible compressions, while others allow for more possible compressions.

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Ask students to design an experiment to investigate how the energy (U_{spring}) stored in the spring depends on the distance by which it is compressed. Specifically, if you increase the compression by a factor of 2, what happens to the U_{spring}?

Part II (A): Qualitative Investigation of Potential Energy

Instruct students to design an experiment to qualitatively describe the relationship between compression of the spring and the gravitational potential energy. Students should be prepared to present a convincing argument and defend their results. Again, have small groups create a presentation to be shared with the whole class (individual poster-size whiteboards work well).

Part II (B): Quantitative Investigation of Potential Energy

Instruct students to design an experiment in which they collect data in order to quantitatively support their claim. Students should complete their experiment and share their results with the class.

Part III: Improving the Experimental Design

There are a number of potential experimental errors. If students did not take these into account as they conducted their experiments in Part II, they should now consider the following:

1. What role does friction play in the experiment? How can you minimize or take into account the frictional effects?
2. If the spring could only be compressed by two values (or if the spring could be compressed for multiple values), how would your experiment change?
3. How does the amount of compression of the plunger change the manner in which you measure the distance the cart moved and/or the maximum height?

Extension

There are a number of possible extensions to this investigation that students can choose from as well as extensions they can create on their own, including:

1. How would the results change if the angle of the ramp were to change?
2. Should the experiment be done at multiple angles?
3. Which angle produces the most reliable results?
4. Do the wheels have an impact on the experimental results? Would the experiment work better with large wheels or small wheels?
5. Does the mass of the cart affect the experimental results? Which mass car would produce the most reliable results?

A more complex extension would be to have the cart descend the ramp and hit the spring. With this setup, students can investigate how much the spring compresses. They can also investigate at which point the cart is traveling the fastest.

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Common Student Challenges

Part I:
Students should observe that changing the angle of the ramp will change the distance traveled, the acceleration of the cart, and the elapsed time to reach the top. They then design a way to accurately measure the distances the cart travels since the cart is only at its peak for a moment. Changing the angle will not have a large effect on the height above the ground that the cart reaches. It will not be obvious to many students why the most important variable is the one variable (height) that does not change, or why it does not change.

Part II:
Since Part I should confirm that the gravitational potential energy gained by the Earth-cart system was always the same for the same compression, students should be comfortable with using the final gravitational potential energy as the quantity for the initial elastic potential energy. As students vary the compression distance, the observation should be that the cart's final height is directly related to the compression; however, the relationship will not be linear. If students have only two possible compressions, they should try to look for a mathematical pattern with the two data points (linear or not linear). If there are multiple compressions permitted with the apparatus, then students should make a graph and find that it is not linear.

Analyzing Results

Part I:
Having students report on large individual whiteboards is ideal. Since this investigation is qualitative in nature, students need only present their general findings. As small groups present, be sure to call particular attention to the presentations that include convincing data (especially graphic data). There are a number of variables that could have been studied (e.g., velocity, distance traveled, height attained, and elapsed time). If these have not been investigated by any group, ask them for their predictions and an explanation for that prediction.

If no team chose to investigate the height attained (and you did not encourage a team that identified height as a variable to measure it), then it will be necessary to have them do so now. Some students may wonder why you did not just tell them at the outset that height is the important variable instead of letting them "waste time" on variables that, in effect, are not as helpful. But doing so would have prevented you from being able to tap into students' sense of what variables matter and what should determine their design of the experiment; it might also have misled them into thinking that their variables were just as valuable as height attained. Telling students which variables to study limits the inquiry-based methodology being encouraged.

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In reviewing the experimental design and results, you should once again discuss whether multiple measurements should have been made for each angle and, if so, how many measurements would be sufficient. You should also ask if one angle change was sufficient or if multiple angle changes should have been made.

At this point you should also raise the question of the role of friction in the experiment: if there was much more friction, how would the results have changed?

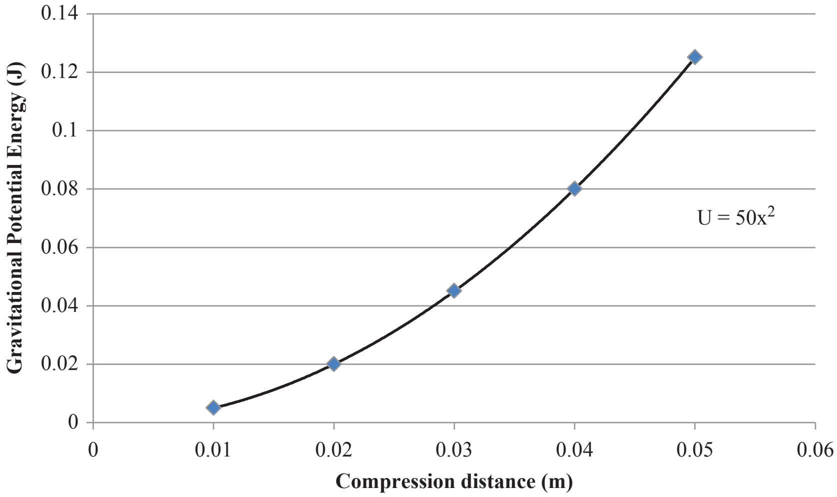
Part II:

If the apparatus has only two settings for the spring compression, that will prevent a graph from being useful. Students can still investigate if twice the compression changes the U_g by a factor of 2 or more than 2. If the apparatus allows for multiple spring compressions, then students should consider the value of making a graph.

Students should create a presentation that will provide a convincing argument supporting their findings. Have students present and discuss what was observed. Each presentation should be followed by questions from the other groups challenging the experimental technique and asking how different factors were taken into account. Encourage students to come up with alternative interpretations of the data. While the whiteboard is useful for displaying procedure, data, and graphs in a way that can be easily shared, students should use a graphing program (calculator or computer software) to evaluate the trend-line; and if linear, include the equation of the line with their graph.

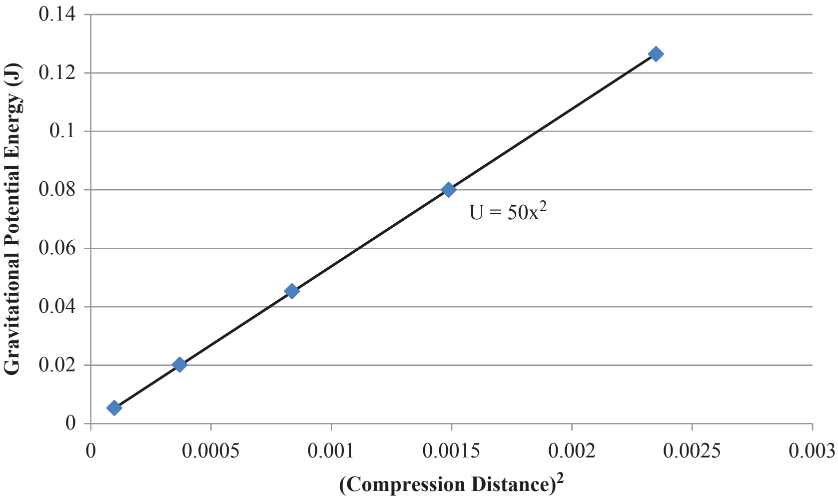
Students should include enough detail so that other groups could perform their experiment. This includes the mass of the cart, description of the ramp, measurement of the angle of the ramp, and description of how the compression of the spring and the final height (for U_g calculation) were measured.

When the compression distance is varied, students should observe that height increases, but the relationship is not linear, as shown in Graph 1. This function behaves as $y = x^2$, so plotting the compression distance squared vs. the gravitational potential energy will yield a linear relationship.



Graph 1: Gravitational Potential Energy vs. Compression Distance

An example of how to graph the compression distance vs. cart height is shown above in Graph 1, and an example of how the linearized data would appear is shown below in Graph 2:



Graph 2: Gravitational Potential Energy vs. (Compression Distance)^2

Students more familiar with approaches to making a graph linear may choose to make a log-log plot of the gravitational potential energy vs. the compression distance. They will find that the log-log plot is linear and the slope is equal to 2, which can be interpreted as the quadratic relationship. You will have to decide whether graphs should be completed by hand or by using a computer (spreadsheet or graphing program) or calculator.

With fewer than four data points, it is not possible to disprove a linear relationship graphically. With few data points, even if more than four, the graphs may not reveal the relationship that gravitational potential energy is proportional to the square of the compression distance. This can lead you to have the students investigate the uncertainties inherent in each of their measurements. What is the uncertainty in your measurement of height? How does this lead to uncertainty in the calculation of gravitational potential energy? Similarly, what is the uncertainty in the measurement of the spring compression?

Part III:
This part speaks to subtleties in the interpretation of experimental results. As extensions, students can perform additional experiments and/or explain how they would respond to these questions and/or how they would design experiments to test them.

Students should record the final product of the experiments in either their lab journal, portfolio, or on a whiteboard display. Have students examine the best examples and give an opportunity to move around the room and record the general procedure, data and graph, and discuss the results.

Assessing Student Understanding

Part I:
After completing this part of the investigation, students should be able to make the following statement regarding the transfer of energy from the spring to the cart:

For a given compression of the spring, the energy transferred from the spring to the Earth-cart system produced a consistent height traveled by the cart regardless of the angle of the incline.

Part II:
Given a reminder about the calculation of U_g and the assumption that energy is conserved, students should be able to explain the energy decrease in the spring was equal to the energy gain by the Earth-cart system. After completing this part of the investigation, students should be able to make the following statements regarding the transfer of energy from the spring to the cart:

- *When the compression of the spring is increased, the resulting height traveled by the cart increases nonlinearly.*
- *A doubling of the compression more than doubled the maximum height of the cart.*

Students should be able to conclude that it is a quadratic relationship. They should be able to recognize that compressing the spring changed the value of the spring's potential energy (U_{spring}). Students should also see that a quadratic relationship between spring compression and U_{spring} could account for the experimental results.

Assessing the Science Practices

Science Practice 2.2 The student can *apply mathematical routines* to quantities that describe natural phenomena.

Proficient	Using multiple data points, creates a new graph of the square of the compression vs. the gain in U_g, and determines the equation for this straight line as well as the significance of the slope and *y*-intercept. Using only two data points (due to limitations of the apparatus), illustrates that the relationship between compression of the spring and the energy the spring can provide a cart is not linear. Calculates the U_{spring} and the U_g from the data.
Nearly Proficient	Using multiple data points, creates a new graph of the square of the compression vs. the gain in U_g and determines the equation for this straight line. Using only two data points (due to limitations of the apparatus), illustrates that the relationship between compression of the spring and the energy the spring can provide a cart is not linear. Calculates the U_{spring} and the U_g from the data.
On the Path to Proficiency	Using multiple data points, graphs the compression vs. the gain in U_g and determines that it is not linear. Using only two data points (due to limitations of the apparatus), illustrates that the relationship between compression of the spring and the energy the spring can provide a cart is not linear; several errors may be present in the illustration. Identifies the values needed to calculate the U_{spring} and the U_g from the data; attempted calculations contain several errors.
An Attempt	Using a few data points, graphs the compression vs. the gain in U_g. Using only two data points (due to limitations of the apparatus), illustrates that an increase in the compression of the spring increases the energy the spring can provide a cart; several errors may be present in the illustration. Explains the quantities expressed by variables in the equation; no calculations of U_{spring} and U_g are attempted.

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Science Practice 3.1 The student can *pose scientific questions.*

Proficient	Makes a claim regarding angle size and distance traveled, and provides a quantitative estimate for its justification. Makes a quantitative statement about the ratio of the compression of the spring, U_g, and the measured height. Poses scientific questions based on the translation of their claims and quantitative statements.
Nearly Proficient	Makes a claim regarding angle size and distance traveled. Makes a quantitative statement about the ratio of the compression of the spring, U_g, and the measured height; the statement contains minor errors. Poses scientific questions based on a claim or quantitative statement.
On the Path to Proficiency	Makes a claim regarding angle size and distance traveled, but several errors are present. Makes a statement regarding an increase in the spring compression and the increase in gravitational potential energy. Poses scientific questions based on a claim.
An Attempt	Makes an incomplete claim regarding angle size and distance traveled; major errors are present. Attempts to identify the relationship between the compression of the spring and how it may affect the height that the cart attains; several errors in logic are present.

Science Practice 4.1 The student can *justify the selection of the kind of data* needed to answer a particular scientific question.

Proficient	Demonstrates how to best measure the compression of the spring and the change in gravitational potential energy, and provides justification for measuring each. Explains why at least three trials should be taken for each compression of the spring and how more will be needed if the data has too much spread.
Nearly Proficient	Demonstrates how to best measure the compression of the spring and the change in gravitational potential energy. Explains why at least three trials should be taken for each compression of the spring.
On the Path to Proficiency	Identifies that measurements of the compression of the spring must be made along with the change in gravitational potential energy. Explains why multiple trials and measurement readings are made.
An Attempt	Describes the type of data being collected.

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Science Practice 4.4 The student can *evaluate sources of data* to answer a particular scientific question.

Proficient	Identifies and describes that the transfer of energy is due to the work done by frictional forces. Explains how the results would differ if friction were somehow eliminated. Describes the relationship between friction and the energy considerations of the experimental design.
Nearly Proficient	Identifies that the transfer of energy is due to the work done by frictional forces. Explains how the results would differ if friction were somehow eliminated.
On the Path to Proficiency	Articulates that there is a transfer of energy. Describes the impact of friction on the data.
An Attempt	Makes a statement regarding the presence of friction; some errors may be present.

Science Practice 5.1 The student can *analyze data* to identify patterns or relationships.

Proficient	Demonstrates that the data are not linear and that a change of axes could produce a linear relationship. Observes the graph to be quadratic and draws a new graph with the square of the compression distance on the *x*-axis. Using the regression line, writes an equation (for this line) and determines the spring constant. Demonstrates how a quadratic relationship is supported by the data.
Nearly Proficient	Observes that the data are not linear and that a change of axes could produce a linear relationship. Observes the graph to be quadratic and draws a new graph with the square of the compression distance on the *x*-axis. Draws the regression line.
On the Path to Proficiency	Observes that the data are not linear and that a change of axes could produce a linear relationship.
An Attempt	Observes that the data are not linear but cannot demonstrate why.

Science Practice 6.1 The student can *justify claims with evidence.*

Proficient	Makes a claim regarding the motion of the cart up different inclines, and provides experimental evidence and reasoning to support or refute the claim; the evidence is based on experimental data; the reasoning includes the concepts of energy transfer and the role of frictional forces.
Nearly Proficient	Makes a claim regarding the motion of the cart up different inclines, and provides experimental evidence and reasoning to support or refute the claim; the evidence is based on experimental data; minor errors are present.
On the Path to Proficiency	Makes a claim but provides insufficient evidence; the evidence is based on a statement referring to possible data.
An Attempt	Makes a claim.

Science Practice 6.4 The student can *make claims and predictions about natural phenomena* based on scientific theories and models.

Proficient	Applies the conservation of energy, and explains how the spring's compression can be used to calculate the spring potential energy. Uses the height the cart attains to calculate the gravitational potential energy from the data.
Nearly Proficient	Defines the principle of conservation of energy, and explains how the spring's compression can be used to calculate the spring potential energy. Identifies that the height the cart attains can be used to calculate the gravitational potential energy from the data; calculations are attempted with several errors.
On the Path to Proficiency	States the principle of conservation of energy, and identifies that the spring's compression is one measure of energy and that the height the cart attains represents the gravitational potential energy from the data.
An Attempt	States the principle of the conservation of energy with minor errors, and identifies that spring potential energy and gravitational potential energy are both present in the system.

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Science Practice 7.2 The student can *connect concepts* in and across domain(s) to generalize or extrapolate in and/or across enduring understandings and/or big ideas.

Proficient	Connects the concepts of spring potential energy, the kinetic energy, and the gravitational potential energy to the big idea of conservation of energy. Tracks the total energy, the spring potential energy, the kinetic energy, and the gravitational potential energy at all points on the incline. Explains where energy losses occur and/or what energy has not been accounted for in the experiment. Provides upper limits to the loss of energy, and makes reasonable predictions of how the system would behave if the frictional forces were eliminated.
Nearly Proficient	Connects the concepts of spring potential energy, the kinetic energy, and the gravitational potential energy to the big idea of conservation of energy. Tracks the total energy, the spring potential energy, the kinetic energy, and the gravitational potential energy at many of the points on the incline with minor errors. Explains where energy losses occur and/or what energy has not been accounted for in the experiment.
On the Path to Proficiency	Connects the concepts of spring potential energy, the kinetic energy, and the gravitational potential energy to the big idea of conservation of energy with minor errors. States where each energy is a maximum. Describes the sources of energy losses.
An Attempt	Articulates the relationship that exists between spring potential energy, kinetic energy, and gravitational potential energy with several errors in logic. Identifies that energy losses occur.

Supplemental Resources

“Elastic Potential Energy.” HyperPhysics. Georgia State University. Accessed September 1, 2014. http://hyperphysics.phy-astr.gsu.edu/hbase/pespr.html. [*This website provides a basic explanation of the energy stored in a spring.*]

Froehle, Peter, and Charles H. Miller. “Student Misconceptions and the Conservation of Energy.” *The Physics Teacher* 50, no. 6 (2012): 367–368.

“Gravitational Potential Energy.” Zona Land Education. Accessed September 1, 2014. http://zonalandeducation.com/mstm/physics/mechanics/energy/gravitationalPotentialEnergy/gravitationalPotentialEnergy.html. [*This website provides a good definition of gravitational potential energy. It shows a basic derivation of the equation from work. There are also sample problems to solve.*]

“Potential Energy.” The Physics Classroom. Accessed September 1, 2014. http://www.physicsclassroom.com/class/energy/u5l1b.cfm. [*This website outlines many applications of the conservation of energy.*]

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AP Physics 1 Investigation 5: Impulse and Momentum

How are force and impulse related to linear momentum and conservation of momentum?

Central Challenge

In this multipart investigation, students investigate concepts of impulse and momentum both qualitatively and quantitatively. After they explore the basic concepts of momentum, they gather the data needed to calculate changes in momentum and impulse, make predictions about motions of objects before and after interactions, and determine whether momentum is conserved.

Background

Linear momentum describes the translational motion or motion of the center of mass of an object or system in terms of its mass and velocity ($\vec{p} = m\vec{v}$). Momentum is a vector quantity that has the same direction as the velocity. A net external force exerted on a body or system will change its momentum; this change in momentum is called impulse ($\Delta\vec{p}$). The rate of impulse, or impulse divided by time of interaction, is equal to the net force exerted on the object or system. Newton's third law of motion, then, arises from the conservation of momentum and describes interactions in terms of impulse and force: the impulse one object or system exerts on another is equal in magnitude and opposite in direction to the impulse the second object or system exerts on the first object or system.

$$\vec{F} = \frac{\Delta\vec{p}}{\Delta t}$$

The area between the plot line and the *x*-axis for a graph of force exerted on an object as a function of time is the change in momentum of the object. For example, if a force is exerted by a tennis racket while serving a tennis ball, the force exerted by the racket on the ball is forward, and the increase in momentum of the ball is also forward. The force exerted by the ball on the racket is equal in magnitude to the force exerted by the racket on the ball, and the impulse delivered by the ball to the racket is equal and opposite to the impulse delivered by the racket to the ball.

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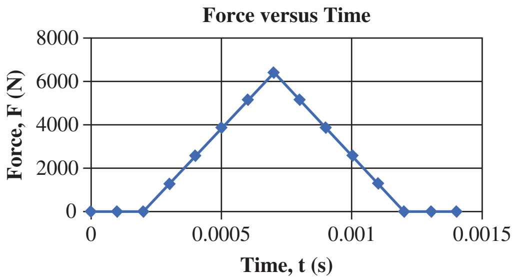
In Graph 1 below, the area between the graph line and the time axis (a triangular shape here) represents the change in momentum ($\Delta\vec{p}$) of the object on which the force is exerted. If this area is divided by the mass (m) of the object, the change in velocity ($\Delta\vec{v}$) of the object can be determined.



Graph 1

Linear momentum is always conserved. This means that if no net external force is exerted on the system, the linear momentum of the system cannot change. So, total linear momentum of objects within a system prior to an interaction of those objects is equal to the total linear momentum of the objects after the interaction when there is no external force acting on the system during the interaction. For example, if two carts on a level, frictionless track collide, the total momentum of both carts prior to the collision ($m_{1o}\vec{v}_{1o} + m_{2o}\vec{v}_{2o}$) is equal to the total momentum of both carts after the collision ($m_{1f}\vec{v}_{1f} + m_{2f}\vec{v}_{2f}$). In isolated collisions, momentum is constant and if the collision is elastic, kinetic energy is also restored, so that the final is equal to the initial for momentum and kinetic energy.

Real-World Application

Sports provide a lot of real-world applications regarding momentum and impulse. In boxing or karate you can talk about the differences between a quick jab, which produces a large change in momentum over a short time and so a large force, or a follow-through punch, which may deliver the same change in momentum, but over a longer time so a smaller force. In baseball, you can talk about how the bat changes the momentum of the ball.

Seatbelts and airbags are designed to increase the amount of time it takes a body to stop, thus decreasing the amount of force exerted on a body by the car, since the impulse exerted on a body is always equal to its change in momentum. Similarly, crumple zones in cars are also designed to increase the amount of time over which a collision occurs, thus reducing the amount of force being exerted on objects as they come into contact during the collision.

Inquiry Overview

In this lab students first pursue a qualitative examination of interactions between objects, making predictions and observations about the motions of objects before and after interactions in response to these three questions:

- How do forces exerted on an object by another object change the linear momentum of the object?

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- What is impulse?
- How are force and impulse related to conservation of linear momentum?

Depending on equipment selected (or available), students design their investigations to include collisions of two moving carts of equal and of unequal mass — both elastically and inelastically. If possible, they include an "explosion" where two carts connected by a spring and at rest are released so that the carts move apart. They use the terminology that includes linear momentum, force, impulse, and conservation of momentum to write their observations. They then share those observations with the larger group, refining their descriptions in readiness for the quantitative part of the lab.

The quantitative portion of the lab is guided inquiry, where the teacher provides the recommended equipment and sets some parameters, such as providing the purpose and setting the requirement that the analysis should include at least one graph. Students then meet in small working groups to decide how to gather and record data for the same situations they have observed qualitatively. Students decide how to make the necessary measurements of the speeds of the carts, set experimental controls, and process the data in order to answer the central question: How are force and impulse related to linear momentum and conservation of linear momentum?

Connections to the AP Physics 1 Curriculum Framework

Big Idea 5 Changes that occur as a result of interactions are constrained by conservation laws.

Enduring Understanding	Learning Objectives
5.D The linear momentum of a system is conserved.	**5.D.1.1** The student is able to make qualitative predictions about natural phenomena based on conservation of linear momentum and restoration of kinetic energy in elastic collisions. (Science Practices 6.4 and 7.2)
	5.D.1.6 The student is able to make predictions of the dynamical properties of a system undergoing a collision by application of the principle of linear momentum conservation and the principle of the conservation of energy in situations in which an elastic collision may also be assumed. (Science Practice 6.4)
	5.D.2.1 The student is able to qualitatively predict, in terms of linear momentum and kinetic energy, how the outcome of a collision between two objects changes depending on whether the collision is elastic or inelastic. (Science Practices 6.4 and 7.2)
	5.D.2.4 The student is able to analyze data that verify conservation of momentum in collisions with and without an external friction force. (Science Practices 4.1, 4.2, 4.4, 5.1, and 5.3)

[**NOTE:** In addition to those listed in the learning objectives above, Science Practice 4.3 is also addressed in this investigation.]

Skills and Practices Taught/Emphasized in This Investigation

Science Practices	Activity
4.1 The student can *justify the selection of the kind of data* needed to answer a particular scientific question.	Students meet in advance of the experiment to determine the data they need to collect in order to calculate change in momentum and impulse. They also decide what data they need to determine whether linear momentum is conserved. They may decide, for example, to collide carts moving on a track, measuring cart velocities before and after the collision and measuring the carts' masses to determine change in momentum in order to determine impulse.
4.2 The student can *design a plan* for collecting data to answer a particular scientific question.	Students make decisions in their small working groups about how to conduct the experiment to gather the necessary data to answer the question. They decide how many trials are appropriate and the method(s) they will use to gather data. For example, students may decide to use motion sensors at each end of the track to record and plot velocities.
4.3 The student can *collect data* to answer a particular scientific question.	Students collect the data they have determined they need, using the collection method(s) available to them. If motion sensors are available, students may decide to use them to plot velocities. If a camera and computer analysis tools are available, they may use this method to find velocities and changes in velocity. In the absence of these tools, students may need to use distance–time measurements to directly calculate velocities.
4.4 The student can *evaluate sources of data* to answer a particular scientific question	From the results of their experiment, students may compare momenta before and after a collision, with the goal of demonstrating that linear momentum is conserved. If the results are different, students examine sources of uncertainty in the experiment. For example, if momentum seems to have been lost or gained due to the collision, students may consider how carefully they derived values from motion sensor graphs or may re-evaluate whether friction played a role in exerting an external force on the system.
5.1 The student can *analyze data* to identify patterns or relationships.	In the quantitative portion of this lab, students answer the experimental questions by calculating impulse, force, changes in momentum, and whether momentum is constant for the system. They also determine what data can be used to create a plot that reveals meaningful results. If they have used motion sensors or video analysis, they have to use the velocity–time plots to determine changes in momentum and to assign correct signs to the quantities measured, based on direction of motion.
5.3 The student can *evaluate the evidence provided by data sets* in relation to a particular scientific question.	After calculating and graphing data, students compare results to predictions to determine whether the data produced reasonable results. For example, in making calculations related to conservation of momentum, students need to decide whether differences between original and final momentum are within reasonable limits and uncertainties to conclude that momentum is constant.

6.4 The student can *make claims and predictions about natural phenomena* based on scientific theories and models.	From the qualitative portion of the lab, students gather observations and are introduced to terminology that they will use to make predictions about results from the quantitative portion. They then evaluate their predictions, comparing the qualitative data to their predictions.
7.2 The student can *connect concepts* in and across domain(s) to generalize or extrapolate in and/or across enduring understandings and/or big ideas.	In the final analysis, students should extrapolate their findings to other experiments that might be performed to gather further data. As a required part of each analysis they also discuss practical applications of the lab. For example, students might decide to discuss how the collision of carts on a track reveals information about how cars collide on a road, particles collide in a cloud chamber, or meteorites collide with planets.

[**NOTE:** Students should be keeping artifacts (lab notebook, portfolio, etc.) that may be used as evidence when trying to get lab credit at some institutions.]

Equipment and Materials

Per lab group (three to five students):

- Two spring-loaded carts
- Track
- Bubble level
- Known calibrated masses (three to four per station, in the range of 200–500 g) and at least two objects with unknown mass (also in the range of 200–500 g)
- Calculator
- Meterstick
- Stopwatch
- Computer with Internet access
- (Optional) Video camera and analysis software
- (Optional) Force sensor
- (Optional) Motion sensor with calculator or computer interface

Most schools have access to spring-loaded carts, but a simple substitution can be contrived using any two similar objects with wheels on a track or level surface. The wheeled carts can be launched by constructing a rubber band launcher (similar to a sling shot) at each end of the track area. By pulling each cart back and releasing, the carts will move toward each other and collide. Moving carts on an air track will also work for this experiment.

Timing and Length of Investigation

- **Teacher Preparation/Set-up:** 15 minutes

  Setting up the equipment for Parts I–III should take about 15 minutes.

- **Student Investigation:** 70 minutes

  Prelab should take under 5 minutes if you choose to talk about each section of the lab individually. You can have students work on the entire lab and turn it in when completed, at which time it will take about 10 minutes to go over the lab, or you can have them all work on Part I, then stop, discuss it, and move on to Parts II and III.

  Part I: 10 minutes (Qualitative Construction of Momentum)

  Part II: 30 minutes (Collisions of Carts)

  Part III: 20 minutes (Explosions of Carts)

- **Postlab Discussion:** 25 minutes

  Student presentation time after Part I (the qualitative section) should take about 25 minutes. If you have eight groups, each group should have about 2–5 minutes to present their findings. After Parts II and III, the postlab presentation of results might take about the same: 2–5 minutes per group. It will most likely be necessary to split the lab up over two or three days, at which time Part I and the postlab discussion might take one full class period, and Parts II and III could be done the next day with their presentations on the third day.

- **Total Time:** 85 minutes to 2 hours

  [**NOTE:** This lab could be split into two class periods, with the setup, collisions, and qualitative analysis on the first day and the explosions of carts and postlab discussion on the second day. Of course, the amount of time spent depends on how you choose to setup the prelab, quantitative report-out, and postlab discussions.]

Safety

Safety is of minimal concern with this lab. Make sure students do not have the carts going excessively fast. The biggest risk may be to equipment; students should be warned not to allow carts to hit the motion sensors or the camera, or to allow carts to roll onto the floor from a raised track. The carts should NOT be considered skateboards by students trying to ride on them.

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Preparation and Prelab

Prior to this lab, students should have an introduction to linear momentum, with definitions and equations related to linear momentum, force, impulse, and conservation of momentum so they can more effectively design investigations related to those concepts. This might include just a single day or a single lesson, with students assigned a set of related problems from the textbook prior to the lab. You may decide to use one or more of the recommended resources listed as portions of assigned work to help develop concepts. However, the lab itself should be the vehicle for clarification of these concepts, so that students are truly "investigating" the meaning of concepts.

The Investigation

Students begin the lab with a qualitative investigation of the basics of momentum in Part I, where they examine the movements of carts and learn to apply the vocabulary to a description of momentum, force, and impulse. In Parts II and III, students design investigations to qualitatively gather data to examine force and impulse when carts interact — followed by measurements they then use to examine conservation of linear momentum. You should keep the pulse of how the student groups progress, depending on student proficiency. It may be necessary to convene for small-group reporting between each part in order to ensure that students understand the concepts before proceeding to the next part. This may be particularly important after Part II, which is longer and requires the application of several different concepts.

Part I: Qualitative Introduction (~10 minutes)

The first part of this activity is a qualitative introduction to the concept of momentum and how objects interact when they collide.

Have students in each group use their hands to stop as quickly as possible two identical carts rolling towards them (both carts should be stopped at roughly the same time). One cart should be moving about twice as fast as the other. Then they should repeat this procedure with both carts moving at the same speed but with one having additional masses on it. To achieve these nearly identical speeds, students can push both carts simultaneously by placing each cart against a bent ruler that then acts like a spring launcher. (Commercial devices will have a rubber band or spring launcher at each end of a track.) This part can be done on one track or on parallel tracks. The students should then discuss which cart was more difficult to stop and why they think it was more difficult to stop. Ask the students to come up with a way that would enable them to demonstrate that the force needed to be exerted on the cart in order to stop it can differ depending on the means used to stop it (e.g., by using a spring and then a ball of clay to stop each cart, and then comparing the compression of the spring to the indentation made in the clay as a means of distinguishing the force necessary to stop the cart). The point is for students to get a qualitative understanding of the stopping force as a function of cart speed and cart mass.

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The property of the carts students are describing and observing is called *momentum*. Students should also discuss the forces and impulses exerted by their hands (and by the springs or the clay) on the carts in the process of stopping them. In which case(s) does time play a role?

Part II: Colliding Carts (~30 minutes)

In this activity students design an experiment in which two carts gently collide with each other in different ways.

Task the students to calculate the velocities of the carts before and after different types of collisions. They should report their methods and their uncertainty, and then calculate the momentum of the system before and after each type of collision. Students should design at least four different variations, with the collisions ranging, for example, from one with a moving cart colliding with a parked cart, to one with two carts moving towards each other, to one with two carts moving in the same direction where the faster cart collides with the slower cart. Students should record masses and velocities before and after each collision.

Acting as a facilitator, help students to understand and realize the importance of running trials numerous times. Students should ultimately run the trials multiple times to look for patterns in the data which indicate the role mass plays in each collision. Then they should look at the data and see if they can come up with a rule for each collision. For example, they could collide equal mass carts and unequal mass carts, and see if they notice a pattern. After that they can play around and see if the rule still holds when both carts are moving. The student groups should then make presentations to the larger group that include discussions of the forces the carts exert on each other as well as the impulses delivered.

Part III: Explosions (~20 minutes)

In this activity, students build up to the idea of conservation of momentum via "explosions" of two objects moving away from one another. This can be done with two identical carts with a spring compressed between them. Releasing the spring will cause the carts to move apart, and students then calculate the momentum of both carts. [**NOTE:** Students may need to be prompted with the idea that since the center of mass of the spring does not accelerate, the force exerted by the spring on one cart is equal and opposite to the force exerted by the spring on the other cart.] Prior to releasing the carts, have the students predict what they expect to happen when the carts are released. They should come up with the expectation that if the carts start at rest, the final total momentum of the two carts should be zero. Then have students extend the activity to carts of unequal mass to again show that total momentum is constant.

Guide students to consider another way of looking at their data (i.e., using the conservation of momentum of the cart, to calculate the ratio of the two velocities during the trials where carts of unequal mass were used). For example, for one velocity to be twice the size of the other, they would need to double the mass of the other cart. Have the students write up their procedures and their experiments.

[**NOTE:** If students calculate velocities using direct distance–time measurements, their results may not show conservation as clearly as if the velocities are determined using motion sensors or video analysis methods.]

Extension

Students could follow up with an experiment where they stop a moving cart with a rubber band attached to a force sensor. That would show the force/time/impulse relationship. The cart is attached by a string/rubber band combination to a force sensor: it moves away from the sensor, extends the rubber band, stops, and then moves backward toward the force sensor. If used in conjunction with a motion detector, a full force/time/impulse/momentum analysis can be done. Several commercial types of equipment include motion sensors and force sensors that can be used for this extension. Students could produce from this data a "Force vs. Time" graph and use the area under the graph to calculate impulse.

Common Student Challenges

One of the large challenges students face with momentum is that they think momentum and inertia are the same thing. They think that larger objects will always have a larger momentum, which is not necessarily the case. In terms of conservation of momentum, students tend to place a higher value on the velocity aspect. If a small object moving quickly hits a larger object, they might expect that the larger object would move fast, because they don't realize that objects can be moving at different speeds and still have the same momentum. Students also tend to believe that conservation of momentum is only true in elastic collisions or (better but still wrong) in isolated systems. The difference between constant and conserved is often lost.

Another challenge with this topic is that students tend to think that force and impulse are synonymous. They do not realize that impulse also involves how long the force is acting on an object. Students should be required to create and/or analyze a plot of force vs. time to determine impulse (and change in momentum), either as a part of this lab or as a follow-up assignment, to reinforce this concept. If a force sensor is available, comparing the area under the curve (either by counting squares or using the computer to calculate it) of the force acting on a cart vs. time to the change in momentum of the cart is a powerful way to show that impulse on an object is the change in momentum of that object.

Particularly important is the demonstration (along with calculations) of the vector nature of change in momentum (e.g., a ball hitting a wall and bouncing back) to emphasize that the change in direction generates a much larger change in momentum (and thus larger force) than a ball that hits the wall and stops. It is important here for the teacher to emphasize the vector property of momentum by pointing out that if the ball hits the wall horizontally moving at a velocity $\vec{v}$, after an elastic collision with the wall the ball bounces back with a velocity $-\vec{v}$. The change in momentum of the ball is proportional to its final velocity minus initial velocity:

$$\Delta\vec{p} = m\Delta\vec{v} = m(\vec{v}_f - \vec{v}_o) = m(-\vec{v} - \vec{v}) = -2m\vec{v}$$

On the other hand, if the ball hits the wall and stops, the change in momentum of the ball is less:

$$\Delta\vec{p} = m\Delta\vec{v} = m(\vec{v}_f - \vec{v}_o) = m(0 - \vec{v}) = -m\vec{v}$$

Analyzing Results

Part I:
In the first part of this lab, students qualitatively explore and report (either verbally to their partners and/or in a journal) how hard they had to push on the cart — or how much force was exerted by another object, as designed by the students earlier — to make it stop. Once the two procedures in this part are done, you might want to confirm their understanding by asking if it was possible for the larger mass cart to require the same force to stop as the smaller mass cart (which was moving faster). Students should use the terms *momentum*, *force*, and *impulse* correctly in their reporting from this part in readiness for Parts II and III. If time allows, small student groups can prepare 2–3 minute presentations to the class, with you acting as facilitator, to gain feedback on improvement in procedure and correct use of terminology before proceeding to the quantitative measurements.

Questions to ask students might include:

- What quantities affect momentum?
- How is force related to change in momentum?
- When two carts collide, how do the forces they exert on each other compare?
- How is impulse related to force and to change in momentum?
- When two carts collide, how do the impulses they deliver to each other compare?

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Part II:
In the second part, students develop a method to calculate velocity. Ask them about uncertainty in the experiment. The largest will be a reaction-time error, such as a delay in starting and stopping the stopwatch. Have the students create a data table where they calculate the momentum of a system before and after each collision. Students should calculate the theoretical value for total momentum of both carts after each collision, based on the total momentum of the carts before the collision, and compare that calculated value to the experimental value for total momentum after collision, based on measurements after each collision.

Students should discuss possible sources of difference in the two values. If motion sensors or video analysis are used, students should also be able to determine the time of collision and from that calculate the impulses and forces the carts exert on each other.

Part III:
In this part the goal is to see if the previous pattern (initial momentum equals final momentum) still holds true in a situation where there is an explosion (i.e., two carts are held stationary with a compressed spring between them). The same procedures as in Part II can be used to measure and determine total momentum after the explosion to compare to the theoretical value of zero, since that was the total momentum prior to the collision. Students will have some difficulty here, as they may lose a sense of the magnitude of the uncertainty. Additionally, poor measurement techniques for both carts may, in fact, yield an answer near to the "correct" sum of zero.

You might want to provide prompts, such as: "What conclusions can be drawn about the change in momentum of cart 1 compared to that of cart 2?" Be sure to discuss how force, time, mass, and velocity play a role in your observations.

If students have been required to create at least one meaningful graph that can be used in analysis, the graph produced might be a "Velocity vs. Time" graph for one or both of the moving carts produced to show change in sign with change in direction before and after collision. If motion sensors or data analysis equipment are used, these graphs can be selected from those produced on the computer. Students should realize and comment that the amount of uncertainty in their measurements will depend upon the measurement methods employed (e.g., students using motion sensors may have a smaller amount of uncertainty than students making direct measurements with marked distances and stopwatches).

Assessing Student Understanding

After completing this investigation, students should be able to use the terms *momentum*, *force*, and *impulse* correctly to describe the motions of a system of objects before and after interactions. They should also be able to explain the meaning of conservation of linear momentum and the conditions under which momentum is constant.

Students should be able to:

- Design an experiment to show that in either an explosion (where a single object becomes multiple objects) or a collision (where multiple objects come into contact and exert forces on each other) the total momentum before the collision or explosion has to equal the total momentum afterward (providing there are no net external forces acting on the system);
- Demonstrate situations in which different forces are required to stop objects with different momenta;
- Calculate momentum and impulse (and also force if data analysis or sensors are used);
- Use calculations to show that linear momentum is conserved; and
- Produce a graph that can be used to show meaningful relationships related to momentum, such as force vs. time or velocity vs. time.

Assessing the Science Practices

Science Practice 4.1 The student can *justify the selection of the kind of data* needed to answer a particular scientific question.

Proficient	Uses the velocity–time data accurately to calculate forces and impulses and also to calculate conservation of momentum in Parts II and III of the investigation.
Nearly Proficient	Uses the data to calculate cart velocities, forces, and impulses but has some errors in calculations.
On the Path to Proficiency	Connects the concepts of spring potential energy, the kinetic energy, and the gravitational potential energy to the big idea of conservation of energy with minor errors. States where each energy is a maximum. Describes the sources of energy losses.
An Attempt	Gathers data for cart collisions but data interpretation is not present.

Science Practice 4.2 The student can *design a plan* for collecting data to answer a particular scientific question.

Proficient	Designs an experimental plan that is well communicated and leads to values for cart velocities that can be used to accurately calculate forces, impulses, and momentum conservation values.
Nearly Proficient	Designs an experimental plan to collect data for cart velocities before and after interactions that might prove effective; however, the plan is not clearly communicated or has a flaw that will produce errors.
On the Path to Proficiency	Designs an experimental plan to determine cart velocities but makes multiple errors in the plan that will lead to erroneous values.
An Attempt	Designs an experimental plan to determine cart velocities, but the design will not prove effective in answering the experimental questions.

Science Practice 4.3 The student can *collect data* to answer a particular scientific question.

Proficient	Collects data in such a way to as to minimize uncertainty; the data collected is adequate to make all calculations for cart velocities, forces, and momentum before and after interactions.
Nearly Proficient	Collects data that can be used to determine cart velocities, but does not follow through with additional data necessary to complete all calculations for force, impulse, and momentum.
On the Path to Proficiency	Collects data but collection methods are such that uncertainty is so large that calculated values will not be meaningful.
An Attempt	Collects data but the data collected will not answer any portion of the questions posed.

Science Practice 4.4 The student can *evaluate sources of data* to answer a particular scientific question.

Proficient	Addresses assumptions in the experimental design effectively, and discusses uncertainties in data gathering appropriately. If electronic methods are used to gather data, selects appropriate ranges from graphs produced by the computer, for example.
Nearly Proficient	Discusses uncertainties in the measurements in gathering data, but the discussion is incomplete or has flaws; for example, a systematic error such as a nonlevel track is evident but not realized or addressed.
On the Path to Proficiency	Collects data that can be used to calculate cart velocities, but attempts at explanations of uncertainty in the measurements are flawed.
An Attempt	Collects data but does not address uncertainty in their measurements.

Science Practice 5.1 The student can *analyze data* to identify patterns or relationships.

Proficient	Determines values for velocity, force, impulse and momentum in each scenario correctly. Constructs a correct graph from the data, such as the use of force vs. time to verify impulse calculations from velocities.
Nearly Proficient	Makes accurate calculations and graphical representation. Calculates cart velocities, forces, impulses, and momenta, but there may be errors in calculations or the graphical representation is attempted but has an error.
On the Path to Proficiency	Attempts the calculations and the graphical representation, and then makes attempts to calculate forces and impulses, but there is confusion in how the terms are used or there are errors in the calculations and the graphical representation is incorrect.
An Attempt	Unable to calculate forces, impulses, and momenta correctly from the data collected. Does not attempt a graphical representation.

Science Practice 5.3 The student can *evaluate the evidence provided by data sets* in relation to a particular scientific question.

Proficient	Reports correct relationships among force, momentum, and impulse in all three parts of the experiment, and demonstrates insight into these concepts during postlab discussions.
Nearly Proficient	Makes correct conclusions about force and momentum that need only minimal correction during postlab discussion. Makes correct conclusions about relationships between force and momentum, or about conservation of momentum during collisions, needing only minimal refinement.
On the Path to Proficiency	Makes some correct conclusions about force and momentum that need correction during postlab discussion. Makes incorrect conclusions about relationships between force and momentum or about conservation of momentum during collisions.
An Attempt	Unable to make correct conclusions about force and momentum. Unable to make correct conclusions about relationships between force and momentum or about conservation of momentum during collisions.

Science Practice 6.4 The student can *make claims and predictions about natural phenomena* based on scientific theories and models.

Proficient	Makes meaningful predictions about momentum concepts that are effectively applied to the quantitative portions of the lab.
Nearly Proficient	Makes meaningful predictions after the qualitative portion of the lab, and needs only minimal guidance on how to proceed during the quantitative portions.
On the Path to Proficiency	Makes only a few meaningful predictions after the qualitative portion of the lab, and needs guidance on how to proceed during the quantitative portions.
An Attempt	Unable to make meaningful predictions during the qualitative portion of the lab that will apply to quantitative measurements.

Science Practice 7.2 The student can *connect concepts* in and across domain(s) to generalize or extrapolate in and/or across enduring understandings and/or big ideas.

Proficient	Describes a practical application in the analysis section of the lab report that is complete and accurate.
Nearly Proficient	Describes a practical application in the analysis section of the lab report that is generally correct but is not complete or contains an incorrect step.
On the Path to Proficiency	Makes an attempt to describe a practical application in the analysis section of the lab report that is partially correct, but the connection contains some incorrect physics.
An Attempt	Makes an attempt to describe a practical application in the analysis section of the lab report but the connection is flawed.

Supplemental Resources

“Collision Lab.” PhET. University of Colorado Boulder. Accessed September 1, 2014. https://phet.colorado.edu/en/simulation/collision-lab. [*This simulation could be assigned work to follow up the investigation.*]

ComPADRE. Accessed September 1, 2014. www.compadre.org. [*This website has a free collection of resources and publications for physics educators. Click on “Classical Mechanics” under the “By Topic” section in the lower left of the landing page; then click on linear momentum which provides a list of physics education research documents relating to momentums.*]

“Elastic and Inelastic Collision.” Walter Fendt. Accessed September 1, 2014. http://www.walter-fendt.de/ph14e/collision.htm. [*This is an applet to help simulate the results of collisions and can help differentiate between elastic and inelastic collisions.*]

“Learning Cycle on Newton’s Third Law using the Momentum Approach.” Rutgers Physics and Astronomy Education Research Group. Accessed September 1, 2014. http://paer.rutgers.edu/pt3/experimentindex.php?topicid=3&cycleid=4. [*A series of videos highlighting Newton’s Third Law from a momentum approach. Though these videos are set up for Newton’s Third Law, they revolve around momentum. You will find the “Happy and Sad Ball” experiment here.*]

O’Brien Pride, Tanya, Stamatis Vokos, and Lillian C. McDermott. “The Challenge of Matching Learning Assessments to Teaching Goals: An Example from the Work–Energy and Impulse–Momentum Theorems.” *American Journal of Physics* 66, no. 2 (1998): 147–157.

Rosengrant, David, and Mzoughi, Taha. “Preliminary Study of Impulse Momentum Diagrams.” Paper presented at the Physics Education Research Conference, Part of the PER Conference series, Edmonton, Canada: July 23–24, 2008.

Singh, Chandralekha, and David Rosengrant. “Students’ Conceptual Knowledge of Energy and Momentum.” Paper presented at the Physics Education Research Conference, Part of the PER Conference series, Rochester, New York: July 25–26, 2001.

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AP Physics 1 Investigation 6: Harmonic Motion

What factors affect the motion of a pendulum?

Central Challenge

In this investigation, students explore the motion of a pendulum in two parts. In the first part, students experimentally determine what factors affect the period of a pendulum. In the second part, students create the motion graphs resulting from the periodic motion.

Background

A simple pendulum is a system that can be modeled as a point mass (m) at the end of a string of negligible mass and of length (L). The pendulum executes oscillatory motion because gravity (or more specifically, the component of gravity perpendicular to the string) provides a restoring force that pulls the pendulum back toward equilibrium at every point in its motion. The gravitational force is dependent on the mass of the pendulum bob, and since $\vec{a} = \frac{\Sigma \vec{F}}{m}$, the acceleration of the bob is independent of its mass, and so the period of the pendulum is independent of its mass. For small angles of oscillation, the period is also independent of the amplitude, so the motion approximates simple harmonic motion. So the period of a simple pendulum depends only on its length and the acceleration due to gravity (g).

An example of a system that exhibits simple harmonic motion is an object attached to an ideal spring and set into oscillation. The spring's restoring force depends on the displacement from equilibrium but not on the mass of the object in oscillation. The period can be shown to be equal to $T = 2\pi\sqrt{m/k}$ if the mass of the spring can be neglected. Refer to any calculus-based introductory physics textbook for the derivation of this period (and for a pendulum) from a second-order linear differential equation.

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Real-World Application

The most obvious real-world application of harmonic motion for students is the idea of time keeping. Everything from traditional grandfather clocks to atomic clocks use periodic oscillations to keep time. For those who study music, metronomes are a type of pendulum that keeps time. A child on a swing in the playground is a reasonable approximation of a simple pendulum, assuming he or she does not swing too high (i.e., at too great an amplitude). A good discussion could be had about under what conditions a child on swing acts like a simple pendulum, and under what conditions he or she does not. The period of an old-fashioned metronome depends on the position of the mass on the vertical post. Having students first visualize these types of oscillations is a useful way to start discussion about this investigation.

Inquiry Overview

In this investigation, students explore the motion of a pendulum in two parts.

In Part I, students experimentally determine which quantity or quantities affect the period of a pendulum. This part of the lab can be more open inquiry if implemented at the start of a simple harmonic motion unit; or it can have more structured, guided inquiry if implemented as the first lab (or very early in the course), in accordance with the modeling curriculum out of Arizona State University (see Supplemental Resources). As the first lab it would then serve to teach students about designing experiments, making measurements, and calculating or estimating uncertainties.

In Part II, students create the harmonic motion graphs resulting from the periodic motion. This part of the lab can either be open inquiry, allowing students to determine how to graph the motion of the pendulum as a function of time, or it can be more guided inquiry depending on the experience and sophistication of your students at the time you decide to implement this lab.

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Connections to the AP Physics 1 Curriculum Framework

Big Idea 3 The interactions of an object with other objects can be described by forces.

Enduring Understanding	Learning Objectives
3B Classically, the acceleration of an object interacting with other objects can be predicted by using $\vec{a} = \frac{\Sigma\vec{F}}{m}$	**3.B.3.1** The student is able to predict which properties determine the motion of a simple harmonic oscillator and what the dependence of the motion is on those properties. (Science Practice 6.4) **3.B.3.2** The student is able to design a plan and collect data in order to ascertain the characteristics of the motion of a system undergoing oscillatory motion caused by a restoring force. (Science Practice 4.2) **3.B.3.3** The student can analyze data to identify qualitative or quantitative relationships between given values and variables (i.e., force, displacement, acceleration, velocity, period of motion, frequency, spring constant, string length, mass) associated with objects in oscillatory motion to use that data to determine the value of an unknown. (Science Practices 2.2 and 5.1)

[**NOTE:** In addition to those listed in the learning objectives above, Science Practice 4.3 is also addressed in this investigation.]

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Skills and Practices Taught/ Emphasized in This Investigation

Science Practices	Activities
2.2 The student can *apply mathematical routines* to quantities that describe natural phenomena.	Students graph the period as a function of mass, angle, and length. Students derive an equation relating the period of a pendulum to the length of the pendulum using their data.
4.2 The student can *design a plan* for collecting data to answer a particular scientific question.	Students design a plan for collecting data to determine what factors affect the period of a simple pendulum.
4.3 The student can *collect data* to answer a particular scientific question.	Students collect data while varying several factors (mass, length, angle) to determine which affect the period of a pendulum and how they affect it.
5.1 The student can *analyze data* to identify patterns or relationships	Students analyze the data for period vs. mass, length, and angle to identify which factors affect the period of a pendulum and to determine the mathematical relationship from the data.
6.4 The student can *make claims and predictions about natural phenomena* based on scientific theories and models.	Students predict the period of a pendulum based on its length, mass, and angle of release.

[**NOTE:** Students should be keeping artifacts (lab notebook, portfolio, etc.) that may be used as evidence when trying to get lab credit at some institutions.]

Equipment and Materials

Per lab group:

Part I:

- String
- Set of calibrated masses (20–500 g)
- Stopwatch or timer
- Meterstick
- Protractor
- Support rod
- (Optional) Pendulum clamp

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Part II:

- Paper and tape (to create a scroll)
- Leaking bob (can be made by placing a paper towel in a small funnel and soaking it with colored water)
- (Optional) Constant speed buggy
- (Optional) Motion detector, software, and computer
- (Optional) Video camera and analysis software

Extension:

- Spring

Timing and Length of Investigations

- **Teacher Preparation/Set up:** 10–15 minutes
- **Student Investigation:** 170 minutes
- **Part I:** 85 minutes

  Prelab/demonstration/discussion: 10 minutes

  Student-centered investigation: 45 minutes

  Student-led discussion of results: 15 minutes

  Postlab data linearization activity and discussion: 15 minutes
- **Part II:** 85 minutes

  Prelab/demonstration/discussion: 10 minutes

  Student-centered investigation: 45 minutes

  Student-led discussion of results: 15 minutes
- **Postlab Discussion:** 15 minutes
- **Total Time:** approximately 3.5 hours

Safety

General lab safety should be observed. Instruct students to exercise special caution as they swing masses in the classroom. Make sure all members of the group are clear of the swinging area and are aware when the pendulum will be released.

In addition to student safety, motion sensors and other equipment should be protected when using springs. If you decide to do the extension to this lab, warn students to keep attached masses small enough and amplitudes small enough that springs are not extended beyond their elastic limits. Also, if motion sensors are used, they should be protected from swinging pendulums or masses falling from springs.

Preparation and Prelab

Part I:
This part of the investigation works well for introducing the skills of taking data and modeling the behavior of an object graphically and mathematically.

If used at the beginning of the course, it will probably be necessary to discuss how to reduce uncertainties in timing measurements. You can start with a short pendulum, around 30 centimeters long, and distribute the stopwatches among students in the class. Then, as a class, time one complete oscillation of the pendulum. Record the values and have students calculate the average and uncertainty in the period. Given that the period is less than 1 second, human reaction time error will be a large percentage of the period. Have the class time ten complete oscillations. If all goes well, the uncertainty in the time measurement will be the same, but the percentage uncertainty of the total time will decrease. For example, 0.25 second reaction time uncertainty is 25 percent of a 1.0 second period measurement, but only 2.5 percent of a 10 second measurement of ten oscillations of the same pendulum.

How much instruction you give students before the lab will depend on when you choose to implement it. If this lab is done as a first lab, it might be useful to have a class discussion brainstorming factors that might affect the period of a pendulum. Since this is an easy system to analyze, students have many ideas about what might affect the period. They will make suggestions like "the force with which you push it to start the motion" and "air resistance." This can lead to a discussion about how to make measurements, what we can actually measure, and what factors we can control. Air resistance may play a role, but at the speeds of these pendula, it is not large, nor can it be controlled or eliminated. A discussion of measuring the initial force should lead students to realize that this is not easy to measure, and it is directly related to the amplitude of the pendulum, which is much more easily measured and thus controlled.

A class discussion such as this can help new physics students narrow the field of possibilities to mass, length, and angle as factors that may affect the period of a pendulum. This narrows the scope for them and makes the task more manageable. If you choose to implement this lab later in the year when students will have more experience designing labs, then you may wish to skip the class discussion and let students decide for themselves how to narrow the scope of the investigation. Depending on how much time you have for this lab, you may also choose to have several groups study the effect of angle on period, several groups study mass, and several groups study length; then have the groups share the data and results with each other.

Part II:
If Part I is done as the first experiment of the year, then this part can be delayed until the oscillations unit, which should come after the kinematics and forces units. Part II works well at the beginning of a unit on simple harmonic motion. At that time, you can revisit the period vs. length experiment and have students do a more mathematical analysis.

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The Investigation

Part I:

Start with a demonstration of a pendulum about 30–50 centimeters long. Pull the pendulum bob back and release, and catch the bob when it returns. Explain to students that the time for the pendulum to complete one complete cycle is referred to as the *period*. Next, pose the question, “What factors affect the period of a pendulum, and what factors do not?” You could also phrase the question as, “What could we change about this system that would change the time it takes the pendulum to swing back and forth once?”

At this point, you could choose to have a class discussion or release them to their groups to discuss and plan their data-taking strategies. If you choose to have a whole-class discussion, help students focus on what can be measured and what tools they will use to measure the quantities they decide to measure. This would be a good time to discuss the benefits of timing multiple periods. Students should also refrain from having multiple students involved in the timing such that one person says “start” and another person releases the pendulum. To reduce uncertainty, the pendulum should be set in motion, and then the student with the stopwatch starts timing at some point in the motion, and stops when it returns to that position after multiple periods. It is up to you how much guidance you want to give before releasing students to their groups to design and execute their plan.

As you circulate, remind students to manipulate one variable at a time and record their data neatly in tables, and encourage students to display the data in the best way to represent the relationship. Students will usually choose to vary the mass of the pendulum, the angle of release from the equilibrium position, and the length of the pendulum. Consideration needs to be taken as to where to measure the length of the pendulum: the top of the bob, the middle, or the bottom. You might want to provide some guidance by reminding them a simple pendulum models the object as a point mass and asking them where the point would be (center of mass).

Most guidance in an inquiry lab should take the form of questions to students as to what they are doing and why they are doing it that way. Make them articulate what they know about best scientific practice, and remind them to engage in that.

At the conclusion of the first part of this investigation, students should observe, from their data, a significant relationship between length and time (period). They may assume that the mass affects the period as well, given that they are not likely to get the exact same value for the period for each different mass. At this point, encourage students to plot the period as a function of mass and period as a function of length and observe the results. As students analyze the relationships, they should ultimately linearize the data in order to determine the relationship between time (period) and length.

Have students present their results to the class (or otherwise share and discuss them) before proceeding to the next part.

Part II:
To start this part of the investigation, ask students to consider, "How could we graphically express periodic motion?"

Challenge students to draw a prediction of the position-vs.-time graph of the motion of the pendulum for one full period. Suggest the equilibrium position to be where $x = 0$. Once students have completed their graph prediction, instruct students to design an experiment that allows them to directly record position and time.

This part may be very challenging to many students. Allow each group enough time to discuss and brainstorm some ideas for collecting this data. Students may choose to use a motion detector to collect this data or, depending on their level of sophistication, video analysis. A lower-budget alternative is a leaking bob, which drops colored liquid onto a moving piece of paper, although this introduces a small error in the length of the pendulum and will concern those students who believe mass is a factor.

Students should design a method of tracing the position of the swinging pendulum bob (the leaking bob) onto a constantly moving scroll/paper. Remind them to use small amplitude, based on their results from Part I. Encourage them to think carefully about how to move the paper at a constant rate.

Extension

Another example of periodic motion is an object oscillating at the end of a spring. Ask students to determine mathematically the relationship between period and mass for an object oscillating at the end of a spring hung vertically from a support rod. Depending on when you choose to implement this lab, you could hand students a spring and a stopwatch and ask them to find the spring constant of the spring. In this case, they should already know the relationship, $T = 2\pi\sqrt{m/k}$ where m is the mass of the object and k is the spring constant. They can compare the spring constant obtained using the slope of a graph of T^2 *vs.* mass to one using the relationship using $F = k\Delta x$, where F is the force applied to the spring and Δx is the spring extension from equilibrium.

Common Student Challenges

Part I:
One common challenge for students is how to measure the length of the pendulum, and how to keep the length of the pendulum constant while varying mass, angle, or other factors they might choose to study. If a set of hooked masses is used, the different masses will have different heights, and thus the length of the pendulum from support point to the bottom of the mass will vary by up to 5 centimeters depending on which masses are used. This will most likely present itself in a slight increase in the period of a pendulum with increasing mass. Address this uncertainty with each group individually as you circulate, or address it in a class discussion when students present their results.

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A robust discussion of measurement uncertainty can now take place. Some guiding questions for this discussion include:

- How much longer was the pendulum with the 500-gram mass compared to the 50-gram mass?
- Was it 10 times longer or only about 5 percent longer?
- Was the length of the pendulum really constant when the mass was varied?
- To which part of the hooked mass should you measure when you measure the length of a pendulum?
- If the length of the pendulum was 30 cm, and the 500-gram mass was 5 cm taller, how much uncertainty does this introduce?

Another common student challenge relates to timing. Students regularly time by having one student watch the motion and say "start" and "stop," while another student operates the stopwatch. Students should learn that the person operating the stopwatch is the one who observes the motion and counts the oscillations. This results in less human reaction-time error. Sometimes timing demonstrations work to show students the added uncertainty. It might suffice to tell them that each time one student says "start" and the other one reacts, they introduce more reaction-time uncertainty. A good point of discussion may be whether to measure from the bottom of the arc or the top of the arc and why.

When graphing, a common mistake students make is to limit the vertical axis range on graphs to the range of data collected. If they do this, in particular for the period-vs.-mass data, they will miss the fact that the period is independent of the mass. During the analysis portion, encourage students to start each axis at zero and continue beyond the greatest value of their data.

Part II:

If students have a difficult time drawing a position vs. time graph for the pendulum, ask them if they have ever seen a polygraph (lie detector) or seismograph record data (see Supplemental Resources for videos of these devices). Both of these have a piece of paper moving under a needle that writes. Have students imagine a pen attached to the pendulum that writes on a moving piece of paper.

Marking the position of the pendulum on the paper becomes the second challenge. Some will want to attach a marker to the pendulum to draw on the paper; however, this causes both friction and incomplete data, because the marker will not keep contact with the paper as it swings. The "leaking bob" will mark the position without adding external forces to the system. If students find it challenging to pull the paper scroll at a constant rate, suggest attaching it to a constant speed buggy (available online for under $10).

Analyzing Results

Part I:
Due to the investigative nature of this experiment, having students report on individual large whiteboards is ideal (or large poster/bulletin board paper). The whiteboards are useful for displaying procedure, data, and graphs in a way that they may be easily shared. Having students sketch a graph of their data in a large format that can be shared during the discussion will give students the opportunity to see how others approached the investigation.

The following are prompts for discussion as students analyze their results. It is up to you which factor to address first. If the students agree that length affects period, then they might see that the objects with more mass are longer/taller and thus add to the length of the pendulum. Thus, small variations in period as the mass is changed could then be attributed to the variations in sizes of the different objects used to vary the mass.

1. *Does mass matter?* Many students will expect that it should. Students will probably obtain slightly different periods for different masses, and some will think these differences are "real." This is an opportunity, not a problem. After the small groups have presented their results, challenge the whole class — including groups that measured other quantities — to take the shared data and make the best argument they can that period depends on mass, or that it doesn't depend on mass. This could lead to an authentic discussion about uncertainty. In particular, as mentioned above, students should consider whether, when changing mass, the length of the pendulum changed significantly.

Sample student data for this part of the experiment is as follows:

Mass (g)	Period (s)
20	1.451
100	1.46
200	1.473
500	1.515
1000	1.542

Table 1

Students frequently claim that the period for the 1000-gram mass is greater than that for the 20 grams, and thus the mass affects the period. This is when to ask the question, "You increased the mass by a factor of 50 (for example), by how much did this increase the period?"

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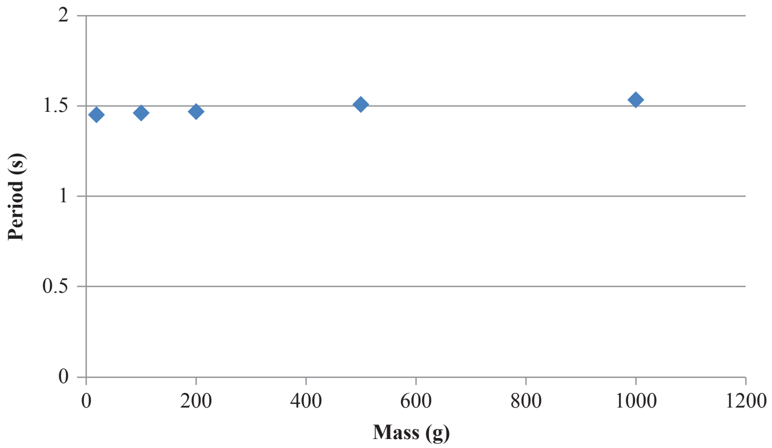
A graph of the data yields the following:



Graph 1

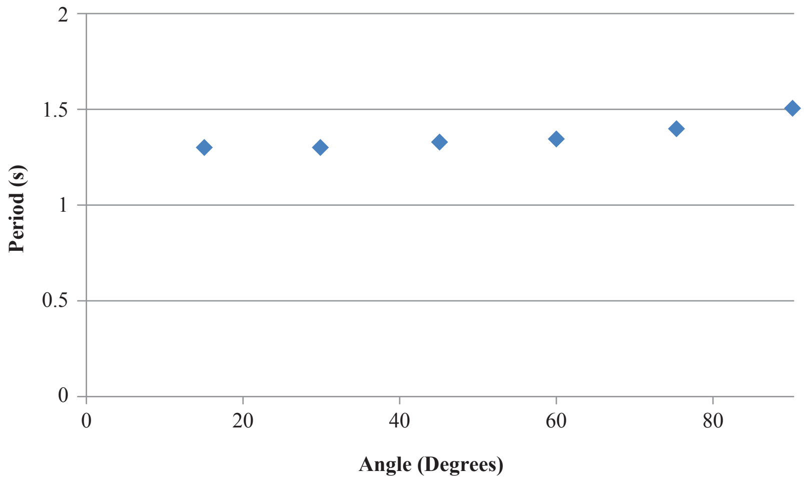
Groups using graphical representations will make the most convincing arguments, and you can turn this into a meta-discussion: What way of showing the data best clarifies whether the differences between the measured periods for different masses are significant vs. the result of measurement uncertainty? One source of measurement uncertainty that comes in to play if students are using hooked masses is that that larger hooked masses are much taller than the smaller hooked masses. Thus, if the students merely keep the string length constant while changing masses, they will be inadvertently changing the length as well as the mass when they change to larger masses. This can be pointed out to explain the slight increase in period for larger masses. Or students could be advised to keep the length the same and always measure to the center of mass of the pendulum bob.

2. *Does angle matter?* This discussion can go the same way as the discussion above, except now ask students from all small groups to represent the data in a way that best helps decide the issue, as decided in question #1 above. This discussion could also take advantage of another teachable moment regarding conceptual learning. Students will likely come to consensus that the angle doesn't matter, or only matters a little for larger angles, but they will likely find this result counterintuitive.

Typical data for this section:

Angle (degrees)	Period (s)
15	1.308
30	1.305
45	1.335
60	1.35
75	1.404
90	1.512

Table 2



Graph 2

Students can see from this graph that for angles less than about 30 degrees, the period is relatively constant; but as the angle increases, the period increases. The mathematics of the dependence of period on angle beyond 30 degrees is too complex for students at this level, so instruct them to make sure that for future pendulum measurements, as long as the angle is less than 20–30 degrees, the period is relatively constant. If you wish for more precision in your students' data, instruct them to do further investigations of the dependence of period on angle. Specifically, they could measure the period for many different angles between 0 and 30 degrees to see the variation in that range. The sine of an angle (in radians) is within 10 percent of the angle for angles less than 30 degrees, and within about 2 percent for angles less than 20 degrees.

It is a useful exercise to have students put their calulator in radian mode and compare the sine of an angle to value of that angle for angles less than one radian. For example, 20 degrees is equal to 0.35 radians. The sine of 0.35 radians is 0.343, which is approximately 2 percent smaller than 0.35.

3. *Does length matter?* Since students will quickly agree that string length *does* matter, the discussion can quickly transition to focus on figuring out what the relation is. Is it linear, square, square root, or something else? At this point a discussion of straightening graphs is imperative, if it has not already been done. Once again, the "Modeling Instruction" website has excellent resources for this discussion.

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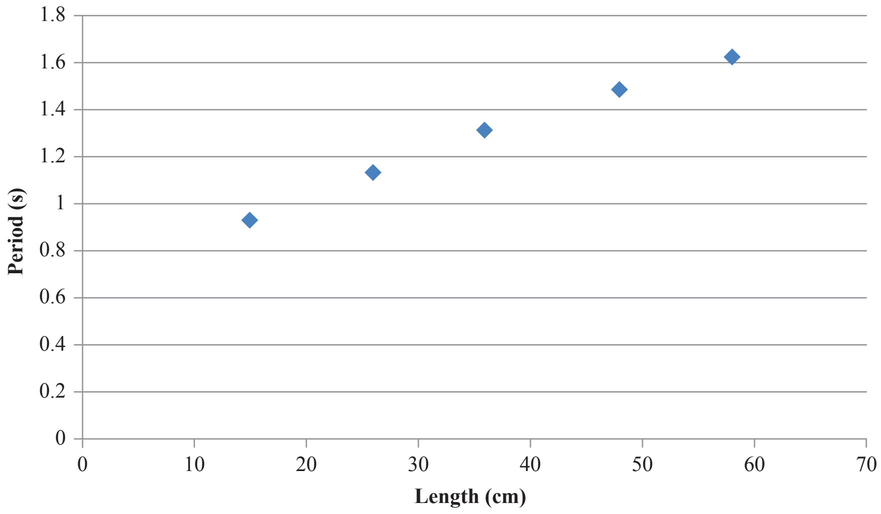
Sample data for this section:

Length (cm)	Period (s)
15	0.93
26	1.14
36	1.32
48	1.49
58	1.63

Table 3



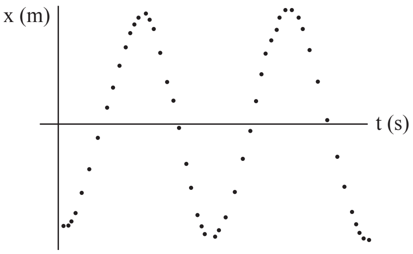
Graph 3

The range of the data shown above is very small, and demonstrates the fact that, for small ranges of data, the difference between a straight line and a square-root curve can be difficult to see. Encourage students to take a much larger range of data if their results look like Graph 3. As an alternative, lead a class discussion and time the period of a pendulum that is much longer (1.5–2 meters or more depending on the height of your classroom). This data can then be added to the data set to more clearly illustrate the nature of the curve. It is difficult to time the period of a pendulum shorter than 10 centimeters, since even the smallest masses are 2–3 centimeters tall. However, it might be worth attempting, in order to extend the data range even more; and the length measurements can be made more accurate by measuring to the center of mass of the pendulum bob in each case.

Students should then linearize the data by plotting one of the following: period vs. square root of length or period squared vs. length. From this graph, the mathematical relationship between period and length can be determined. Depending on when you decide to implement this lab, you can continue with a comparison to the equation for the period of a simple pendulum, $T = 2\pi\sqrt{L/g}$. You might want to wait to do this until the unit on simple harmonic motion. When this is done, students use the slope of their linearized graph to calculate the numerical value of g and compare to the accepted value. Additional discussions about uncertainty and whether they got the "right" answer can occur at this point as well.

As a summative assessment, ask students to use their data to make predictions about the period of a pendulum with a given mass, angle, and length. They can either interpolate/extrapolate from their graphical data, or use the equation they obtained for period vs. length to calculate a value. Make sure the angle you provide for them is less than 30 degrees.

Part II:



Graph 4

How you proceed with the analysis of this section depends on when you implement this lab. If Part I of this lab is implemented at the beginning of the course, it is necessary to wait until the study of oscillatory motion, after the study of forces and kinematics, to complete Part II.

Have students consider the following:

- Does their position vs. time graph match the predictions they made, and if not, why not?
- What is different about the motion, and what is the same as their prediction?
- What role does uncertainty play in their graphs?

Once an acceptable graph of the position vs. time of the pendulum has been established, as shown in Graph 4, students should use this graph to sketch a graph of velocity vs. time. If they struggle with this, remind them of their knowledge of kinematics and the relationship between position vs. time graphs and velocity vs. time graphs.

Some of the tasks you can ask students to do include:

- Explain the relationship between the velocity and the slope of the position vs. time graph.

- Identify when and where the bob reaches the maximum and minimum velocities.
- Explain how to construct the acceleration vs. time graph from the velocity vs. time graph and locate when and where maximum and minimum accelerations occur.

Once they have made these predictions, they check their predictions using a motion detector and computer interface (assuming this equipment is available). Ask them to label all the points of zero speed on both the position graphs and velocity graphs and to comment on similarities between the multiple points. They should comment on the relationship between the acceleration graph and the position graph. Students should notice that the acceleration graph is the negative of the position graph.

Assessing Student Understanding

Part I:
After completing this investigation, students should be able to:

- Design an experiment to determine the effect of mass, angle, and length on the period of a pendulum;
- Measure the period of a pendulum by timing multiple oscillations; and
- Determine the relationship between mass, angle, or length and the period of a pendulum by examining data in the form of tables or graphs.

Part II:
Students should also be able to:

- Graph the position of a pendulum as a function of time;
- Determine an equation relating the period of a pendulum to its length;
- Predict the period of a simple pendulum given the length, mass, and release angle; and
- Draw the graph of velocity vs. time and acceleration vs. time from their graph of position vs. time.

Assessing the Science Practices

Science Practice 2.2 The student can *apply mathematical routines* to quantities that describe natural phenomena.

Proficient	Uses and applies mathematical routines to detect and describe patterns in the data, and compares the period of a pendulum in terms of its length, mass, and/or amplitude (in terms of angle).
Nearly Proficient	Uses and applies mathematical routines that describe the patterns in the period of a pendulum in terms of its length with only occasional or minor errors.
On the Path to Proficiency	Uses and applies mathematical routines to describe the period of a pendulum in terms of its length with some inconsistency and/or errors.
An Attempt	Incorrectly identifies patterns in the mathematical data or incorrectly applies routines to describe them, and description contains major errors.

Science Practice 4.2 The student can *design a plan* for collecting data to answer a particular scientific question.

Proficient	Designs a plan that will allow a determination of the factors that affect the period of a pendulum.
Nearly Proficient	Designs a plan for measuring the period of a pendulum in terms of angle but cannot articulate how that plan will lead to a rule for the period.
On the Path to Proficiency	Designs a plan to measure the period of a pendulum but it's not clearly defined or articulated — it doesn't take into account varying length, mass, or angle.
An Attempt	Presents an incomplete design for a plan that attempts to measure period as a function of other variables; makes errors in identifying variables.

Science Practice 4.3 The student can *collect data* to answer a particular scientific question.

Proficient	Collects appropriate, adequate, and accurate data in a methodical way, and presents the data in an organized fashion.
Nearly Proficient	Collects appropriate and adequate data; some minor errors are present, and/or the presentation is logical but lacking in an organized format.
On the Path to Proficiency	Collects inadequate or irrelevant data with significant gaps or errors, and presents data in a way that is disorganized and lacks logic.
An Attempt	Collects irrelevant, inaccurate, or incomplete data and doesn't provide any organization for this data.

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Science Practice 5.1 The student can *analyze data* to identify patterns or relationships.

Proficient	Constructs a graph to analyze the data and accurately determine the effect of mass, angle, and length on the period of a pendulum. Uses the graph of period squared vs. length to derive a mathematical relationship between period and length.
Nearly Proficient	Constructs a graph to analyze the data and qualitatively determine the effect of mass, angle, and length on the period of a pendulum, but unable to derive a mathematical relationship between period and length.
On the Path to Proficiency	Identifies patterns in the data for the period of a pendulum, but unable to form a complete conclusion from this analysis.
An Attempt	Forms some accurate analysis of the graphs of the period of a pendulum, but unable to come to an accurate conclusion.

Science Practice 6.4 The student can *make claims and predictions about natural phenomena* based on scientific theories and models.

Proficient	Predicts the period of a pendulum accurately from a graph of period squared vs. length or period vs. square root of length.
Nearly Proficient	Uses a graph to make a prediction, but fails to take the square root of the period.
On the Path to Proficiency	Makes estimates of the period of a pendulum based on data, but cannot make accurate calculations using an equation or a graph.
An Attempt	Makes incorrect predictions about the period of a pendulum using the data collected and graphed.

Supplemental Resources

Carvalhaes, Claudio G., and Patrick Suppes. “Approximations for the Period of the Simple Pendulum Based on the Arithmetic-Geometric Mean.” *American Journal of Physics* 76, no. 12 (2008): 1150–1154. [*This article discusses methods for approximating the period for large angles; a resource for the teacher only, and only if the teacher enjoys a good challenge*.]

“Cut the Rope Trailer.” YouTube. Video, 1:22. Accessed September 1, 2014. http://www.youtube.com/watch?v=8xPUdFaraoQ&. [*This trailer for a “Cut the Rope” video has several good instances of pendulum motion.*]

“How a Seismograph Works.” YouTube. Video, 1:04. Accessed September 1, 2014. http://www.youtube.com/watch?v=Gbd1FcuLJLQ. [*Good video of how a seismograph works*.]

"I Didn't Know That - Beating a Lie Detector Test." National Geographic. YouTube. Video, 4:37. Accessed September 1, 2014. http://www.youtube.com/watch?v=JcDr7O-Wmuk. [*A video describing the pendulum motion and marking of a Lie Detector Machine*.]

Kuhn, Jochen, and Patrik Vogt. "Analyzing Spring Pendulum Phenomena with a Smart-Phone Acceleration Sensor." *The Physics Teacher* 50, no. 8 (2012): 504. [*Alternative methods for measuring the properties of a pendulum*.]

"Modeling Instruction." Arizona State University. Accessed September 1, 2014. http://modeling.asu.edu/.

Mires, Raymond W., and Randall D. Peters. "Motion of a Leaky Pendulum." *American Journal of Physics* 62, no. 2 (1997): 137–139.

"Properties of Periodic Motion." The Physics Classroom. Accessed September 1, 2014. http://www.physicsclassroom.com/class/waves/u10l0b.cfm. [*This website is a good source for properties of periodic motion*.]

"Simple Pendulum." Walter Fendt. Accessed September 1, 2014. http://www.walter-fendt.de/ph14e/pendulum.htm.

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AP Physics 1 Investigation 7: Rotational Motion

What physical characteristics of an object affect the translational speed of the object after it has rolled to the bottom of an incline?

Central Challenge

This investigation introduces students to concepts of rotational motion as they analyze how characteristics of objects such as mass, radius, and shape affect the linear speeds of those objects at the bottom of a ramp. This lab provides instructions for both qualitative and quantitative investigations in rotational motion, giving you the option of choosing which type of investigation is best for your students. If time permits, you might choose to have them complete both investigations.

Background

Without friction, an object at the top of an incline would slide down the incline without rolling, resulting in only linear (or translational) motion. A friction force exerts a torque on the object, allowing it to roll down the incline. Basic kinematic equations already familiar to students can describe the linear (or translational) motion of the center of mass of the object as it changes position, but rotational motion equations must be incorporated to describe the rotational motion of each object as it rolls without slipping down the ramp. Additionally, the way in which an object rotates depends upon the rotational inertia of the object. Although students will not calculate rotational inertia in this course, they will use the concept of rotational inertia in calculations of quantities such as torque and rotational kinetic energy. This lab helps to provide a conceptual understanding of the physics properties of an object that define the object's rotational inertia.

Real-World Application

It is not difficult for students to visualize numerous everyday objects that rotate. Understanding how an object's properties impact rotational motion allows students to critically examine designs used for rotating objects. For example, bicycle racers will choose wheel designs that have properties that can enhance their racing performance. Wheels that are fairly uniform from hub to rim with light rims have low rotational inertia, so they start quickly for a short race. However, bicycle wheels with light spokes and heavier rims have higher rotational inertia, which make the bicycle more difficult to start, but once these wheels are turning they are less influenced by other forces and require more torque to stop — better for a long race or for stability on a rough terrain.

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Another common example is a spinning skater. The skater can exert a torque by pushing on the ice with an extended toe. Once the skater starts rotating, bringing legs and arms in close to the spin axis causes a faster spin. Extending the arms or a leg slows the spinner down to a stop. With arms and legs spinning close to the body (and close to the spin axis), the skater has a lower effective radius of spin and lower rotational inertia. Since angular momentum is the product of rotational inertia and angular speed, angular momentum is conserved when that product remains constant. If no external torque is exerted on the skater, reducing the rotational inertia results in a faster angular speed (and faster spin), and extending to increase the rotational inertia results in lower angular speed.

Inquiry Overview

Students are provided with materials to setup a ramp and objects of various shapes, sizes, and masses to design an experiment to test how objects rotate as they roll down a ramp. If students are provided with a large assortment of objects and options to create the inclined plane, they are given more opportunity for guided inquiry that approaches open inquiry, which is recommended. Students should be given latitude to make decisions about which objects to use, how many trials are adequate, how to make measurements to determine the speed of the object at the bottom of the ramp, and how to analyze their results. Students should be provided with the opportunity prior to actual lab time to meet in groups to design their lab procedure (even though some directions are provided). It adds to the inquiry process for students to report out their procedural plans to the other groups in order to gain feedback about oversights or gain suggestions prior to actually conducting the experiment. This can also happen postlab, giving students the opportunity to engage in critical discussions with the other groups.

Initially, student groups will make qualitative predictions about how object shape, size, and mass will affect the speed of the object as it reaches the bottom of the ramp. These predictions will be discussed and compared in small student groups and recorded. Then students will run the trials and make qualitative observations. Finally, students will design methods to make measurements of the speeds of the objects at the bottom of the ramp to compare to their predictions and observations.

Connections to the AP Physics 1 Curriculum Framework

Big Idea 3 The interactions of an object with other objects can be described by forces.

Enduring Understanding	Learning Objectives
3.A All forces share certain common characteristics when considered by observers in inertial reference frames.	**3.A.1.1** The student is able to express the motion of an object using narrative, mathematical, and graphical representations. (Science Practices 1.5, 2.1, and 2.2) **3.A.1.2** The student is able to design an experimental investigation of the motion of an object. (Science Practice 4.2) **3.A.1.3** The student is able to analyze experimental data describing the motion of an object and is able to express the results of the analysis using narrative, mathematical, and graphical representations. (Science Practice 5.1)

Big Idea 4 Interactions between systems can result in changes in those systems.

Enduring Understanding	Learning Objectives
4.C Interactions with other objects or systems can change the total energy of a system.	**4.C.1.1** The student is able to calculate the total energy of a system and justify the mathematical routines used in the calculation of component types of energy within the system whose sum is the total energy. (Science Practices 1.4, 2.1, and 2.2)

Big Idea 5 Changes that occur as a result of interactions are constrained by conservation laws.

Enduring Understanding	Learning Objectives
5.E The angular momentum of a system is conserved.	**5.E.2.1** The student is able to describe or calculate the angular momentum and rotational inertia of a system in terms of the locations and velocities of objects that make up the system. Students are expected to do qualitative reasoning with compound objects. Students are expected to do calculations with a fixed set of extended objects and point masses. (Science Practice 2.2)

[**NOTE:** In addition to those listed in the learning objectives above, Science Practice 4.3 is also addressed in this investigation.]

Skills and Practices Taught/Emphasized in This Investigation

Science Practices	Activities
1.4 The student can *use representations and models* to analyze situations or solve problems qualitatively and quantitatively.	Students include diagrams of objects and experimental setups in order to describe procedures, and they provide qualitative explanations and/or mathematical calculations as part of their analysis.
1.5 The student can *re-express key elements of natural phenomena across multiple representations* in the domain.	Students support work with written observations of the objects' motion as part of the analysis, and they include diagrams as part of background and analysis. If the quantitative method is used, students also express the motion with equations and calculations.
2.1 The student can justify *the selection of a mathematical routine* to solve problems.	If the quantitative method is selected, students use equations and calculations to support predictions about which objects move with greater translational speed at the bottom of the ramp.
2.2 The student can *apply mathematical routines* to quantities that describe natural phenomena.	Students apply selected mathematical routines to the calculations of speed if the qualitative method is selected.
4.2 The student can *design a plan* for collecting data to answer a particular scientific question.	Students make decisions about which objects to test, how to design ramps, how to measure translational speed at the bottom of the ramp, and how to appropriately analyze the data.
4.3 The student can *collect data* to answer a particular scientific question.	Students use observations in the qualitative method or numerical measurements in the quantitative method.
5.1 The student can *analyze data* to identify patterns or relationships.	Students decide what methods will be used to analyze the data, such as graphing speed at the bottom of the ramp as a function of object radius for objects of the same mass and shape.

[**NOTE:** Students should be keeping artifacts (lab notebook, portfolio, etc.) that may be used as evidence when trying to get lab credit at some institutions.]

Equipment and Materials

Per lab group (three to four students):

- Objects of different shapes, masses, and diameters (that can roll down an incline)
- Inclined plane or inclined grooved track (with sufficient coefficient of friction that chosen objects only roll and do not slide)

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- Objects to prop up the inclined plane (books, bricks, pieces of wood, clamps on ring stands, etc.)
- Metersticks
- Rulers
- Stopwatch
- Mass scale
- (Optional) Motion sensor or video analysis tools

The number of different shapes of objects to roll down the incline is up to you, but it is recommended to have at least three. Examples of the most common shapes used include a hoop (a PVC or metal pipe cut into thin pieces); spherical cylinder (small sections of pipe or small metal cans); solid cylinder (samples of different metal cylinders available from lab supply density sets, or 1- and 2-inch wooden dowels cut into short sections); hollow sphere (ping pong ball, tennis ball); and solid sphere (ball bearings, marbles, or wooden balls of various diameters found in craft stores). Other shapes can be included depending on time and teacher preference.

[**NOTE:** On a larger scale, students can take identical small empty food cans and refill them with various types of solid material (wood putty, cement, marshmallow fluff) and then reseal them. To make objects of the same diameter and radius but different mass, pieces of PVC pipe can be cut and materials can be stuffed inside the PVC pieces to create different masses. **NOTE ALSO:** If the distribution of mass inside the PVC is different or if the mass inside the PVC can move, that introduces another factor in rotational inertia. The distribution of material inside the pipe pieces must be uniform.]

For the qualitative investigation you will need the same shapes with varying masses and radii so that students have enough to test their predictions and to come to the appropriate conclusions. Examples include a set of spheres of the same diameter but different masses (such as 1-inch steel or wooden balls) or the small metal cans as described above.

The inclined plane can be any material that allows a smooth path for the objects to roll. However, since this investigation may require timing the objects, it is recommended that the length of the inclined plane be more than 1 meter to minimize timing errors. Examples of materials you can use to construct an inclined plane include boards, ring stands, and wood strips or metersticks (which can be taped to the boards to create channels of different widths for objects to roll through). Aluminum sliding door C-channel track also works (it can be cut with a hacksaw and bent into tracks for balls to roll on).

If the analysis is done correctly, students should realize that the masses and radii of the objects do not affect the final linear speed of the objects, making the mass scale and ruler unnecessary.

Timing and Length of Investigation

- **Total Time:** 3.5–4.5 hours
- **Teacher Preparation/Set-up:** 5–10 minutes

Most of this time will be spent gathering the equipment. If you also setup the equipment for students (not highly recommended) or if you need to saw dowels, etc., into sections, more time will be needed.

Qualitative Investigation

- **Total Student Time:** 135–200 minutes
- **Part I:** 45–65 minutes

Prediction/Setup/Observation Time: 10–15 minutes

Data Collection/Calculations: 15–20 minutes

Discussion: 20–30 minutes

- **Parts II and III:** 30–45 minutes each

Prediction/Setup/Observation Time: 10–15 minutes

Data Collection/Calculations: 15–20 minutes

Discussion: 5–10 minutes

- **Part IV:** 30–45 minutes

Prediction/Setup/Observation Time: 5–10 minutes

Answer questions: 15–20 minutes

Discussion: 10–15 minutes

Quantitative Investigation

- **Total Student Time:** 45–65 minutes

Prediction/Setup/Observation Time: 10–15 minutes

Equation Derivation: 15–20 minutes (or more), depending on students' algebraic abilities; includes equations involving energy analysis and kinematic analysis

Data Collection/Calculations: 15–20 minutes; includes both energy and kinematic calculations

Error Analysis: 5–10 minutes

[**NOTE:** If you need to fit this investigation into a shorter class period (50–55 minutes), have the prediction/observation portion done on one day, assign the equation derivation portion as homework, and complete the rest of the investigation the next day.]

Safety

There are no specific safety concerns for this lab. However, all general lab safety guidelines should always be observed.

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Preparation and Prelab

Students should have previously studied and developed proficiency with applications of kinematics equations to solutions of problems on linear motion. They should also have had previous laboratory experience determining speeds of objects in linear motion. The “Ladybug Revolution” interactive simulation on the PhET web site (see Supplemental Resources) has teacher materials available that provide ideas for student assignments and “clicker” questions to assess students’ understanding of the differences between translational and rotational motion.

Students may have previously done a similar lab for an object sliding down a ramp in which they used a rolling ball. In that situation, they may have already noticed that the ball consistently had less linear speed than predicted by energy calculations. This is an opportunity to build on that lab by giving students the opportunity to rethink the uncertainty in that previous experiment in terms of the rotational kinetic energy of the ball.

If students are still learning experimental protocols, it may be necessary to point out that they need to think about the independent and dependent variables and control other variables. For example, if students are examining the effect of mass on speed of the object, they should keep the object’s radius constant.

The Investigation

You have significant leeway here in how to proceed with this investigation. If time is limited, have students proceed with only the qualitative or the quantitative investigation. To make the lab more inquiry based, simply set out a variety of objects (disks, hoops, spheres, etc.) and have students design their own investigation to determine which factors affect the speed of an object after it has rolled to the bottom of a ramp.

QUALITATIVE INVESTIGATION

The qualitative investigation is divided into three parts as follows. Each part may be conducted by each lab group or different parts may be assigned to different lab groups, with the groups sharing their observations in a larger group discussion for final analysis.

Part I:
Students investigate the question, “How does the mass of a rolling object affect its final speed at the bottom of an incline if radius and shape are held constant?”

Have students make a prediction about whether heavy or light objects will reach the bottom with more speed. After their predictions are made, they design an experiment with the provided equipment that can be used to answer the investigation question.

This part of the investigation should lead to two sets of good discussions, within the groups and also in a whole-class debrief. The first set is about control of variables: When comparing heavier versus lighter, did students hold the shape of the objects constant? The second set is about uncertainty: The heavy and light objects will not reach the bottom with exactly the same speed. How do students decide whether the small difference in speed is a "real" difference? Depending on how deep you want to go, this issue could lead to students taking more data, representing the dispersion in their data points in some way, and using those representations to make arguments about whether the differences are real. Ideally, students would choose which representations to create and the all-class discussion would be about which representations most convincingly supported the related arguments. In any case, the class should arrive at a consensus about the (non) effect of mass on final speed before proceeding to Part II. In both the small group and all-class discussions, students should also explain why the mass didn't matter.

Part II:
Students investigate the question, "How does the radius of a rolling object affect its final speed at the bottom of an incline if mass and shape are held constant?"

Students repeat the same type of procedure as in Part I and come to a conclusion. There will be less discussion about controlled variables and uncertainty, since time may have already been spent on this in Part I. More discussion can be devoted to why the radius doesn't affect the final speed.

Part III:
Students investigate the question, "How does the shape of a rolling object affect its final speed at the bottom of an incline if radius and mass are held constant?"

Students again repeat the same type of procedure as in Part I and come to a conclusion. However, this time there should be an obvious difference in the final speeds of the different shapes. More discussion can be devoted to why the shape does affect the final speed and this discussion can lead into energy concepts and the difference between translational and rotational kinetic energies.

Part IV:
Students investigate the question, "Would a cart that has four solid disks for wheels have a final speed that is *greater than*, *less than*, or *equal to* the final speed of a single disc that has the same mass as the cart and wheels?"

Students then reason through the question, using observations and conclusions from Part I. In their small groups they work through the following questions to help guide their thinking, and then come together as a class to discuss their results and answers:

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1. Suppose a cart with four wheels and a disk whose mass is equal to the total mass of the cart roll down the ramp. Which, if either, has more gravitational potential energy at the top?
2. Which of those objects has more kinetic energy at the bottom? Why?
3. Imagine the disk just spinning in place instead of rolling. Would it have kinetic energy? Why?
4. Why does the cart have more speed at the bottom even though it doesn't have more kinetic energy than the disk? Build upon your answers to questions 1 and 2 to answer.

QUANTITATIVE INVESTIGATION

Give students several objects of different shapes (not necessarily the same mass or radius, but they can be) that are capable of rolling down an incline. Then pose the question: "If each of these objects were rolled down an incline, each starting at the same height, how would their linear speeds compare at the bottom of the incline?" Ask the students to predict the results of the investigation before the investigation is performed.

After the predictions are made, students setup the equipment and allow the different shapes to roll down the incline, finding an appropriate method to measure the speed of each object at the bottom. (For example, students might decide to use a motion sensor to determine speed or might decide to allow each object to roll onto a level section and measure distance and time on that section to calculate linear speed.) After these initial observations are made, students must then take the necessary measurements and complete the required calculations to support their observations.

Students must use the law of conservation of energy to derive equations for the linear speed of each object at the bottom of the incline. To do this, rotational inertia equations (see Equations 1–4 below) for each object and the relationship between linear speed and angular speed will be needed; you can choose to provide these or have students research and find the equations themselves.

Next, ask the students to calculate the linear speed of each shape using kinematics. (For example, students may allow each object to roll off a table onto the floor and measure the range to determine initial speed. Or they may leave a long level section at the bottom of the ramp and measure distance and time for the object after it leaves the slope and rolls along the level section to calculate a velocity.) Necessary measurements need to be made and calculations shown for the kinematic analysis. Students should then do an error analysis since the linear speed should be the same whether it was determined experimentally using kinematics or calculated using measurements and energy conservation. After the investigation is complete, ask students if their predictions were correct. If not, have them explain why the prediction did not match the observations (i.e., resolve inconsistencies).

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The following equations should be provided to students for this investigation:

$$I_{cylinder} = \frac{1}{2}mr^2$$

[Equation 1]

$$I_{hoop} = mr^2$$

[Equation 2]

$$I_{hollow\ sphere} = \frac{2}{3}mr^2$$

[Equation 3]

$$I_{solid\ sphere} = \frac{2}{5}mr^2$$

[Equation 4]

$$v_{cm} = r\omega$$

[Equation 5]

$$E_{initial} = E_{final}$$

[Equation 6]

Students should be able to derive these equations for their use in the experiment:

$$\Delta U_{gravitational} = \Delta K_{translational} + \Delta K_{rotational}$$

[Equation 7]

$$mg\Delta h = \frac{1}{2}m(\Delta v)^2 + \frac{1}{2}I(\Delta\omega)^2$$

[Equation 8]

The following equations are the results students should get for the final speed of each shape at the bottom of the incline using conservation of energy. (You might prefer students to derive an equation for an object of $I = kmr^2$ and then substitute k for each of the shapes.)

$$v_{cylinder} = \sqrt{\frac{4}{3}gh}$$

[Equation 9]

$$v_{hoop} = \sqrt{gh}$$

[Equation 10]

$$v_{hollow\ sphere} = \sqrt{\frac{6}{5}gh}$$

[Equation 11]

$$v_{solid\ sphere} = \sqrt{\frac{10}{7}gh}$$

[Equation 12]

Extension

Small student groups select an activity or sport that operates on wheels, such as bicycle racing or skateboarding. Each group researches wheel design and how it relates to the performance in that activity. They should incorporate conclusions from the lab in their final reporting to the class. Each group should also include a diagram of the wheels and how the structure relates to the activity, estimating rotational inertia, if possible.

A more challenging experiment might be for students to repeat part or all of their investigative procedures with objects and ramp heights where the object slides and rolls (rolls with slipping) down the ramp. This would be a qualitative investigation, as the quantitative measurements and calculations are beyond the scope of the course.

Common Student Challenges

First, students may have an incomplete understanding of the role of friction. If each object rolls without sliding, the friction force is necessary to provide the torque to roll the object. Without friction, the object would not roll but would slide down the ramp. Since both investigations presented in this lab introduce a new topic, they are not designed to deal with specific misconceptions or conceptual challenges. However, they will demonstrate that different shapes roll/rotate differently, and that the final speed of the object at the bottom of the incline does not depend on the mass or radius of the object, which may be surprising to many students.

Since they have been taught and have learned that objects of different mass fall at the same rate with no air resistance and that those same objects will slide down an incline at the same rate if there is no friction, students may predict that all the objects will roll down the incline and reach the bottom at the same time and with the same speed. The observations made in either investigation may surprise some students. It is important to bring the discussion around to what causes objects to roll and help students justify that what they have learned in the case of no friction is still valid.

In the quantitative investigation, the biggest challenge students face is the derivation of the equations for the final speeds of the objects using the conservation of energy, because they are usually not yet familiar with the rotational inertia equations and the equation that relates linear speed to angular speed. A possible way to help students is to show them an example of a derivation in class, using the equations from the AP Physics 1 Equations Sheet using a shape not involved in the investigation. Then students may derive the necessary equations for those shapes that will be used. You might want to limit the number of shapes so that students can spend more time on developing an understanding of the underlying concepts rather than getting bogged down in the algebra. It depends on how much class time you have to devote to the investigation and how comfortable your students are with algebraic manipulation.

Analyzing Results

Qualitative Investigation:

One method of having students analyze their results is to compare their observations to the predictions made at the beginning of the investigation. If the prediction does not match the observations, then ask students to explain/resolve the inconsistencies. This means that students need to provide an explanation of WHY they obtained the observed results. This can be done in small groups and then reported to the larger group for discussion and refinement prior to conducting the quantitative investigation.

Quantitative Investigation:

Students should calculate the percent difference between the theoretical energy analysis and the experimental finding using the kinematic method.

$$\%\ \textit{difference} = \frac{\left|\textit{result from energy analysis} - \textit{result from kinematic method}\right|}{\textit{average of the two results}} \times 100\%$$

You could also ask them to identify sources of uncertainty in measurement, identify the source of the largest uncertainty, and explain what can be done to minimize the uncertainty if the experiment were performed again.

Possible questions you could ask during postlab discussions, or provide for students to consider during their laboratory analysis, include:

1. How well do the final linear speeds, calculated using the theoretical energy analysis and determined by experiment, compare to your predictions? Does one method consistently produce a larger or smaller value? Why?
2. What specific evidence from this investigation supports your answer?
3. How does the rotational inertia of a rolling object affect its final speed at the bottom of an incline?
4. What specific evidence from this investigation supports your answer?
5. Suppose you repeated the experiment with objects of the same radius but larger masses. Would the results of this investigation change? If so, how? If not, why not?
6. Suppose you repeated the experiment with objects of the same mass but larger radii. Would the results of this investigation change? If so, how? If not, why not?
7. If the objects in this investigation were not rolling down an incline, but were each just rotating on their own stationary, fixed axis located through the center of the object, would the mass of the object have an effect on the rotational inertia of the object? Why?
8. If the objects in this investigation were not rolling down an incline, but were each just rotating on their own stationary, fixed axis located through the center of the object, would the *radius* of the object have an effect on the rotational inertia of the object? Why?

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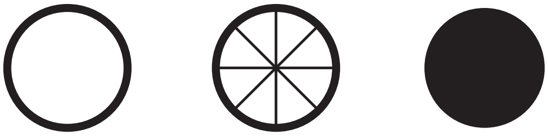
9. Based on your observations in this investigation, rank the following objects, which all have the same mass, in terms of rotational inertia, largest to smallest. Explain the reasoning for your ranking.



10. If you were to allow these three objects to roll from rest down an incline simultaneously, in what order would they reach the bottom? Why?

You can either create or find other qualitative and quantitative questions and problems, such as TIPERS (ranking tasks) that would be an effective measure of students' understanding (see Supplemental Resources).

Assessing Student Understanding

Qualitative Investigation:

After completing this investigation, students should be able to:

- Demonstrate an understanding of what rotational inertia means;
- Explain how and why different shapes roll/rotate differently using evidence from the investigation;
- Develop ideas and questions about how and why the location of the mass of a rotating object affects the ease or difficulty of rotating that object and experimental means of verifying this; and
- Design and analyze an experiment to test the rotational properties of objects of various shapes, masses, and radii.

Quantitative Investigation:

Students will also be able to:

- Use measurements to calculate the speed of an object after it rolls to the bottom of a ramp based on conservation of energy principles;
- Use an experimental method to determine the speed of an object after it rolls to the bottom of a ramp based on kinematic principles; and
- Relate calculations of speed of a rolling object at the bottom of a ramp to a specific aspect of the physical properties of the object, when other factors are held constant.

The quantitative investigation is not intended to get students to derive rotational inertia equations. The equations are given to students or students acquire them for the purpose of using them in the investigation.

Assessing the Science Practices

Science Practice 1.4 The student can *use representations and models* to analyze situations or solve problems qualitatively and quantitatively.

Proficient	Uses diagrams of objects of various shapes to describe the rotational motions of those objects, both verbally and mathematically.
Nearly Proficient	Uses diagrams of objects to determine rotational motions of those objects in most cases, and/or uses equations to describe rotational motions for most shapes.
On the Path to Proficiency	Partially applies diagrams to the analysis of the rotational motion of several shapes or applies equations to the analysis of the motions of several shapes.
An Attempt	Uses only one type of model — either a diagram or kinematic equation — to analyze the motion of a shape.

Science Practice 1.5 The student can *re-express key elements of natural phenomena across multiple representations* in the domain.

Proficient	Applies qualitative observations accurately for all the shapes used, and/or correctly makes mathematical derivations for all the shapes provided.
Nearly Proficient	Applies qualitative observations to correct conclusions for most of the shapes provided or to derive most mathematical relationships correctly for the quantitative method.
On the Path to Proficiency	Applies qualitative observations to correct conclusions for several shapes or to derive more than one (but not most) mathematical relationship(s) correctly for the quantitative method.
An Attempt	Applies qualitative observations to correct conclusions for only one shape or to derive only one mathematical relationship correctly for the quantitative method.

Science Practice 2.1 The student can *justify the selection of a mathematical routine* to solve problems.

Proficient	Selects all the appropriate equations applying to various shapes and correctly relates variable for linear and rotational motion in a conservation of energy statement.
Nearly Proficient	Selects all the appropriate equations but unable to connect them to all the correct shapes. Possibly addresses conservation of energy with some errors in the derivation.
On the Path to Proficiency	Selects some appropriate equations but unable to connect them to all the correct shapes. Possibly addresses conservation of energy with equations, but not correctly.
An Attempt	Selects some appropriate equations but unable to connect them to the correct shapes. [Applies to quantitative method only.]

Science Practice 2.2 The student can *apply mathematical routines* to quantities that describe natural phenomena.

Proficient	Correctly calculates rotational inertia for all shapes, and correctly applies these calculations to determination of velocity using conservation of energy.
Nearly Proficient	Calculates rotational inertia for most shapes, and calculates velocity from energy conservation with minor errors.
On the Path to Proficiency	Makes some calculations, but they are incomplete; for example, missing some shapes or with a consistent error throughout.
An Attempt	Unable to make complete or correct calculations for any of the shapes, though an attempt is made for each shape. Conservation of energy calculations are missing or incomplete. [Applies to quantitative method only.]

Science Practice 4.2 The student can *design a plan* for collecting data to answer a particular scientific question.

Proficient	Designs a plan that is appropriate, clearly thought out, and clearly described. Presents an oral or written laboratory report that has all of the following elements: labeled diagram of the setup, succinctly outlined procedure, multiple trials, clearly shown derivation of mathematical model used (if qualitative treatment is pursed).
Nearly Proficient	Designs a plan that is well thought out with good experimental controls but with a weakness that may affect one set of conclusions or is not clearly described. Offers an oral or written laboratory report that is missing one of the following elements: labeled diagram of the setup, succinctly outlined procedure, multiple trials, clearly shown derivation of mathematical model used (if qualitative treatment is pursed).
On the Path to Proficiency	Designs a plan for the assignment given (quantitative or qualitative or a particular shape) that is generally well thought out but has a flaw (e.g., trying to compare 1″ steel balls to 2″ wooden balls) that will affect results. Offers an oral or written laboratory report that is missing a significant number of essential elements and contains many errors in labeling, identification, mathematical calculations, and derivations.
An Attempt	Fails to think out the design plan for the assignment given (quantitative or qualitative or a particular shape) well enough to get relevant results from the experiment.

Science Practice 4.3 The student can *collect data* to answer a particular scientific question.

Proficient	Collects all relevant data on all rotational shapes, organized in a data table with appropriate units.
Nearly Proficient	Collects all relevant data with some important element missing (e.g., units).
On the Path to Proficiency	Collects data but some relevant data is missing or there are not an appropriate number of trials.
An Attempt	Collects minimal data and presentation is not coherent.

Science Practice 5.1 The student can *analyze data* to identify patterns or relationships.

Proficient	Presents a complete analysis, addressing all aspects of the data, includes analysis of sources of uncertainty, and compares results using the energy method to the kinematic method.
Nearly Proficient	Presents a mostly complete analysis with only some flawed conclusions or final calculations, or doesn't make an attempt at error analysis.
On the Path to Proficiency	Clearly states data and/or observations but analysis methods are somewhat incomplete or contain some flawed conclusions.
An Attempt	Attempts an analysis but the approach is flawed.

Supplemental Resources

Hieggelke, Curtis, J., David P. Maloney, Steve Kanim, and Thomas L. O'Kuma. *TIPERs: Sensemaking Tasks for Introductory Physics.* Boston: Addison-Wesley, 2013.

"Ladybug Revolution." PhET. University of Colorado Boulder. Accessed September 1, 2014. http://phet.colorado.edu/en/simulation/rotation. [*In this simulation students can move a ladybug to different locations on a rotating disk and observe the rotational speed and rotational inertia of the system, as well as several other variables.*]

Rolling Ranking Tasks Solutions. College Board. Accessed September 1, 2014. http://apcentral.collegeboard.com/apc/public/repository/ap07_Rolling_Ranking_Tasks_Solutions.pdf.

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AP Physics 2 Investigation 1: Boyle's Law

How can the relationship between pressure and volume for a confined gas be determined experimentally, and how do changes in pressure and volume relate to the work done on this gas?

Central Challenge

The purpose of this investigation is to reinforce the concept of pressure and introduce the idea that the area under a pressure-versus-volume graph of a gas is the work done on or by the gas. The intent is to have students perform this investigation after the topics of pressure and atmospheric pressure have been covered, but before the introduction of pressure-versus-volume (PV) diagrams.

Background

Pressure is a scalar quantity defined as force exerted on a defined area ($P = F/A$). In physics, pressure is usually measured in pascals (Pa, or N/m^2) when force is measured in newtons and area is measured in square meters. In a gas with temperature (T), the molecules are in constant motion. Collisions of the molecules with a container wall result in forces between the wall and the molecules, resulting in changes in the motion of the molecules. The force exerted by the wall on a molecule is equal to the change of the momentum of the molecule with respect to time as defined by Newton's second law ($F = \Delta p/\Delta t$). By Newton's third law, we know that the force on the wall due to the molecule must be equal in magnitude.

The average pressure on a wall of the container is then equal to the average force on that wall due to all of the molecules striking it divided by area of the wall. A change in the temperature of the gas also changes average kinetic energy of the gas ($K = 3/2\ nRT$). Thus, because of the relation between kinetic energy and momentum, gas pressure is related to temperature. The ideal gas law ($PV = nRT$) relates gas pressure for a confined gas to the number of moles of gas (n), volume (V), and absolute temperature (T). Gas pressure is exerted equally in all directions and exists within the gas system itself as well as on the walls of the container.

Work done on or by a gas changes the total energy of the gas. In a situation where no energy is allowed to enter or escape a confined gas by thermal processes ($Q = 0$), the work done on or by a gas will result in change in internal energy of the gas, according to the conservation of energy/first law of thermodynamics ($\Delta U = Q + W$). Work done on a gas is defined as a positive quantity in AP Physics, resulting in a compression of the gas ($W = -P\Delta V$). Thus, work done on a confined gas with no thermal energy allowed to enter or escape would result in increased energy of the molecules of the gas. After positive work is done on the gas, its molecules have higher average velocity and momentum and exert larger forces on the walls of the container.

Atmospheric pressure at Earth's surface is due to the combined effect of the gravitational force on the molecules of the atmosphere and is determined to be 1.01×10^5 Pa.

Real-World Application

The concepts of how pressure and volume of a gas relate to each other and to the work done on or by that gas can be used to connect to isobaric, isothermal, and adiabatic processes. A common illustration of thermodynamic processes is the gasoline engine. In each cylinder, a complete cycle consists of four steps or strokes: intake, compression, power, and exhaust. During the intake stroke, the vaporized gasoline–air mixture is drawn into the engine as the piston moves down to increase the space in the cylinder, with the intake valve open and exhaust valve closed. During the compression stroke, both valves are closed, so as the piston moves upward, the gas increases in temperature with the decrease in volume and the spark plug then is timed to ignite the gasoline–air mixture. During the power stroke, the "explosion" of the gasoline vapor from the previous stroke causes rapid expansion of the gas, doing work on the piston to move it downward. In the last stroke, the piston is moved back up with only the exhaust valve open, so the gasoline combustion products are expelled.

Another example that can be demonstrated to students uses a bicycle tire pump. As the handle of the pump is pushed downward to push air slowly into a tire, the handle does work on the gas in the cylinder of the pump, causing the temperature of the air in the pump to increase as the air is compressed. Students can feel the change in temperature of the pump.

A simple illustration of pressure related to volume during an isothermal process occurs when a sealed bag or container becomes visibly greater in volume when the bag is brought from a lower elevation where atmospheric pressure is higher to a higher elevation where atmospheric pressure is lower.

In the study of weather, adiabatic cooling explains cloud formation. When warm, moist air over a body of water on a sunny day rises due to convection, a quick rise in elevation causes rapid expansion of the air due to lower pressure — with no addition or removal of energy. As the air temperature decreases due to rapid expansion, the moisture condenses to form a cloud.

Inquiry Overview

In this investigation, students use a Boyle's law apparatus (see Figure 1) to investigate the relationships between pressure, volume, force, and work done on a gas. Students use different known forces to compress the gas and measure the change in volume as more force is used to compress the gas. The data will be used to produce and analyze a graph of gas pressure as a function of volume, using the assumption that the air in this experiment approximates an ideal gas.

Equipment is provided, along with fairly specific directions needed for data gathering, so this lab is on the "directed" side of guided inquiry. Students should still be given as much latitude as possible in determining how to make measurements and design their analysis methods. However, without some clear directions, they can easily make major errors that lead to confusion. Open inquiry is preferable, but this guided-inquiry lab can reinforce important concepts on a challenging topic. Allow latitude while maintaining vigilance, watching and making suggestions to steer students in the right direction as they do their prelab conferencing and as they work.

Connections to the AP Physics 2 Curriculum Framework

Big Idea 5 Changes that occur as a result of interactions are constrained by conservation laws.

Enduring Understanding	Learning Objectives
5.B The energy of a system is conserved.	**5.B.5.4** The student is able to make claims about the interaction between a system and its environment in which the environment exerts a force on the system, thus doing work on the system and changing the energy of the system (kinetic energy plus potential energy). (Science Practices 6.4 and 7.2)
	5.B.5.5 The student is able to predict and calculate the energy transfer to (i.e., the work done on) an object or system from information about a force exerted on the object or system through a distance. (Science Practices 2.2 and 6.4)
	5.B.5.6 The student is able to design an experiment and analyze graphical data in which interpretations of the area under a pressure-volume curve are needed to determine the work done on or by the object or system. (Science Practices 4.2 and 5.1)
	5.B.7.1 The student is able to predict qualitative changes in the internal energy of a thermodynamic system involving transfer of energy due to heat or work done and justify those predictions in terms of conservation of energy principles. (Science Practices 6.4 and 7.2)

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Enduring Understanding	Learning Objectives
	5.B.7.2 The student is able to create a plot of pressure versus volume for a thermodynamic process from given data. (Science Practice 1.1) **5.B.7.3** The student is able to use a plot of pressure versus volume for a thermodynamic process to make calculations of internal energy changes, heat, or work, based upon conservation of energy principles (i.e., the first law of thermodynamics). (Science Practices 1.1, 1.4, and 2.2)

[**NOTE:** In addition to those listed in the learning objectives above, Science Practice 4.3 is also addressed in this investigation.]

Skills and Practices Taught/Emphasized in This Investigation

Science Practice	Activities
1.1 The student can *create representations and models* of natural or man-made phenomena and systems in the domain.	Students create diagrams of their experimental piston setup as part of the lab report. They create a free-body diagram of the piston in order to examine the forces exerted by the atmosphere by the added weights and by the gas on the piston They also create at least one pressure–volume graph to represent the change in volume with increased pressure.
1.4 The student can *use representations and models* to analyze situations or solve problems qualitatively and quantitatively.	Students analyze their pressure–volume graph to determine the work done on the system. An extension might include analyzing the meanings of the slope and intercept of the PV graph as well. They use the free-body diagram representation to examine and calculate force.
2.2 The student can *apply mathematical routines* to quantities that describe natural phenomena.	Students use mathematical equations to calculate the area of the piston and the volume of gas for each change in height. Students use area under the PV graph to calculate work done on the system by the external force.
4.2 The student can *design a plan* for collecting data to answer a particular scientific question.	Though clear directions need to be provided, students make decisions about measurement and graphing techniques. They determine in what increments measurements will be made in order to produce a meaningful graph.
4.3 The student can *collect data* to answer a particular scientific question.	Students make multiple measurements of force to determine pressure and measurements of change in column height to determine change in volume.
5.1 The student can *analyze data* to identify patterns or relationships.	Students analyze the data graphically, determining the meaning of graph area to find work done on the system.

Science Practice	Activities
6.4 The student can *make claims and predictions about natural phenomena* based on scientific theories and models.	As part of the background and preparation of the lab, students make predictions about how gas volume changes with increasing pressure on the piston. Then, as part of the analysis, students discuss at least one everyday situation in which gas pressure and volume variations are important (such as in the piston-cylinder system of an automobile).

[**NOTE:** Students should be keeping artifacts (lab notebook, portfolio, etc.) that may be used as evidence when trying to get lab credit at some institutions.]

Equipment and Materials

Per lab group (two to four students):

- Boyle's law apparatus (see Figure 1 below)
- Mass scales
- Rulers
- Graph paper or graphing calculator
- Objects with mass large enough to compress the air in the syringe (such as books, small bricks, or any other objects that are heavy enough and can be stacked on top of the Boyle's Law apparatus)
- (Optional) Pressure sensor that can be inserted directly into the lower chamber of the syringe, allowing students to measure pressure changes directly to plot against volume

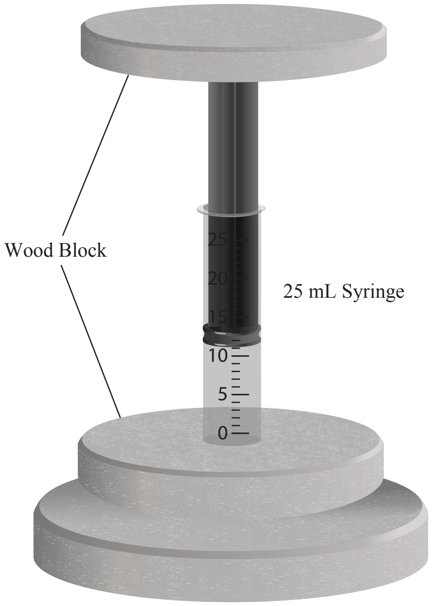

Figure 1: Boyle's Law Apparatus

[**NOTE:** The Boyle's law apparatus can be purchased from most science supply companies for less than $15 depending on the supplier and quality of the apparatus. It is also possible to build your own by looking at Figure 1 and other designs shown online.]

There is an alternative setup that involves the masses hanging from the syringe instead of being stacked on top. This apparatus is available from several science supply companies for less than $15.00 and is illustrated in Figure 2.

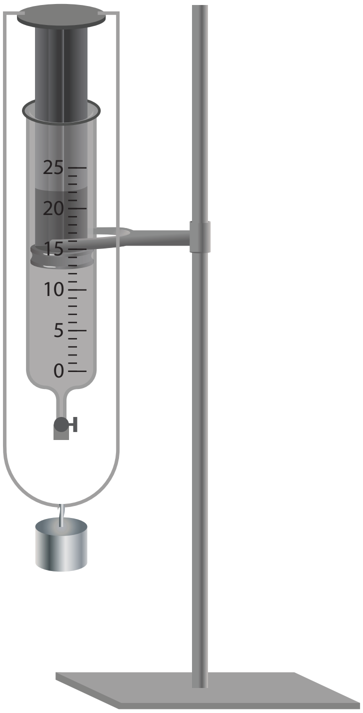

Figure 2: Boyle's Law Apparatus (Alternate)

[**NOTE:** This investigation works best when the piston of the Boyle's law apparatus has been well lubricated. If not, there will be too much friction between the piston and the walls of the syringe, which will greatly affect the results.]

Timing and Length of Investigation

- **Teacher Preparation/Set-up:** 5–10 minutes

  This time is for gathering the equipment. More time will be needed if you also setup the equipment for students.

- **Student Investigation:** 35–50 minutes

  Prediction/Setup/Observation Time: 10–15 minutes

  Data Collection/Calculations: 15–20 minutes. This time includes weight, piston area, and pressure calculations.

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Graphing/Calculations: 10–15 minutes, depending on whether the graph is done by hand or on a graphing calculator (includes calculations of area under the graph).

- **Postlab Discussion:** 10–15 minutes

  If fitting this investigation into a shorter class period (50–55 minutes) is an issue, have the Prediction/Setup/Observation portion done on one day (or assign the writing of the prediction and observations as homework), and complete the rest of the investigation the next day (or assign graphing and analysis of the graph as homework).

- **Total Time:** approximately 1 hour

Safety

The largest safety issue is the danger of the objects that are stacked on top of the Boyle's law apparatus falling over. The more objects stacked on top, the less stable the apparatus becomes. Students should take care in preventing this from happening as they add objects.

Preparation and Prelab

Prior to the lab, students should understand the concept of gas pressure and how it relates to the kinetic theory of gases, as well as how to calculate pressure on a surface in terms of force and area using the equation $P = F/A$. They should know how atmospheric pressure is determined and be ready to apply the concept of equilibrium to the initial condition where atmospheric pressure and gas pressure are equal. The basic relationships among pressure, volume, and temperature as defined by the ideal gas law should be familiar enough that students can use $PV = nRT$ to make descriptions and calculations. You might need to prompt students with the reminder that the pressure of the gas in the cylinder is the sum of the pressure produced by the weight added to the cylinder and atmospheric pressure. (In other words, when no additional weight has been added to the top of the piston, the gas pressure is equal to the atmospheric pressure. Students will need to decide whether it is reasonable to assume that the piston itself has a small enough mass to neglect.)

You could assign practice problems from the textbook that include calculations with the ideal gas equation and with pressure and area concepts and calculations. Additionally, "Gas Properties" is an interactive simulation that would help students develop an understanding of the relationship between pressure and volume at constant temperature (see reference to the PhET web site in Supplemental Resources). The PhET site also has teacher-prepared student activities that can be used as homework or used in class for formative assessment in preparation for the lab.

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The Investigation

Select from the options below the method that might best fit the equipment available in your school or the experience level of students (or yourself). It is up to you to determine whether students should choose a method themselves or should be directed to a specific method.

For a student-designed, inquiry-based lab:

Each lab group should decide how they will setup the equipment to gather the necessary data, how many data sets they need, and the increments in which they will measure the data to create the best graphical representation of that data. Students should think about experimental control (i.e., how they will prevent energy transfer into or out of the apparatus during data gathering). If students decide to use pressure probes, they need to consider how the probes can be used without allowing gas molecules to enter or leave the gas during the experiment.

Encourage students to examine the situation and then draw a free-body diagram of the forces on the piston in the Boyle's law apparatus in order to realize that if the piston reaches equilibrium after each additional mass is added to the top of the piston, the total force on the top (Mg + atmospheric pressure) must be equalized with the gas pressure from the inside of the container.

Students also need to consider how they will add weight to the top of the piston so that stability is maintained. Most will decide to carefully select and add calibrated masses to the top of the piston, recording the value of the added mass (m) for each trial as they carefully measure the change in height (Δh) of the piston. Later, they will use the change in piston height, along with radius of the piston, to calculate change in volume of the gas ($\Delta V = \pi R^2 \Delta h$).

Consider giving students the following guiding questions: "What is the relationship between pressure and change in volume for a gas?" "What does the area under a pressure vs. volume (PV) graph of a gas represent?" Whatever experimental design is chosen, students must collect data and evidence to answer the questions and come up with their own procedure and draw their own conclusions based on the results. Students must then construct a graph to analyze data and answer the experimental question regarding work. They will need to do research to determine what equations are necessary for analysis of data.

[**NOTE:** Many students have had experience with the Boyle's law apparatus in their chemistry classes, so they may decide to use that apparatus and come up with their own design, with little prompting. Others that are aware that the school has pressure probes may decide to measure gas pressure directly.]

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For a more directed student inquiry:

Depending on the experience and skill level of the group, a more directed student inquiry may be more appropriate. You might want to do a quick demonstration of how the apparatus is setup. Then give students directions, as needed, providing further prompts as necessary for each group to progress. Show students the equipment to use, and have them do the following:

1. Measure the radius of the plunger in the syringe.
2. Measure the mass of each object as the objects are stacked on top of the Boyle's law apparatus.
3. Measure the volume of the air in the syringe as the objects are added.
4. Record all data.

Extension

Once students have discovered the inverse relationship between pressure exerted on the gas and volume of the gas, ask them how they might graph the data to create a linear graph. Once they decide that a plot of pressure versus the reciprocal of volume would create a linear graph, ask about the meaning of the slope of the graph and the intercept. Since $PV = nRT$, the slope would be nRT, assuming temperature stays constant, and the intercept (where volume is zero) is also the point where the pressure is zero. (Students should not extend their experiment line to the origin, since that would indicate a zero volume and pressure.)

Students may investigate how temperature varies with the changes in pressure and volume during this experiment by installing a thermometer or temperature probe directly into the chamber of the piston. For this extension, they pull the apparatus apart and use a drill or hot soldering iron to create a hole just barely larger than the thermometer or probe. After the probe is inserted into the chamber, they carefully seal the opening with wax so that air molecules cannot enter or escape during the compression process. Then they reassemble the apparatus and conduct the experiment as done previously, with temperatures recorded during each step along with pressure and volume.

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Common Student Challenges

The largest challenge to students (or cause of the most common error made by students) is not including the atmospheric pressure in their calculations for the pressure on the plunger in the syringe. If this is not included there is a large error in their results. Ask students, "What is the pressure of the gas in the cylinder before any objects are added to the top of the piston?" If it is pointed out that the gas and piston are, indeed, in equilibrium, students are more likely at each step to realize that the gas pressure is equal to the total pressure exerted on it, which is the atmospheric pressure plus the additional pressure exerted by the mass added to the top of the piston. This is a very common source of error for students, and something you might need to bring out both in prelab preparation and in the postlab discussion.

When students get ready to graph their results, they might need to be reminded that the pressure is not zero when the piston is in its starting position. The pressure is equal to the atmospheric pressure.

This challenge could also be addressed by having students draw a free-body diagram of the apparatus in equilibrium with air in the syringe before any masses are added along with a discussion about what forces are acting on the gas without any masses stacked on top.

Students who have large margins of error in their measurements will have difficulty in connecting the area under the PV graph to the work done by gravity pulling on the object sitting on the piston, which is equivalent to the work done on the gas in the cylinder. Encourage students and/or remind them to take measurements as accurately and precisely as possible — and in small increments — to ensure greater success in their results.

If students are advised that the apparatus without weights added to the top is a system in equilibrium, then it may be easier for them to understand that they are taking data for *changes* in the system (i.e., as they add more weight to the top of the piston, the volume changes until a new equilibrium is established).

Students may have some difficulty with the requirement to graph pressure versus volume, since they are actually using change in applied pressure as the independent variable. However, they will graph gas pressure as a function of volume, which is the conventional way of analyzing these changes.

Students may have some difficulty recognizing that the graph of pressure versus volume is a hyperbola. Depending on student level, you might need to include, at the least, a reminder of what such a curve means in terms of the variables ($P \propto 1/V$), and, if needed, a brief lesson on how to determine that a curve is a hyperbola (i.e., that the product of coordinates at each point is constant). It may be easier for students to understand if they rearrange the equation $PV = nRT$ into the linear $y = kx + b$ form. By putting P on the y-axis, the equation takes the form $P = (nRT)/V$. It is assumed that the changes in pressure and volume take place slowly enough that the temperature of the gas remains constant — an isothermal process. Then it's easier for students to see that by graphing P versus $1/V$, the resulting graph is linear, with the values of n, R, and T assumed to be constant. [**NOTE:** You can decide how much of this prompting is necessary prior to the lab and how much can occur as students process the data.]

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Analyzing Results

After the lab, students meet in small groups to develop strategies to answer the questions posed by the experiment. After calculating the new volume of the gas and the new pressure on the gas each time mass is added to the top of the piston, they decide how to graph the pressure–volume data to make conclusions about the relationship between pressure and volume. This should be left to students, though they may need prompting if they get stuck on the hyperbolic curve shape and how to interpret it (as mentioned earlier).

Once they find the curve, they design a plan to find the area under the curve. Solutions may range from "counting squares" to making a best fit line that includes and excludes approximately equal areas to using a graphing program like Excel to find and integrate the equation. Again, you can decide how much prompting is necessary for students to succeed in this.

Students should include the following steps in their calculations:

1. Calculate the pressure exerted on the plunger by the objects as each object is added, using gravitational force on the objects and the area of the plunger. [**NOTE:** Students will likely assume the mass of the piston is negligible.]
2. Plot the data of pressure and volume on a PV graph with the pressure on the *y*-axis and volume on the *x*-axis and draw a best-fit curve through the data.
3. Calculate/estimate the area under the graph. This could be done by hand on a sheet of graph paper and counting squares to obtain the area or with a graphing calculator and using the calculator to find the area. Calculate the total change in gravitational potential energy of the added objects during the experiment and the total change in volume of gas to calculate a comparison of work done on the gas (force times distance).
4. Then compare the area under the graph to the change in gravitational potential energy (work done by the mass) and notice that they are approximately the same.

The following equations should be provided for this investigation:

$$P = \frac{F}{A}$$

[Equation 1]

$$A = \pi r^2$$

[Equation 2]

$$F_g = Mg$$

[Equation 3]

$$\Delta U_g = Mg\Delta y$$

[Equation 4]

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The following equation is the result students should derive for calculating the pressure on the syringe plunger due to the added objects:

$$P = \frac{Mg}{\pi r^2}$$

[Equation 5]

Ask students to predict the shape of the resulting graph by sketching a graph. You can also ask them to make a prediction about what quantity they think would be represented by the area under the PV graph. Another discussion that should be part of the lab analysis is how students accounted for atmospheric pressure on the apparatus and the effect of the weight of the plunger.

Students should compare their observations to the predictions made at the beginning of the investigation. If their prediction does not match their observations, ask them to explain/resolve the inconsistencies. You can also ask the students to identify errors, identify the largest source of error, and explain what can be done to minimize the identified error(s) if they were to perform the investigation again. This is the time to ask students how they took into account the mass of the plunger in their experimental design.

As time permits after the lab experiment is performed, small student working groups can report their results to the group at large, obtain feedback from the other groups, and close the "learning loop." Whether done in this group format or in individual lab reporting, students should be required on every lab to summarize sources of uncertainty (not necessarily calculations) and describe how those uncertainties might have affected final calculations or analyses. For example, in this lab, the syringe may leak air during the compression, reducing the number of moles of gas in the syringe during the experiment. This would cause a larger reduction of volume with each compression as the experiment progresses.

Assessing Student Understanding

After completing this investigation, students should be able to:

- Calculate pressure in terms of force and area;
- Calculate change in volume in terms of change in height of piston and area;
- Describe the equilibrium condition of the piston using a free-body diagram;
- Describe the equilibrium condition of the piston without added objects as a balance of gas pressure with atmospheric pressure;
- Recognize the hyperbolic shape of the PV curve as an inverse relationship;
- Determine the area under a PV graph;
- Relate the area under the PV graph to the work done on or by the gas;
- Explain how they got values for the pressure of the gas (unless it was measured directly);

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- Recognize why they must take into account atmospheric pressure; and
- Recognize the role of friction between the piston and the cylinder wall as a source of uncertainty.

[**NOTE:** Some assessment questions could be based on the "Gas Properties" simulation (see Supplemental Resources). Also on the PhET website are additional activities as well as clicker questions that relate to this investigation.]

Assessing the Science Practices

Science Practice 1.1 The student can *create representations and models* of natural or man–made phenomena and systems in the domain.

Proficient	Creates an accurate and appropriate graph of the relationship between pressure exerted on the gas and volume of the gas, and creates an accurate free-body diagram of the forces exerted on the piston.
Nearly Proficient	Creates a free-body diagram that contains one or more erroneous forces, or creates a graph of pressure and volume that has a flaw in the representation of the relationship between pressure exerted on the top of the piston and volume change in the piston; one of the representations is correct.
On the Path to Proficiency	Creates a free-body diagram, but it has a conceptual error. Creates a graph of pressure and volume that is a flawed or an incomplete representation of the relationship between pressure exerted on the top of the piston and volume change in the piston.
An Attempt	Unsuccessfully attempts either the free-body diagram or the graph.

Science Practice 1.4 The student can *use representations and models* to analyze situations or solve problems qualitatively and quantitatively.

Proficient	Creates and uses both the free-body diagram and the pressure–volume graph to correctly analyze forces on the piston and correctly determine work done on the gas.
Nearly Proficient	Creates a correct free-body diagram but doesn't use the diagram to make correct inferences about forces on the piston, or makes the pressure–volume diagram but makes errors in using the area to determine work done on the gas.
On the Path to Proficiency	Creates a correct free-body diagram but doesn't use the diagram to make correct inferences about forces on the piston. Creates a pressure–volume diagram but makes errors in using the area to determine work done on the gas.
An Attempt	Makes an attempt but doesn't use the free-body diagram to make correct inferences about forces on the piston. Unable to use a pressure–volume diagram to correctly determine area and the meaning of area.

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Science Practice 2.2 The student can *apply mathematical routines* to quantities that describe natural phenomena.

Proficient	Correctly calculates all pressures for data gathered and correctly plots the pressure–volume graph and the area under the graph.
Nearly Proficient	Calculates pressure on the piston in terms of area and calculates area under the pressure–volume graph, but there are mathematical errors in one of the calculations.
On the Path to Proficiency	Makes an attempt to calculate pressure on the piston in terms of area and calculate area under the pressure–volume graph, but there are some mathematical errors in both calculations.
An Attempt	Makes an attempt to calculate pressure but there are significant errors.

Science Practice 4.2 The student can *design a plan* for collecting data to answer a particular scientific question.

Proficient	Creates a design that is thorough, producing meaningful data and a plan for analysis that will answer the questions posed by the experiment; for example, explains how values for the pressure of the gas (which may not be measured directly) were obtained, recognizes why atmospheric pressure must be taken into account in the design, and recognizes the role of friction of the piston as a source of error.
Nearly Proficient	Creates an experimental design that is generally good but with an omission that makes graphing or calculations difficult, such as omitting units on measurements. Partially addresses the experimental factors important to the results (e.g., atmospheric pressure, determination of pressure from other measurements, and the uncertainty produced by friction).
On the Path to Proficiency	Creates an experimental design that is generally useful but has numerous flaws; for example, forgets to record units on measurements and neglects to record initial volume of the gas, making calculations from those measurements impossible.
An Attempt	Attempts a setup for this experiment but the experimental design will not produce useful data; for example, the design may include gathering data for mass and height with no plan to calculate pressure and volume.

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Science Practice 4.3 The student can *collect data* to answer a particular scientific question.

Proficient	Collects adequate data for added weight and volume change, and analysis includes relevant, accurate discussion of the precision and measurement uncertainty.
Nearly Proficient	Collects adequate data for added weight and volume change and there's some discussion of uncertainty, but the discussion is irrelevant or inaccurate.
On the Path to Proficiency	Collects adequate data for added weight and volume change, but data lacks any discussion of precision or uncertainty in measurements.
An Attempt	Collects some data for added weight and volume change, but data is minimal and not organized well enough to create a meaningful graph.

Science Practice 5.1 The student can *analyze data* to identify patterns or relationships.

Proficient	Creates the pressure–volume plot, correctly determines the inverse relationship between pressure and volume, and correctly determines the area under the plot, relating it to work done.
Nearly Proficient	Creates the pressure–volume plot and determines the inverse (hyperbolic) relationship but is unable to correctly relate this to the mathematical trend; or, is unable to correctly determine the area under the graph.
On the Path to Proficiency	Creates a pressure–volume plot but has not found the hyperbolic relationship, or has the hyperbola but has made no attempt to explain what the shape means in terms of pressure and volume changes.
An Attempt	Attempts a graph, but the data is too scattered or not enough data sets are gathered to produce a meaningful graph.

Science Practice 6.4 The student can *make claims and predictions about natural phenomena* based on scientific theories and models.

Proficient	Makes a prediction based on scientific models, ties it to a real-world example, uses his or her prediction to compare to experiment to identify inconsistencies, and makes a physically significant connection to explain them.
Nearly Proficient	Makes a prediction based on scientific models and may tie it to a real-world example but the connection is not strong, or uses his or her prediction to compare to experiment to identify inconsistencies, but fails to make a physically significant connection to explain them.
On the Path to Proficiency	Makes a prediction based on scientific models but does not tie it to any real-world example or use it to consider inconsistencies to be resolved in the investigation.
An Attempt	Makes a prediction but it seems to be more of a guess, not based on scientific models.

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Supplemental Resources

De Berg, Kevin Charles. “Student Understanding of the Volume, Mass, and Pressure of Air Within a Sealed Syringe in Different States of Compression.” *Journal of Research in Science Teaching* 32, no. 8 (1995): 871–884. [*These papers give some research background into pertinent student ideas*.]

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McDermott, Lillian C., Peter S. Shaffer, and the Physics Education Group at the University of Washington. “Ideal Gas Law. First Law of Thermodynamics.” In *Tutorials in Introductory Physics*. Upper Saddle River, NJ: Pearson, 2012. [*Contains two guided inquiry tutorials based on Michael Loverude’s and Chris Kautz’ Ph.D. dissertations that are intended to address student difficulties with topics of this lab.*]

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AP Physics 2 Investigation 2:
Fluid Dynamics

What is the mathematical relationship between the depth of a fluid in a container and the rate at which the fluid will move through an opening near the bottom?

Central Challenge

In this lab, students establish and investigate the relationship between water level in a container and the rate at which it exits through a hole near the base. The investigation provides practice with experimental design, data collection, and graphical analysis. All of these practices can be accomplished with varying degrees of sophistication depending on the equipment available.

Background

Pressure due to a static fluid is ρgh, depending only on the density and depth of the fluid but not on the total volume of fluid. The total force can be calculated by multiplying pressure by area ($F = PA$). Pascal's principle states that at any depth in a fluid, the pressure is exerted by the fluid equally in all directions. As a consequence, an object fully or partially submerged in a fluid has an upward, buoyant force exerted on it that is equal to the weight of fluid displaced by the object — a statement of Archimedes' principle.

Bernoulli's equation describes conservation of energy as applied to fluids in motion. The sum of the terms $(P + \rho gh + \frac{1}{2} \rho v^2)$ remains constant at every point throughout the fluid. In the equation, P is pressure, ρ is the density of the fluid, g is the constant gravitational acceleration near Earth's surface, h is the depth of the fluid, and v is the speed of the fluid. The conservation of energy statement by this equation becomes more apparent to students if each term is multiplied by volume, so that the first term becomes PV (or work), the second term becomes mgy (potential energy) and the third term becomes $\frac{1}{2} mv^2$ (kinetic energy).

The continuity equation, $\rho_1 A_1 v_1 = \rho_2 A_2 v_{2,}$ describes how the speed of fluid flow changes with cross sectional area for any two points in level fluid flow and is a statement of conservation of mass in fluid flow. If the fluid is incompressible so that density can be assumed to remain constant, the density cancels on both sides, reducing the equation to its simpler form: $A_1 v_1 = A_2 v_2$.

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Real-World Application

Bernoulli's principle (conservation of energy) can be used in medicine to understand blood pressure, helping students to understand that when blood vessels change in diameter the speed of blood flow and pressure change.

Bernoulli's principle applies in engineering to help explain, for example, the lift provided by the wings of aircraft and the driving force on the sails of boats. In both cases, air moving along the more curved surface (such as the upper surface of an airfoil) has a higher speed than the air moving on the opposite surface. Since the fluid in both cases is air, the difference in pressure due to the depth of fluid relative to Earth between the two sides of a wing or sail is negligible. Thus, the only two terms to consider in the equation are the pressure term and the term containing speed. By conservation of energy, then, as the speed increases at one point in the fluid, the pressure decreases. That difference in pressure between two points — the upper and lower surfaces of a wing or the windward and leeward sides of a sail — creates a net force, which is $F = \Delta PA$.

As a fundamental description of fluids in motion, there is almost unlimited application. This law also explains the fact that advanced gasoline engines use pumps rather than "gravity feed," as it would be undesirable to have the rate of delivery of fuel depend on the depth of the fluid in the tank (as exemplified by this experiment). Using gravity feed, the rate of fuel delivery would depend on the depth of fuel in the tank. In newer engines, an electric pump delivers fuel at a constant pressure (regardless of depth of fuel in the tank), increasing engine efficiency.

Inquiry Overview

This investigation is designed to be guided inquiry, with the teacher providing materials and instructions and students taking the responsibility for laboratory setup and the design for data gathering and processing. Using any one (or more) of the three methods described here, depending on equipment available, students design an experiment to measure the rate of flow of water from the container either by (a) measuring the depth of water as a function of time, (b) measuring the speed of the water as it exits the hole as a function of time (using the distance the water lands from the container and doing kinematics calculations), or (c) using video analysis.

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Connections to the AP Physics 2 Curriculum Framework

Big Idea 5 Changes that occur as a result of interactions are constrained by conservation laws.

Enduring Understanding	Learning Objectives
5B The energy of a system is conserved.	**5.B.10.1** The student is able to use Bernoulli's equation to make calculations related to a moving fluid. (Science Practice 2.2) **5.B.10.2** The student is able to use Bernoulli's equation and/or the relationship between force and pressure to make calculations related to a moving fluid. (Science Practice 2.2) **5.B.10.3** The student is able to use Bernoulli's equation and the continuity equation to make calculations related to a moving fluid. (Science Practice 2.2) **5.B.10.4** The student is able to construct an explanation of Bernoulli's equation in terms of the conservation of energy. (Science Practice 6.2)
5F Classically, the mass of a system is conserved.	**5.F.1.1** The student is able to make calculations of quantities related to flow of a fluid, using mass conservation principles (the continuity equation). (Science Practice 2.1, 2.2, and 7.1)

[**NOTE:** In addition to those listed in the learning objectives above, Science Practice 4.3 is also addressed in this investigation.]

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Skills and Practices Taught/Emphasized in This Investigation

Science Practices	Activities
2.1 The student can *justify the selection of a mathematical routine* to solve problems.	Students decide which equations to use to make calculations from their data. For example, if they choose to measure the horizontal distance water travels as it exits a container and also measure the depth of water above the exit, they use a modified version of Bernoulli's equation, along with kinematics equations related to projectile motion.
2.2 The student can *apply mathematical routines* to quantities that describe natural phenomena.	Students apply mathematical routines to make the calculations as described above, and then in the experimental analysis relate results to a natural phenomenon, such as using the distance water lands from the base of a waterfall to the speed of the water at the top.
4.3 The student can *collect data* to answer a particular scientific question.	Students decide what measurements to take, depending on their experimental design, to answer the question of how the speed of the water at the exit hole compares to depth of the water above it and how far horizontally the water travels.
6.2 The student can *construct explanations of phenomena based on evidence* produced through scientific practices.	In the analysis of this experiment, students explain their measurement and conclusions in terms of conservation of energy and conservation of mass.
7.1 The student can *connect phenomena and models* across spatial and temporal scales.	In the analysis, students connect their experimental results on the laboratory scale to at least one common example — either on a smaller scale (such as application to the venturi valve in a carburetor) or on a larger scale (such as using photos of a waterfall to relate height and trajectory of the water to the speed of the water).

[**NOTE:** Students should be keeping artifacts (lab notebook, portfolio, etc.) that may be used as evidence when trying to get lab credit at some institutions.]

Equipment and Materials

Per lab group (three to four students):

- Plastic transparent bottle or discharge container with largely vertical walls (such as soda or juice bottles of at least 1-liter volume; smaller containers might produce issues of precision).
- Masking tape (one roll)
- Metric rulers/metersticks
- Stopwatch or cell phone/computer with a stopwatch program (Timers available on many cell phones allow continuous recording of the times for each measurement of height as the water level falls.)

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- Container to catch the water (large enough so that the stream of water will strike it from the extreme of the trajectory to just below the hole itself; sinks large enough to contain the trajectory are ideal, but large buckets or plastic bins will work)
- Graphical analysis program on a computer or calculator
- (Optional) Video cameras (or smartphones, etc.) and video analysis software

Making the discharge containers

In order to reduce the effects of turbulent flow, the hole should be circular with smooth edges. A round hole with smooth edges can be most easily produced in the lower side of the bottle by using the round tip of a hot soldering iron. You might want to prepare the bottles in this way ahead of time. Care should be taken that the hole is made perpendicular to the bottle so the stream emerges in a horizontal direction (see Figure 1 below). If bottles need to be transported once they are filled with water, you may want to use a stronger tape (e.g., duct tape) to cover the holes, or simply keep the caps on the bottles. (It is left to students to discover that the water won't leave the holes if the cap is on the bottle.)

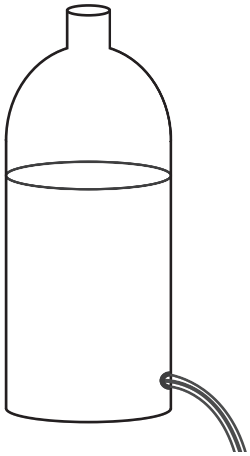

Figure 1

Timing and Length of Investigation

- **Teacher Preparation/Set-up:** 20–30 minutes

  The prep time should be minimal for gathering equipment and materials needed. The holes can be put in 15 bottles with a soldering iron in well under 30 minutes.

- **Prelab:** 10–20 minutes

The prelab should also be minimal because it is assumed that students know in advance how to make and analyze graphs. You can engage students in a discussion of their intuition about the rate of drainage, techniques for timing, etc. With a little more investment in time and money, you can demonstrate the concept qualitatively using 4-inch diameter plastic landscaping drain tile, which comes with predrilled holes and can be set vertically on a tight-fitting cap. With minimal discussion following the demonstration, students may be better prepared to design their own investigation on a smaller scale with the "bottle" method described here.

- **Student Investigation:** 30–45 minutes

  The experiment itself is likely to take less than 45 minutes allowing for practice runs and more than one attempt. Depending on the technology used (video analysis, calculators/computers, or pencil and paper), students' analysis of results could take as little as 15 minutes or as much as 45 minutes. In any event, it is suggested this analysis work be completed outside of class.

- **Postlab Discussion:** 40–50 minutes

  Postlab time should be provided to discuss techniques, results, and conclusions. For a class of 30 students, this could well extend for a full class period.

- **Total Time:** 1.5–2.5 hours

Safety

To avoid slippery floors or damage to equipment from water spills, it is a good idea to have a sponge and a mop available. (If access is available, the measurement phase of this lab could be moved outdoors.)

If it is decided that students are to use the soldering irons to make their own holes in the bottles, they need to be cautioned that the irons remain hot for some time after being turned off or unplugged. The hot irons should be removed to a safe area of the lab — perhaps creating "soldering stations" that have soldering iron holders for the hot irons as they cool.

Preparation and Prelab

Prior to this lab, students should have a basic understanding of fluids and fluid pressure, along with some introduction to fluid flow. Since the purpose of the experiment is for students to derive the relationship between fluid height in a container and the rate at which the fluid flows from the container, students need a conceptual understanding of conservation of energy and conservation of mass principles prior to the experiment, in order to make good decisions about experiment design.

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However, students may not have actually done calculations with Bernoulli's equation. You might want to use demonstrations and assigned student reading to familiarize students with concepts. "Under Pressure" and "Fluid Pressure and Flow" are interactive simulations that help students develop an understanding of fluid pressure (see Supplemental Resources). The PhET website also has teacher-prepared student activities that can be used as homework or in class for formative assessment in preparation for the lab. Actual problem solving with the continuity equation and Bernoulli's equation, for example, would be appropriate follow-up assignments, once students discover the relationship in the lab.

The Investigation

The simple specific understanding for this experiment is that the rate at which liquids leave a container is a function of the square root of the distance of the surface above the hole. However, students should be told in advance only that they are to devise a procedure for gathering data that can be used to show the relationship between the two quantities — the rate at which fluid leaves the container and the height of fluid above the exit hole.

You can have materials ready, but allow students to discuss with each other in small groups how they will use those materials to setup the experiment. Encourage students — within your own comfort level — to devise their own ways to use the provided materials, or add materials if available.

The methods described below are for your benefit. Given a little time, students groups will come up with their own designs and will likely need little prodding to come up with similar methods.

Method I: Measuring depth vs. time

The most direct method students might choose to find the relationship between water depth above the exit hole and time would be to use a vertical piece of masking tape to make a marked scale on the outside of the discharge container. Then they could use a stopwatch to note regular time intervals and read the height at those even intervals in order to produce data regarding the height of the water as a function of time. If students choose this method, you might want to suggest that they mark the tape off in small increments to gather enough data points to make more accurate graphs. A watch or tablet app with a split time mode (or recording with a cell phone) makes this a much easier task. Graphs of this data could then be used to find the rate of change of depth over time.

Method II: Measuring depth vs. velocity

A second method (and perhaps an easier method for students to analyze) that students might choose is the "projectile method." Students would again make a scale for the depth of the water, but this time they would allow the stream to fall a given height into a sink or large pan and record the horizontal distance traveled by the stream from where it leaves the hole to where it hits the pan for the different depths in the container.

Method III: Video analysis

If video equipment is available, (e.g., smartphone, tablet, or computer with a camera) a third data-gathering method that is a hybrid of video analysis with one of the previous approaches can be used effectively. This can be done in two distinctly different ways:

1. Students could simply make a video recording of the experiment described in either of the two methods above, so the data can be gathered from the recording; or
2. While being careful to use a correct perspective for the recording, students could use a video analysis program to directly do a graphical analysis. Using a video of the bottle as the water level falls and as the water exits the hole, students "track" the water as it exits and use distance and time to determine the speed of the water and also track the water level to determine the distance the water level falls in given time intervals. [**NOTE:** Some experience with video analysis is a good prerequisite for use of this method.]

Extension

It would be interesting for students to use the same apparatus and experimental methods to investigate how changing the density of the fluid changes the results. The easiest to use would be salt water, which has a larger density (that can be determined by students by measuring the mass of a small volume of the salt water and using the formula $\rho = m/V$). Other liquids, such as an alcohol/water mixture, which has a lower density, would also be fairly easy to use.

Assign students to groups to design plans that might be used to address questions related to water conservation. For example, one group could be assigned to design a plan to determine the amount of water that leaks from a dripping faucet in one day. Another group might be assigned to design a plan to find the volume of water used to irrigate a specific lawn, where students would have to designate lawn size and frequency of irrigation before making collection measurements and subsequent calculations.

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Common Student Challenges

If students opt to study the height of the liquid in the container as a function of time, they might confuse the velocity of the surface (as it drops in height) with the velocity of the water as it exits the hole. The procedure of using the projectile motion method to obtain the exit speed avoids this confusion.

One form of reasoning that is ubiquitous in science, but that students have difficulty mastering, is to reason by using proportions without directly applying equations. That is, scientists often ask themselves, "If I double *this,* what happens to *that?*" If doubling *this* quadruples *that,* we can guess the form of the equation relating these quantities, and if doubling *this* increases *that* by 1.4, we can guess that equation as well. Moreover, we have some experience relating these relationships to what we might expect a graph to look like. This lab could be an ideal vehicle for encouraging this kind of reasoning.

"Identifying and Addressing Student Difficulties with Hydrostatic Pressure" describes student difficulties with pressure in a fluid and also has research questions that could be used for assessment (see Supplemental Resources).

If students are having difficulty deciding what to graph or how to setup graphs, provide a review. See "Special Focus: Graphical Analysis" in Supplemental Resources for worksheets.

Analyzing Results

Method I: Measuring height vs. time

There are several ways students could approach this analysis. Among these possibilities are:

1. Students could simply graph the data (height of water above the hole on y-axis and time on x-axis) and note that it looks like a section of a parabola. This indicates that a graph of height versus t^2 would yield a straight line. Calculating the slope of the best-fit line of this second graph will give them a mathematical relationship between h and t.

As a second step in the analysis, students should approximate the speed at which the liquid surface drops by calculating the change in height divided by time for small intervals of time, say 0.5 s, at several places over the range of data. They will find that the $\Delta h/\Delta t$, using short time intervals at different times after the experiment has started, will produce water speeds exiting the hole that decrease. This would then be plotted as v vs. t and should yield a linear graph and its corresponding equation. By eliminating t between the two equations, students can get an equation for the velocity of the surface as a function of time. As a challenge, students can use the speed at which the liquid surface drops to calculate the rate at which the volume in the container changes. They can then compare this to the volume of water exiting the hole, which can be calculated from the area of the hole and the speed at which the water is leaving the hole ($V/t = Av$). It will be clear that the exit velocity is proportional to the velocity at which the water surface drops or that the volume per second leaving the hole is equal to the volume per second change within the container.

2. Students use a graphical analysis program on a computer or calculator to go from an h vs. t graph to a v vs. t graph. Of course such a technique is quicker and more elegant but might not provide the same degree of analytical understanding. In any case it should be noted that an attempt to use the continuity of flow equation (see Equation 3) to produce a match with Bernoulli's equation (see Equation 1) will fail because of losses in energy at the hole. Figure 2 provides a sample of the data and resulting graph made with a spreadsheet program:

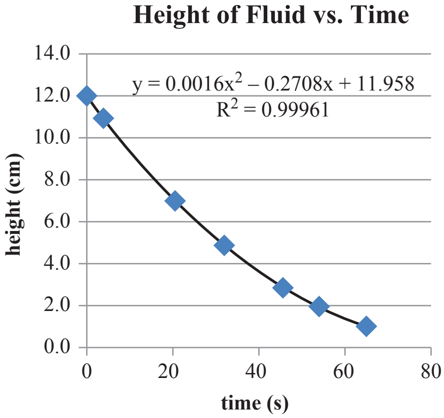

Figure 2

Method II: Measuring depth vs. velocity

Since the distance that the water falls is constant, the time of fall can be calculated and the horizontal velocity determined by the point of impact in the sink/pan. For this analysis, students should graph water depth and velocity. While this method gives a direct correlation of water depth and the square of the velocity, be aware that energy losses at the hole and air resistance will result in numbers for the speed that are smaller than those predicted by the ideal relationship of Bernoulli's equation.

Method III: Video analysis

Students who are familiar with video analysis techniques can use this method to gather the data necessary to either create their own graphs or, more likely, obtain graphs directly from the program used. According to which factors they chose to use in their analysis, they perform one of the analyses above.

Postlab

The results of this investigation should produce a mathematical model that indicates that the water flows from the container at a rate that is proportional to the square root of the distance of the water surface above the hole. The investigation provides an introduction to the relationship established by Bernoulli's law (see Equation 1).

Have students make predictions from their data by asking, "Based on factors you can describe but not control, do you expect your experimental calculations to be higher than or lower than the actual value?" You might have students use the continuity equation (see Equation 2) to compare the experimental results and those given by the theory. While the relation of v and y is confirmed by the experiment, the results for v will be found to be consistently smaller than the predicted values. Students should then be asked to suggest reasons for the systematic discrepancies.

Since this lab is a "discovery" of important mathematical relationships for fluids — the continuity equation and Bernoulli's equation — it is important that you allow students to share their results with other groups. Teacher guidance may be necessary to bring out exactly how data show these relationships if students have difficulty.

Bernoulli's law:

$$P + \rho g y + \frac{1}{2}\rho v^2 = constant$$

Equation 1

Continuity equation:

$$A_1 v_1 = A_2 v_2$$

Equation 2

During the wrap up you might ask the students how they think the size of the hole might relate to the actual speed of the water at the outlet. After they speculate on this, you could do a demonstration with a single bottle with two different size holes. After Bernoulli's law has been treated in class you might want to return to the observation of this discrepancy and ask the student to speculate on the reasons the energy of the water exiting the bottle is not as great as would be predicted from Bernoulli's law and the conservation of energy.

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Another demonstration you might choose would be to attach a tube to the opening of the container and show that water projected vertically does not reach the height of the surface of the fluid. The losses due to friction and turbid flow are then evident.

Assessing Student Understanding

After completing this investigation, students should be able to:

- Design an experiment to collect data related to fluid flow rate;
- State that the speed at which liquids leave a container is a function of the square root of the distance of the surface above the hole;
- Relate experimental observations to continuity of fluid flow and to conservation of energy in fluid flow;
- Discuss and account for the fact that the mechanical energy of the fluid is not completely conserved;
- Compare his or her experimental results to the theoretical predictions and discuss why possible mechanical energy losses are incurred; and
- Discuss why the atmospheric pressure on the surface of the liquid in this experiment can be ignored.

Assessing the Science Practices

Science Practice 2.1 The student can *justify the selection of a mathematical routine* to solve problems.

Proficient	Selects a correct graphical method or correct set of mathematical routines that will correlate change in fluid height to rate of flow.
Nearly Proficient	Graphs data correctly or uses a mathematical routine that will correctly relate water level change to fluid flow, but does not find a way to correctly correlate them.
On the Path to Proficiency	Graphs the data for the experiment, but does not make correct correlations to water flow.
An Attempt	Makes an attempt to calculate rate of fluid flow, but does not select an appropriate graphical or mathematical method to process data.

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Science Practice 2.2 The student can *apply mathematical routines* to quantities that describe natural phenomena.

Proficient	Uses either a graph or set of mathematical routines to correctly determine the square root relationship between rate of flow and change in water level height.
Nearly Proficient	Uses graphical methods correctly or uses the correct mathematical routines to calculate fluid flow and water level height changes, but makes an error in calculation.
On the Path to Proficiency	Correctly calculates rate of flow from the hole and change in water height but does not connect them; or, graphs the data correctly but does not use the graph to make necessary correlations.
An Attempt	Makes an attempt to calculate water velocity, but does not reach conclusions as required by the experiment.

Science Practice 4.3 The student can *collect data* to answer a particular scientific question.

Proficient	Records all relevant data, with units, in a complete and well-organized way.
Nearly Proficient	Records all data in a complete and well-organized way, but misses a key element that can be corrected, such as failing to put units on some measurement.
On the Path to Proficiency	Gathers data in an organized way, but misses some crucial information that will be needed for calculations.
An Attempt	Gathers data but does not record it in a well-organized manner.

Science Practice 6.2 The student can *construct explanations of phenomena based on evidence* produced through scientific practices.

Proficient	Presents a thorough and insightful written analysis of the experiment, with experimental observations clearly connected to the concepts of fluid mass and energy conservation concepts.
Nearly Proficient	Presents a well-prepared written analysis of experimental results but with a key element missing, such as relating the final calculations to conservation of energy for fluids.
On the Path to Proficiency	Explains observations and experimental results in an analysis, but the analysis is incomplete.
An Attempt	Makes an attempt at a verbal explanation in the analysis of the experiment, but the analysis may be disconnected or incomplete.

Science Practice 7.1 The student can *connect phenomena and models* across spatial and temporal scales.

Proficient	Describes a clear, complete connection between how water droplets behave like small projectiles on a micro scale to how a stream of water behaves on a macro scale.
Nearly Proficient	Makes a strong start making connections on a micro and macro scale for fluid flow, but the explanation has missing elements.
On the Path to Proficiency	Makes some connections but does not clearly explain; for example, points out that water flow can be compared to what was learned about projectile motion, but does not explain how single drops are like projectiles.
An Attempt	Makes an attempt to find connections between the experiments on fluid flow, but the connections are not explained.

Supplemental Resources

"Coefficient of Discharge." Denver University. Accessed September 1, 2014. http://mysite.du.edu/~jcalvert/tech/fluids/orifice.htm. [*This website distills the basic treatment of fluid flow through small orifices.*]

"Draining Tank Example." eFunda. Accessed September 1, 2014. http://www.efunda.com/formulae/fluids/draining_tank.cfm#calc. [*This website provides an interactive calculator for the flow from a container. It might be of interest to students who want to compare their results with result of the mathematical model based on Bernoulli's Law, but it should be noted this calculator treats the ideal case and assumes no energy losses.*]

"Fluid Pressure and Flow." PhET. University of Colorado Boulder. Accessed September 1, 2014. https://phet.colorado.edu/en/simulation/fluid-pressure-and-flow.

Loverude, Michael E., Paula R. L. Heron, and Christian H. Kautz. "Identifying and Addressing Student Difficulties with Hydrostatic Pressure." *American Journal of Physics* 78, no. 1 (2010): 75–85.

McDermott, Lillian C., Peter S. Shaffer, and the Physics Education Group at the University of Washington. "Pressure in a Liquid." In *Tutorials in Introductory Physics*. Upper Saddle River, NJ: Pearson, 2012. [*Contains a guided inquiry tutorial based on Michael Loverude's Ph.D. dissertation that addresses student difficulties with topics of this lab.*]

"Special Focus: Graphical Analysis." AP Physics 2006–2007 Professional Development Workshop Materials. College Board. Accessed September 1, 2014. http://apcentral.collegeboard.com/apc/public/repository/AP_Physics_Graphical_Analysis.pdf.

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"Teacher Professional Development Resources." Seattle Pacific University. Accessed September 1, 2014. http://www.spu.edu/depts/physics/tcp/tadevelopment.asp. [*The Open Source Tutorials project has a modifiable tutorial on pressure.*]

"Under Pressure." PhET. University of Colorado Boulder. Accessed September 1, 2014. https://phet.colorado.edu/en/simulation/under-pressure. [*This simulation allows students to see how pressure changes as they change fluids, gravity, container shapes, and volume*.]

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AP Physics 2 Investigation 3: Resistor Circuits

How do conservation laws apply to a simple series or parallel resistor circuit?

Central Challenge

In this investigation, students explore simple series and parallel resistor circuits with a voltmeter and ammeter and encounter Kirchhoff's rules through inquiry.

Background

Of all the conservation laws, the conservation of energy is the most pervasive across all areas of physics and the sciences. Conservation of energy occurs in all physical, chemical, biological, and environmental processes. In circuits, charges gain energy in the battery and then that energy is dissipated in ohmic resistors as thermal energy, and in bulbs as thermal energy and light. Kirchhoff's loop rule states that the energy gained from the battery is equal to the energy loss in the circuit. In particular, potential differences across resistors in series are added together or combined, and the total is equal to the battery potential difference. Potential differences across resistors in parallel are equal to each other.

Conservation of electric charge is another fundamental conservation principle in physics. All processes in nature conserve electric charge. The total electric charge after an interaction or any other type of process always equals the total charge before the interaction or process. A common example is found in electric circuits, in which charge (typically electrons) moves within a circuit. Applying conservation of charge to a single point in the circuit, or through a cross-section of any wire in the circuit leads to Kirchhoff's junction rule. The sum of the currents flowing into any point in the circuit is the same as the sum of the currents flowing out of that point, since charge is neither created nor destroyed. This leads to the rules that govern current in simple series and parallel circuits. The currents are the same for two resistors in series, and the currents for two resistors in parallel with a battery add up to the total current through the battery.

These initial investigations in basic circuit behavior are the foundations for further studies in physics, electrical engineering, and general engineering. Such circuits are used as models for body systems in medical school as well. Proper understanding of basic circuit theory will support students in the more challenging aspects of circuits such as advanced circuits (multiple loops and multiple potential sources) and RC circuits.

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Real-World Application

Asking students to think of some simple, everyday activities that depend on electric current and circuits will yield answers such as using household lighting or laptop computers and watching television. Even students who do not pursue physics in future studies should understand that the outlets and appliances in their homes are connected in a parallel circuit, where all outlets receive the same voltage. It's also useful for them to understand how jump-starting a car requires putting a working battery in parallel with the dead battery to supply the same voltage. Although most circuits that students are likely to encounter are complex, and contain more than just one resistor, an ability to understand basic wiring or basic properties of household circuitry has great future value for all students, even those who have no ambition to go any further in the sciences.

Inquiry Overview

In this multipart investigation, students use a voltmeter and an ammeter to explore the relationships among the potential differences across the various elements in a circuit and the currents through these same elements. This lab is predominantly guided inquiry. Students are presented with a question to answer, they decide which circuits to investigate, and they use the results of their preliminary investigations to make decisions about additional circuits to study.

In Part I, students construct several circuits using D-cell batteries, miniature screw lamps, bulb holders, and wires. They connect the bulbs in series and parallel and use a voltmeter to discover the relationships among the potential differences across the various elements in the circuit.

In Part II, students use an ammeter to measure the current through the various branches of these circuits and devise a rule for the relationships among the various currents.

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Connections to the AP Physics 1 Curriculum Framework

Big Idea 5 Changes that occur as a result of interactions are constrained by conservation laws.

Enduring Understanding	Learning Objectives
5B The energy of a system is conserved.	**5.B.9.1** The student is able to construct or interpret a graph of the energy changes within an electrical circuit with only a single battery and resistors in series and/or in, at most, one parallel branch as an application of the conservation of energy (Kirchhoff's loop rule). (Science Practices 1.1 and 1.4) **5.B.9.2** The student is able to apply conservation of energy concepts to the design of an experiment that will demonstrate the validity of Kirchhoff's loop rule in a circuit with only a battery and resistors either in series or in, at most, one pair of parallel branches. (Science Practices 4.2 and 6.4) **5.B.9.3** The student is able to apply conservation of energy (Kirchhoff's loop rule) in calculations involving the total electrical potential difference for complete circuit loops with only a single battery and resistors in series and/or in, at most, one parallel branch. (Science Practices 2.2 and 6.4)
5.C The electric charge of a system is conserved.	**5.C.3.1** The student is able to apply conservation of electric charge (Kirchhoff's junction rule) to the comparison of electric current in various segments of an electrical circuit with a single battery and resistors in series and in, at most, one parallel branch and predict how those values would change if the configurations of the circuit are changed. (Science Practice 6.4) **5.C.3.2** The student is able to design an investigation of an electrical circuit with one or more resistors in which evidence of conservation of electric charge can be collected and analyzed. (Science Practices 4.1, 4.2, and 5.1) **5.C.3.3** The student is able to use a description or schematic diagram of an electrical circuit to calculate unknown values of current in various segments or branches of the circuit. (Science Practices 1.4 and 2.2)

[**NOTE:** In addition to those listed in the learning objectives above, Science Practice 4.3 is also addressed in this investigation.]

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Skills and Practices Taught/Emphasized in This Investigation

Science Practices	Activities
1.1 The student can *create representations and models* of natural or man-made phenomena and systems in the domain.	Students draw schematic circuit diagrams with meter connections and label the currents and potential differences to represent their experimental setup.
1.4 The student can *use representations and models* to analyze situations or solve problems qualitatively and quantitatively.	Students use schematic circuit diagrams to enhance their data analysis.
2.2 The student can *apply mathematical routines* to quantities that describe natural phenomena.	Students draw conclusions about the sum of the potential differences around a loop in a circuit. Students draw conclusions about the currents in a multibranch circuit.
4.1 The student can *justify the selection of the kind of data* needed to answer a particular scientific question.	Students decide what measurements to make to determine the rules for adding potentials in series and parallel circuits.
4.2 The student can *design a plan* for collecting data to answer a particular scientific question.	Students are provided general directions but make decisions about how to connect bulbs/resistors in series and parallel and how to organize and record data. Students use an ammeter to measure currents and a voltmeter to measure potential differences.
4.3 The student can *collect data* to answer a particular scientific question.	Students collect and record data from their measurements of the potential differences and currents in the various branches of the circuits they analyze.
5.1 The student can *analyze data* to identify patterns or relationships.	Students draw conclusions from their data, based on their measurements of potential differences and currents.
6.4 The student can *make claims and predictions about natural phenomena* based on scientific theories and models.	Students assess the uncertainties in their measurements to help inform the analysis of their data and support their conclusions.

[**NOTE:** Students should be keeping artifacts (lab notebook, portfolio, etc.) that may be used as evidence when trying to get lab credit at some institutions.]

Equipment and Materials

Per lab group (three to four students):

- Four-cell battery holder
- Three D-cell batteries

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- Three to four #14 (round) bulbs and three to four #48 (long) bulbs, plus corresponding bulb holders [**NOTE:** #14 bulbs have a limit of 2.3 volts, so small voltages should be used to avoid burning out too many bulbs. Car brake light bulbs will also work (inexpensive, but 6–12 volts are needed for good measurements) and #40 and #50 miniature screw lamps work as well.]
- Connecting wires (inexpensive alligator clip leads work well)
- Basic multimeters or student single-value meters (voltmeter and ammeter)
- Extra fuses for the ammeters
- (Optional) Basic single pole throw switch

[**NOTE:** If you teach the CASTLE™ curriculum, all of this equipment is part of the student kits except for the multimeters and the switch.]

Timing and Length of Investigations

- **Teacher Preparation/Set-up:** 20–25 minutes

  Check all bulbs, resistors, batteries, and meters to make sure all are in good working order before you start the lab. This may seem obvious, but a blown fuse in the ammeter (which will happen more than you would like) or a blown bulb will create measurement chaos for students.

- **Prelab:** 20 minutes

  A general discussion on the proper use and connection of multimeters, especially ammeters, is crucial.

- **Student Investigation:** 90 minutes

  Part I: 45 minutes to explore the series and parallel circuits

  Part II: 45 minutes to explore the series and parallel currents

- **Postlab Discussion:** 30–60 minutes

  Students share their results with the larger class in a whiteboard sharing session or "circle style meeting," while other students ask follow-up questions and critique student work.

  [**NOTE:** You may wish to have the postlab discussion for Part I before continuing to Part II on a second day, or you may wish to complete both parts in a 90-minute period and then have the discussion on a second day.]

- **Total Time:** approximately 3–3.5 hours

Safety

Safety is of minimal concern with this lab. The potential difference (1.5–4.5 volts) and current involved in the experiments are of no immediate safety concerns to students. If you choose to use a power supply rather than a battery pack, set the voltage to a fixed maximum around 5V. However, you may want to create good laboratory habits by always using a switch in the circuit and emphasizing proper meter usage.

Preparation and Prelab

This activity is designed to follow an introduction to circuits such as Sections 1–4 of the CASTLE curriculum. The student version of the CASTLE curriculum is available as a free download from PASCO. The equipment is also available for purchase from PASCO (see Supplemental Resources) or other sources that may be more economical. See "Modeling Instruction" in Supplemental Resources for the modified version of the CASTLE curriculum available on the website.

The students should already be familiar with the batteries, bulbs, and wires. They should be able to connect two bulbs in series and two bulbs in parallel, and make sure the circuits are in good working order before making measurements.

Give students a thorough tutorial on the use of ammeters and voltmeters in a circuit before beginning this inquiry. If your students are using multimeters, it would be helpful to take a picture of the multimeter setup as a voltmeter and one with it setup as an ammeter. These enlarged images can be projected onto a screen (e.g., in a PowerPoint presentation) for easier viewing. Experience has shown that students in a class of more than 10–12 will not be able to see a demonstration on an average multimeter. These projected images will show students how to set any relevant dials and which inputs to use on the meter for voltmeter and which for ammeter. Also consider color-printing these images for students and having them on laminated sheets or in plastic page covers around the room for students to double check before connecting circuits. It should be reinforced that the ammeter is connected in series, and this connection should be demonstrated for the students. It is highly recommended that students be required to show you their ammeter connections before they close the circuit the first time. This will save a lot of blown fuses.

You may wish to have students use the PhET simulation, "Circuit Construction Kit (DC Only)," to demonstrate that they understand how to connect an ammeter in series (see Supplemental Resources), and then have them use the ammeters with bulbs and batteries and wires. Or the PhET simulation could serve as a homework review after the in-class lab activity.

The Investigation

Part I: Electric Potential Difference in Series and Parallel Circuits

In this first part of the investigation, give students the equipment and the task of determining the relationships between the potential differences across resistors and battery in several different circuits, including the following:

- Two bulbs in series with each other and a battery
- Two bulbs in a parallel with each other and a battery
- Three bulbs in a series–parallel combination: in this case, both one bulb in series with a parallel combination (see Figure 1) and one bulb in parallel with a series combination (see Figure 2).

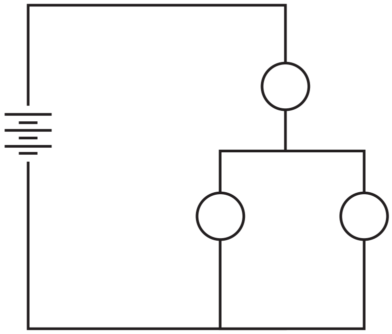

Figure 1

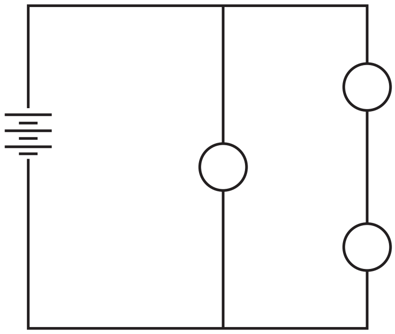

Figure 2

Encourage students to not only investigate the potential differences across each of the bulbs in circuits with two identical bulbs in series or parallel (two #14 round bulbs or two #48 long bulbs) but also in circuits with a long bulb and a round bulb in series, and in circuits with a long bulb and a round bulb in parallel. They should be encouraged to use both types of bulbs in the more complicated three-bulb circuits and possibly to expand to four-bulb circuits. The students should connect the voltmeter in parallel with each individual bulb and with the battery and record the potential differences. It is up to the students to decide how to most efficiently record the data. It is beneficial to students to draw each of the circuits they are studying and record the potential differences next to each circuit element. This helps them see the patterns.

While students are working, circulate and ask them questions to guide their investigation. Some students will record negative potential differences. Question them as to what the meaning of the negative sign is and under what conditions it would be positive. They should be led to see that reversing the leads on the voltmeter gives a positive value for the potential difference.

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Part II: Current in a Circuit Path

In the second part of the investigation, give students multimeters connected as ammeters or single-value ammeters and ask them to explore the relationships among the currents at various points in the circuit. [**NOTE:** You may wish to conduct this portion of the lab on a second day unless you have 90-minute periods.]

Again, encourage students to explore several different circuits, similar to the ones they explored in Part I. Students should keep track of their results by drawing each of the circuits that they evaluate and labeling the currents near each branch in the circuit drawing. Decide if you want to give them direct instruction in this or simply lead them to realize that this organization helps. Experience has shown that students will simply name the type of circuit and list the currents.

Extension

If some students finish early, you could ask them to create larger circuits with three series or three parallel branches and confirm their findings from the circuits with two branches.

Only simple series and parallel circuits are addressed in the AP Physics 1 curriculum, but if you want to provide students with an additional challenge, suggest a circuit that cannot be analyzed using simple series and parallel resistor combinations, such as the one in Figure 3. This will emphasize the need for the more general form of Kirchhoff's laws, rather than the simplified applications to series and parallel circuits. For example, some students report that "potential differences are the same in parallel and they add up in series." These students may need assistance to generalize this to the conservation of energy form Kirchhoff's rule stated as, "the sum of the potential differences around any closed loop is zero." It is important to have students pay attention to the signs of the potential differences in this circuit. If attention is not paid to which end of the middle bulb is at a higher potential, it might appear that the loop rule is not obeyed for several loops in this circuit.

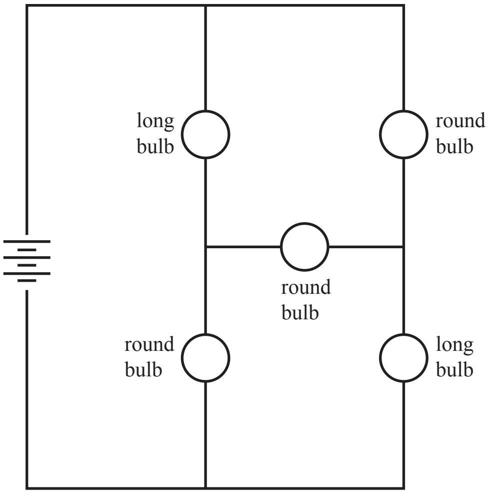

Figure 3

It is critical not to have the bulbs all the same type, as shown in Figure 3, so that different potential differences are measured across each bulb. If all bulbs are of the same type, then no potential difference will be measured across the middle round bulb. The middle bulb could also be a long bulb. The primary constraint here is that the two ends of the bulb cannot be at the same potential or it will not light.

Common Student Challenges

The most common student challenge in this lab is connecting the ammeter in series with the bulb whose current is being measured. Connecting an ammeter in series with just one of two bulbs which are themselves connected in parallel is the most challenging ammeter connection of them all. This activity provides students opportunities to practice these skills in several simple series and parallel circuits.

The most challenging aspect of working with circuits for the first time in a physics lab is training students to use a multimeter or ammeter correctly (i.e., ammeters connected in series with circuit elements and voltmeters connected in parallel with the element being measured). Plan on having plenty of fuses handy! There will be mistakes — just plan on it and be patient with your students. There will undoubtedly be some confusion of the voltmeter usage versus the ammeter usage, and surely a few short circuits with the ammeter will occur as students attempt to measure current by placing the ammeter leads in parallel around a bulb. It is just so tempting to the student!

A very persistent circuit misconception is the idea that current is “used up” in a bulb or resistor. This idea should not find any traction among students if they are asked why the current values are the same at all points around the circuit. A few probing questions to students about this evidence should help to eliminate that persistent student misconception.

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Another aspect of this experiment to keep your eye on is the idea that precision is not that important. The resistors have tolerance (5 percent probably), the bulbs are not always uniform (#14 and #48 bulbs from different batches can have enough variability in the filaments to show slight differences in potential or brightness around multibulb loops), the wires have resistance, the bulbs are non-ohmic, and just general sloppiness in measurement and meter usage can lead students to the incorrect conclusion that electric potential difference can change around a series loop of identical bulbs. You may have to remind students that the measurements of 2.94 volts, 3.03 volts, and 3.10 volts may have slightly different values for these three measurements, but that these small differences are all within the uncertainty of the meters and the uncertainty of the equipment, and they can be generally considered to have equal potential differences. These small variations in potential differences could simply be the result of using six to eight pieces of wire that have a small (but not totally negligible resistance if you are using bulbs) resistance of 0.05 ohms.

If resistors instead of bulbs are used for the experiment, you will probably see higher precision in the measurements, but you would lose the visual value of using the bulbs. Decide what works best for the students' experiments and your teaching style.

The TIPERs book (see Supplemental Resources) has some good conceptual tasks to assess students' understanding and root out alternative conceptions they may have regarding current in series and parallel circuits.

Analyzing Results

How you decide to have students share their results will depend in part on how successful they are at the lab. Have students answer the following guiding questions:

- What can you conclude about how potential differences are related for several resistors in series with a battery?
- What can you conclude about how potential differences are related for several resistors in parallel with a battery?
- How might these conclusions be interpreted from a conservation of energy perspective?
- What can you conclude about how currents are related for several resistors in series with a battery?
- What can you conclude about how currents are related for several resistors in parallel with a battery?
- How might these conclusions be interpreted from a conservation of charge perspective?
- Can you extrapolate these conclusions to more complex circuits with resistors in series and parallel combinations?

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If you see that all of the students have come to the same conclusion, then it may be sufficient to have a class discussion where they summarize the findings and record them in their notes. If students have come to different conclusions, then it is valuable to have them present their findings to the class and argue their positions.

Experience has shown that in Part I of this experiment, students readily come to the conclusion that potential differences add in series and are the same in parallel. They do not always readily come to the conclusion that energy is conserved as a single charge completes a loop in the circuit.

It will probably be necessary to lead them to this conclusion by having them imagine the energy changes for a skier on a hill. The ski lift serves as an analogy for the battery, the skiers for the charges moving in the circuit, and the various ski hills are the different paths around the circuit. Most students have either been skiing or seen skiing on television, so this analogy is very concrete for them. They can imagine walking around the ski hill and observing their changes in potential energy as they go up the ski lift and then walk the various alternate paths down the hill. When they return to their starting point at the bottom of the ski lift, their net change in potential energy is zero for the complete trip. And the same is true for the charges completing a closed loop in a circuit. [**NOTE:** The ski-hill analogy is particularly useful when writing Kirchhoff's loop rule equations and analyzing the potential differences across resistors. But that application isn't used here.]

If skiing proves too unfamiliar or abstract, have them imagine taking the elevator up to the top floor of a building and walking the various staircases back down to the floor where they boarded the elevator. This analogy is slightly less valid, as walking down stairs still requires effort. The simplest example may simply be a playground slide. Students, like the battery, provide energy to climb the ladder, lifting themselves to increase the gravitational potential energy in the system they make with the earth. Then they simply slide back down to their original potential energy level. Brightness can be related to how fast you would speed up, with increased resistance being related to how shallow the slide is.

Uncertainty should be a major postlab topic with the class. Some students may automatically conclude that two potential differences are the same across branches in parallel if they are within 5-10 percent of each other. Other students may conclude they are different. This leads to a discussion of the uncertainty in the meter measurements, resistance of the wires, and uncertainties in the resistances of the bulbs.

Ask students consider the following questions:

- Did the potential difference across the battery vary from circuit to circuit? If so, how did it affect your results?
- How close do two potential difference values have to be in order to be considered equal? What about current measurements?
- What is the maximum allowable uncertainty in the potential and current measurements?

The uncertainty in this lab comes from both the uncertainty in the meters and the fact that the battery is not ideal, and the potential difference across the battery decreases with increasing current. So if three bulbs are in parallel, then there will be a smaller potential difference across the battery than with just one bulb.

It might be helpful at this point to lead a class discussion by creating circuits in the PhET Circuit Construction Kit simulation (see Supplemental Resources). Then the relationships can be observed under ideal circumstances as well as under circumstances of significant internal resistance in the battery. This comparison between ideal and real laboratory results helps reinforce the uncertainties introduced by measurement. The goal is not to quantify the uncertainty to a specific degree, but rather to observe how uncertainties affect the relationships being observed. Thus, a follow-up activity with ideal batteries, resistors, etc., can serve to solidify the relationships between potential difference and current without the obfuscating effects of nonideal circuit elements.

Once the lab is complete, students should understand that current is the same in series elements. Since ammeters measure current, they must be placed in series with the resistor whose current they are measuring. A similar argument can be made for why voltmeters are placed in parallel. The results of this investigation support these requirements for meter connection and should be reinforced at the end of the lab.

Assessing Student Understanding

After completing this investigation, students should be able to:

- Connect several bulbs in various series and parallel combinations;
- Use a voltmeter to measure potential difference;
- Use an ammeter to measure current;
- Describe and apply the relationships among the potential differences around a closed loop in a circuit;
- Explain how conservation of energy is related to the potential differences in a circuit;
- Articulate and apply the relationship between the currents entering any point in a circuit and the currents leaving that same point; and
- Explain how conservation of charge is related to the current flow in a circuit.

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Assessing the Science Practices

Science Practice 1.1 The student can *create representations and models* of natural or man-made phenomena and systems in the domain.

Proficient	Accurately draws circuit diagrams of resistors and bulbs in various combination series and parallel circuits.
Nearly Proficient	Draws circuit diagrams for simple series or parallel circuits, but struggles with combination series–parallel circuits.
On the Path to Proficiency	Draws simple series circuits.
An Attempt	Attempts to draw circuits but the connections to bulbs and/or batteries are incorrect.

Science Practice 1.4 The student can *use representations and models* to analyze situations or solve problems qualitatively and quantitatively.

Proficient	Connects wires, batteries, and bulbs in simple series and parallel combination circuits based on a diagram provided. Identifies which bulbs are in series and which are in parallel.
Nearly Proficient	Connects a simple series or parallel circuit from a diagram but struggles with more complicated circuits.
On the Path to Proficiency	Connects a circuit with two bulbs either in series or parallel based on a diagram.
An Attempt	Incorrectly connects a simple series or parallel circuit from a diagram.

Science Practice 2.2 The student can *apply mathematical routines* to quantities that describe natural phenomena.

Proficient	Uses mathematical routines to detect patterns in the data and compare potential differences and currents in the circuits.
Nearly Proficient	Makes minor mistakes in the mathematical routines that describe the patterns in the current and potential difference data.
On the Path to Proficiency	Needs significant assistance in applying mathematical routines to describe the current and potential difference data.
An Attempt	Unable to accurately recognize patterns in the mathematical data or apply routines to analyze them.

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Science Practice 4.1 The student can *justify the selection of the kind of data* needed to answer a particular scientific question.

Proficient	Measures the appropriate potential differences and currents to make comparisons between series and parallel connections and can justify the choice of measurements thoroughly and accurately.
Nearly Proficient	Accurately selects the appropriate data, but the justification is missing a significant physical principle.
On the Path to Proficiency	Accurately selects the data to measure, but cannot justify the choice that was made based on physics principles.
An Attempt	Makes some relevant measurements, but cannot justify how they will help answer the guiding questions.

Science Practice 4.2 The student can *design a plan* for collecting data to answer a particular scientific question.

Proficient	Designs a plan that will allow for determining the loop rule and the point rule for circuits.
Nearly Proficient	Designs a plan for measuring potential differences or currents, but cannot articulate how that plan will lead to a rule for circuits.
On the Path to Proficiency	Needs significant assistance to design a plan to measure potential differences and currents for a circuit.
An Attempt	Attempts to form a plan to measure potential differences and currents, but the plan is ineffective or flawed.

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Science Practice 4.3 The student can *collect data* to answer a particular scientific question.

Proficient	Collects accurate data in a methodical way and records the data in an organized fashion. Accurately connects a voltmeter to measure the potential differences across various elements in a circuit and the total potential difference across the circuit. Connects an ammeter to measure the current in each branch of the circuit, and then connects the ammeter to measure the total current through the battery.
Nearly Proficient	Collects data that is missing a few minor pieces or is disorganized in its presentation. Accurately connects a voltmeter to measure the potential differences across various elements in a circuit, and connects an ammeter in a simple series circuit to measure the current.
On the Path to Proficiency	Collects data with major gaps, and the presentation lacks any organization. Accurately uses a voltmeter to measure potential difference, but incorrectly uses an ammeter in parallel with the bulb or battery in question.
An Attempt	Collects inaccurate or incomplete data and provides no organization for this data. Connects a voltmeter in series with the circuit to measure the potential difference across each bulb.

Science Practice 5.1 The student can *analyze data* to identify patterns or relationships.

Proficient	Analyzes the data to accurately determine the rules for voltages and currents in resistor circuits.
Nearly Proficient	Identifies patterns in the potential differences and currents, but unable to form a complete conclusion from this analysis.
On the Path to Proficiency	Forms some accurate analysis of the potential differences and currents, but unable to come to an accurate conclusion.
An Attempt	Attempts to analyze the data but there are major errors in his or her analysis.

Science Practice 6.4 The student can *make claims and predictions about natural phenomena* based on scientific theories and models.

Proficient	Predicts changes in the voltages and currents in the various elements in the circuit if more resistors are added in series or parallel.
Nearly Proficient	Makes accurate predictions about how adding bulbs in a simple circuit will affect the potential differences and currents but not in more complex circuits.
On the Path to Proficiency	Makes accurate predictions about changes in potential difference and current but only in the most simple series or parallel circuits.
An Attempt	Makes incorrect predictions about the changes in current or potential difference.

Supplemental Resources

"CASTLE Kit." PASCO. Accessed September 1, 2014. http://www.pasco.com/prodCatalog/EM/EM-8624_castle-kit/.

"Circuit Construction Kit (DC Only)." PhET. University of Colorado Boulder. Accessed September 1, 2014. http://phet.colorado.edu/en/simulation/circuit-construction-kit-dc.

Fredette, Norman, and John Lochhead, "Student Conceptions of Simple Circuits." *The Physics Teacher* 18, no. 3 (1980): 194–198. [*This is one of the classic papers in PER (Physics Education Research) regarding students' understanding of a circuit and is a must read for all physics teachers. It uses a classic question/activity to determine if college freshman electrical engineering majors understand the nature of a complete circuit (lighting a bulb). The article further demonstrates students' misconceptions with case study interviews that reveal some typical struggles students have with circuits.*]

Hieggelke, Curtis J., David P. Maloney, Stephen E. Kanim, and Thomas L. O'Kuma. *E&M TIPERS: Electricity and Magnetism Tasks: Inspired by Physics Education Research*. Boston: Addison-Wesley, 2005.

"Modeling Instruction." Arizona State University. Accessed September 1, 2014. http://modeling.asu.edu/.

Steinberg, Melvin S., and Camille L Wainright. "Using Models to Teach Electricity — the CASTLE Project." *The Physics Teacher* 31, no. 6 (1993): 353–357.

Stetzer, Makenzie R., Paul van Kampen, Peter S. Schaffer, and Lilian C. McDermott, "New Insights into Student Understanding of Complete Circuits and the Conservation of Current." *American Journal of Physics* 81, no. 2 (2013): 134–143. [*This paper confirms that many of the misconceptions that the Fredette and Lochead study brought to light in 1980 still exist in abundance with university physics students, and provides advanced ideas on how to teach students about circuits (multiple battery sources/circuit elements). The paper reveals strongly-held student misconceptions and suggests how to combat them with instructional changes.*]

AP Physics 2 Investigation 4:
RC Circuits

How do the resistors and capacitors in an RC circuit affect the behavior of the circuit?

Central Challenge

In this lab, students perform a series of investigations of RC circuits in order to observe and analyze the relationships that exist when resistors, capacitors, and emf sources are arranged in different ways (series, parallel, or combinations). Students will be building on their prior knowledge of simple, DC circuits from AP Physics 1 (or a similar introductory course) and will now gain an understanding of how to predict the current through a resistor or the charge on a capacitor, and the potential difference across resistors and capacitors under steady-state conditions.

Background

When a capacitor and a single resistor are connected in a simple series circuit with a battery or power supply, there is a transient behavior as the capacitor is filling with charge. The current initially flows freely onto the capacitor, but as the capacitor fills, the rate of flow of charge onto the capacitor (i.e., the current in the circuit) decreases eventually to zero. The capacitor then is as full of charge as the battery can make it; the potential difference across the capacitor being equal to the potential difference across the battery. There is at that point no potential difference across the resistor, and there is no flow of charge through the resistor. When the wires are disconnected from the battery and connected to each other, the capacitor discharges through the resistor until there is no more excess charge stored on the capacitor plates. This transient current can be visualized by charging and discharging a capacitor through small light bulbs, provided the capacitance is large enough for the charging to take a few seconds or more.

When several resistors are connected in a circuit with a capacitor, the transient behavior is more complicated. Initially the capacitor is empty, and all charge flows easily in the branch containing the capacitor. The capacitor momentarily acts as a short circuit to any elements with which it is connected in parallel. However, when the capacitor is full, no more charge can flow in that branch, and the capacitor acts like an open circuit in that branch of the circuit. Depending on how the circuit is constructed, current can still flow in branches that are parallel to the branch containing the capacitor.

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The charge on the capacitor can be determined by the equation $Q = C\Delta V$, where Q is the magnitude of excess charge on each plate of the capacitor (in coulombs), C is the capacitance (in farads), and ΔV is the potential difference across the capacitor (in volts). The resistor in the charging circuit only affects the time it takes the capacitor to charge to the final value Q. It does not affect that final value.

Real-World Application

As students learned in their first physics course, many of their daily activities depend on the use of electric current in complex circuits. The timing effect of the RC circuit is at play in some charging processes in commercial electronics. In particular, RC circuits are used in camera-flash units. One circuit allows the capacitor to charge slowly, and a separate circuit allows the capacitor to discharge very rapidly through a flash bulb. The idea of capacitance changing by changing the geometry or dielectric is also used in some commercial applications (some computer keyboards and accelerometers are examples of this usage). Touch screens on students' phones and tablets use capacitive elements to determine where the user is touching the screen. Students could investigate how this works as one extension of this lab (though not a lab activity itself).

Defibrillators monitor heart rhythm and use the charge on a capacitor to deliver a large amount of energy to restore normal rhythm in the case of a very fast, abnormal heartbeat. Automatic external defibrillators (AEDs) are common in public buildings for use by people without training, since the AED will not deliver a shock if the heart is in normal heartbeat rhythm. Such capacitors vary but may range from 100–200 microfarads at 2–3 kilovolts to deliver the necessary energy.

Inquiry Overview

There are three parts to this investigation, all of which involve guided inquiry, in which students are given a question to answer. Students then design an investigation to answer each question. It is not the intention in this lab for students to develop graphs showing the exponential trends or to actually make final calculations in each situation; rather they are to develop design plans that allow meter measurements to support observations, leading to relationships rather than calculated answers.

In Part I, students investigate how connecting capacitors in series and parallel affects the total capacitance of the circuit. In Part II, they investigate how a resistor connected in series with a single capacitor affects the total charge on a capacitor after it is fully charged by a battery. In Part III, they investigate how the potential differences across the various elements of an RC circuit change as the capacitor charges.

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Connections to the AP Physics 2 Curriculum Framework

Big Idea 4 Interactions between systems can result in changes in those systems.

Enduring Understanding	Learning Objectives
4E The electric and magnetic properties of a system can change in response to the presence of, or the changes in, other objects or systems.	**4.E.5.1** The student is able to make and justify a quantitative prediction of the effect of a change in values or arrangements of one or two circuit elements on the currents and the potential differences in a circuit containing a small number of sources of emf, resistors, capacitors, and switches in series and/or parallel. (Science Practices 2.2 and 6.4) **4.E.5.2** The student is able to make and justify a qualitative prediction of the effect of a change in values or arrangements of one or two circuit elements on currents and potential differences in a circuit containing a small number of sources of emf, resistors, capacitors, and switches in series or parallel. (Science Practices 6.1 and 6.4) **4.E.5.3** The student is able to plan data collection strategies and perform data analysis to examine the values of currents and potential differences in an electric circuit that is modified by changing or rearranging circuit elements, including sources of emf, resistors, and capacitors. (Science Practices 2.2, 4.2, and 5.1)

Big Idea 5 Changes that occur as a result of interactions are constrained by conservation laws.

Enduring Understanding	Learning Objectives
5B The energy of a system is conserved.	**5.B.9.5** The student is able to use conservation of energy principles (Kirchhoff's loop rule) to describe and make predictions regarding electrical potential difference, charge, and current in steady-state circuits composed of various combinations of resistors and capacitors. (Science Practice 6.4)
5C The electric charge of a system is conserved.	**5.C.3.6** The student is able to determine missing values and direction of electric current in branches of a circuit with both resistors and capacitors from values and directions of current in other branches of the circuit through appropriate selection of nodes and application of the junction rule. (Science Practices 1.4 and 2.2)

[**NOTE:** In addition to those listed in the learning objectives above, Science Practice 4.3 is also addressed in this investigation.]

Skills and Practices Taught/Emphasized in This Investigation

Science Practices	Activities
1.4 The student can *use representations and models* to analyze situations or solve problems qualitatively and quantitatively.	Students draw circuit diagrams and label the potential differences for different configurations.
2.2 The student can *apply mathematical routines* to quantities that describe natural phenomena.	Students apply Kirchhoff's loop rule to compare the potential differences across the various elements of the RC circuit.
4.2 The student can *design a plan* for collecting data to answer a particular scientific question.	Students make decisions about how to connect resistors and capacitors in series and parallel and how best to gather, record, and analyze data.
4.3 The student can *collect data* to answer a particular scientific question.	Students collect and record data from qualitative observations of bulb lighting and from direct measurements of potential difference using a voltmeter.
5.1 The student can *analyze data* to identify patterns or relationships.	Students use potential difference and current measurements to analyze how the circuit connections affect the behaviors of resistors and capacitors in the circuit. Students assess the uncertainties in their measurements to help inform the analysis of their data and support their conclusions.
6.1 The student can *justify claims with evidence.*	Students draw conclusions based on their measurements of potential differences and justify those conclusions using their measurements.
6.4 The student *can make claims and predictions about natural phenomena* based on scientific theories and models.	Students make predictions about bulb lighting and potential differences in circuits in order to help inform further circuit designs they will create and observe.

[**NOTE:** Students should be keeping artifacts (lab notebook, portfolio, etc.) that may be used as evidence when trying to get lab credit at some institutions.]

Equipment and Materials

Per lab group (three to four students):

- **Part I:**
- Three D-cell batteries and battery holder or DC power supply
- 8–10 connecting wires
- Four miniature screw lamps (size #40 or #50, with holders)

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- At least two nonpolar 100,000 (or 25,000) microfarad capacitors (These capacitors are expensive, but this type of investigation can only be done with a long "time constant," so these are your best choice if you are using the bulb as a visual timer. The nonpolar capacitors are easier for students to work with and less likely to be damaged than the polar capacitors.)
- **Part II:**

  In addition to the equipment from part I, this part requires:
- Several resistors in the 10–50 ohm range rated at least 1 watt or a resistor decade box with variable resistance
- Stopwatch
- **Part III:**

  In addition to the equipment from Parts I and II, this part requires:
- Voltmeter or multimeter
- Several resistors in the 200–500 ohm range rated at least ½ watt, or a resistor decade box with variable resistance
- Single pole switch

Extension

In addition to the equipment from Parts I–III, this part requires:

- Ammeter or multimeter

[**NOTE:** If you are not using the bulb as a visual timer, then you can opt for more traditional resistors and more traditional capacitor values (e.g., 1000–4700 microfarads, which are very inexpensive) from electronic supply stores, but you will probably sacrifice a little in student understanding. If you are using resistors and capacitors, try to keep your time constant ($\tau = RC$) for each pair of R and C to values between 1–5 seconds. This will ensure that the relevant physics can be measured or observed by students. An example of this would be a resistor value of R = 10,000 ohms and capacitor value of C = 200 microfarads; this would give a time constant of RC = 2.0 seconds. A general rule of thumb is that the charging time for a capacitor is approximately five time constants. (i.e., total charging time equal to about $5RC$).]

Timing and Length of Investigation

- **Teacher Preparation/Set-up:** 15–20 minutes

  This time is to gather materials, if everything has already been purchased.
- **Prelab:** 60 minutes

A suggested prelab activity is a tutorial/lab activity of investigating the capacitors' geometrical/physical properties and the effect of those properties on capacitance. "Capacitor Lab" is one suggested tutorial (see Supplemental Resources). Students should also be given time in small groups prior to the lab to design their procedure and setup, as well as to discuss assumptions and other factors that may affect the lab measurements and observations.

- **Student Investigation:** 120 minutes

  The time for students will vary greatly depending on what issues occur during the students' exploration time. Give students enough time to explore and make mistakes as they try to reconcile their ideas about the behavior of capacitors in these circuits. Many students are fascinated by the actual charging behavior of the RC circuit. That behavior usually challenges their naïve notions about how the capacitor will behave in the circuit, and this conflict in their minds usually leads to discovery and understanding if they are given the chance and enough time to formulate the proper physical reasoning for themselves.

- **Postlab Discussion:** 45 minutes

  Allow enough time for your class to have a vigorous student-led discussion about what they observed in the laboratory investigations. Give enough time for students to have some back and forth questioning about evidence and behavior. You can also provide some driving questions to the discussion that include students' analysis of sources of uncertainty in measurements and how uncertainties may have affected final results.

- **Total Time:** approximately 4 hours

Safety

There are no specific safety concerns for this lab, as long as students do not have access to large sources of potential difference. The 25,000- and 100,000-microfarad capacitors can be charged to a potential difference of 25 volts and 10 volts respectively, and then it can cause harm if a student makes contact with both poles. With only three D-cell batteries it poses little danger to students. Advise students to always handle capacitors carefully, as they can hold charge for a long time. In addition, all general lab safety guidelines should always be observed.

Preparation and Prelab

Circuits do not lend themselves to open inquiry in the same way that topics in mechanics do. Provide students with some guidance on connecting circuits and using meters to prevent them from damaging the equipment, wasting time on fruitless observations or circuits that don't work, and harming themselves. The amount of preparation and prelab required depends on students' prior experience with circuits involving bulbs and capacitors.

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Students should have experience charging and discharging capacitors through bulbs using the equipment. The first few sections of the CASTLE™ curriculum, which pertain more to AP Physics 1, are worth having students do, if they have not done it in a prior course. They should have seen that the bulbs light up for a longer period of time when charging a larger capacitor. They should also have seen a capacitor charged with a three-battery pack, and then charged more by adding another three-battery pack in the circuit in series. When the capacitor is eventually discharged, the brighter bulb lighting and longer time of bulb lighting (the time constant is the same but the length of time there is sufficient current to observe a lit bulb is longer) is indicative of the fact that more charge has been stored on the capacitor when the second battery pack was added. They can use the time of bulb lighting during capacitor discharging as an indicator of the amount of charge stored on the battery pack, so long as each time they discharge through the same bulbs. One of the sections of the CASTLE curriculum outlines this process for students, but in this type of inquiry, in a second-year course, students should be allowed to discover this for themselves (see "Using Models to Teach Electricity — the CASTLE Project" or the PASCO site in Supplemental Resources).

Students also must be familiar with the use of a voltmeter in a circuit. Hopefully, prior labs, either in AP Physics 1 or AP Physics 2, have provided them with this experience. If not, you must demonstrate the proper use of voltmeters in circuits.

Circuits labs are very teacher-intensive labs and students require lots of supervision and direction. This is exacerbated by the addition of capacitors to the circuits and the necessity of reaching the steady-state condition to make observations. Be patient with students and give them proper time and assistance to make these observations.

The Investigation

Part I: Capacitors in Series and Parallel

The purpose of this part of the activity is to have students investigate the charge accumulated on capacitors connected in series and in parallel. Present students with two large (100,000-microfarad or 25,000-microfarad) nonpolar capacitors, a battery pack, connecting wires, and #40 bulbs with sockets. Ask the students to design an experiment to see how combining the capacitors (in series and in parallel) affects the total capacitance. You can further refine the question by asking them to determine if adding capacitors in parallel or series increases the total capacitance, or charge storing capability, of the system of capacitors. Students may need some direction to come up with an appropriate plan. One method they can use is to observe the charging time for various capacitor combinations. They should then be able to reason that longer charging times implies more charge on the capacitor. And more charge on the capacitor implies a larger capacitor (if the battery used to charge the capacitor is constant).

Part II: Resistors in RC Circuits

The purpose of the second part of the experiment is to determine if the resistor affects the amount of charge on a capacitor in a circuit, and if so, how does it affect it? In other words, will a larger resistor allow more charge onto the capacitor, less charge onto the capacitor, or the same charge onto a capacitor as a smaller resistor does? Give students various resistors to use in a charging circuit. The range of resistances should complement the size of the capacitors such that the charging time is less than 10–60 seconds or so. With a 100,000-microfarad capacitor, the range should be between 5–100 ohms. For a different capacitor, set the time constant to about 1/5 of the total time you want the charging to take and use the equation $\tau = RC$ to calculate an appropriate resistance range. Give students at least four to five different resistors to choose from. If only two different resistance values are available, then they combine them in series and parallel to get different resistances. For example, two 10-ohm resistors in series create a 20-ohm resistor, and in parallel they create a 5-ohm resistor. This exercise can provide a good review for calculating equivalent resistances.

Students then should discharge the capacitor in each case through the same light bulb(s). A #40 or #50 bulb works well. They should decide how to use lighting and dimming of the light bulbs during discharging to determine if the capacitors have been charged to the same amount or different amounts with different resistors. For example, students charge the capacitor using a 20-ohm resistor. Then they discharge it through lightbulbs and observe the lighting time. Then they charge the capacitor using the 100-ohm resistor, and again discharge it through the same bulbs. Longer bulb lighting time would indicate more charge had been stored on the capacitor. Depending on the level of your students, you may want to engage the whole class in a brainstorming session about this before you break them into groups, or you may want to work individually with each group as needed.

It is unlikely that a bulb in series with a resistor and capacitor will light up significantly, so they cannot use the bulb lighting to determine if the capacitor is fully charged or not. So students should be instructed to connect the circuit and wait an appropriate amount of time to make sure the capacitor is fully charged. This may be 15–30 seconds, or up to a full minute, depending on the time constant of the circuit. A rule of thumb is that a capacitor is “fully” charged in about five time constants (or $t = 5RC$). To avoid a potential misconception, students should allow each combination to charge for the same amount of time, so set the suggested time as that for the largest RC combination value.

If students are having a difficult time making observations with the bulbs lighting, they could use an ammeter to make observations of the current. Some students have found this more effective and, no pun intended, enlightening.

Part III: Potential Difference in an RC Circuit

In this part of the lab, students investigate how the potential differences change across the various circuit elements as a capacitor charges. Give them a voltmeter and a switch in addition to the capacitors, batteries, and wires from the previous parts. Ask them to create several circuits, each of which contains a battery, a switch, at least one resistor, and a capacitor (see Figure 1). They should use the voltmeter to observe the potential differences across the capacitor and resistor. For this experiment, you want the charging time to be roughly 30 seconds, so with a capacitance of 25,000 microfarads, resistances of 200–500 ohms are appropriate. Students should observe the potential differences across the resistor, across the capacitor, and across the battery from when the switch is closed until no more changes are observed in the voltmeter reading. They should be encouraged to see how the potential differences change if the single resistor in the circuit is in series with the capacitor or if it is in parallel with the capacitor.

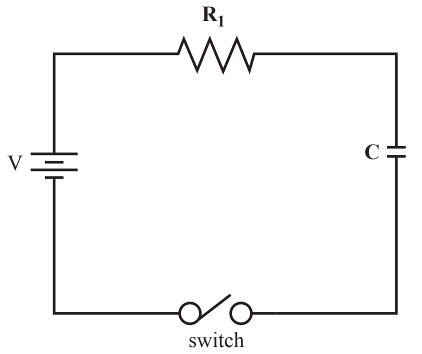

Figure 1

Encourage students to put more than one resistor in the circuit. For example, they could observe the potential difference changes across the various circuit elements in the circuit in Figure 2 below. Depending on how much time you have for this investigation, you can have students observe three or four such circuits.

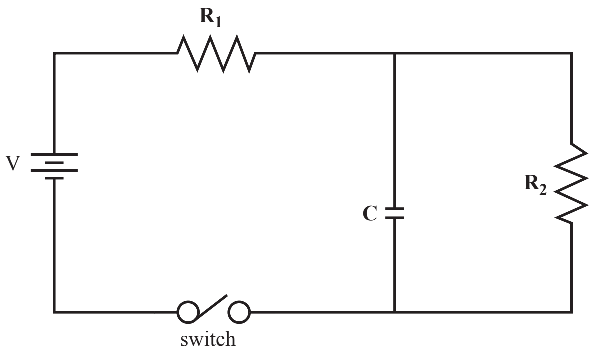

Figure 2

Extension

If additional time is available, or an additional challenge is desired, students could use an ammeter to investigate the initial and steady-state currents in RC circuits (especially those with more than one resistor, such as in Figure 2 above). A careful tutorial is required on the proper use of ammeters. Students have a tendency to use them like voltmeters, connecting them in parallel and immediately blowing a fuse. Have plenty of spare fuses on hand. This is particularly prevalent when multimeters are used both as ammeters and voltmeters.

Common Student Challenges

Capacitor behavior (in series with other capacitors and in series with a resistance) has always been a difficult concept for physics students. You can help students work through common misconceptions (e.g., no current "through" a capacitor or "more capacitors means more capacitance") by making sure they stick to the rules and models that have been developed in this course (e.g., Kirchhoff's loop rule and Kirchhoff's junction rule). The advanced ideas (capacitors in an alternate branch of the circuit) can be better managed by you and better understood by students if the fundamental charging behavior of a capacitor is fully understood. The exponential decay relationship of charge and current in a capacitor can be avoided and replaced with the total time to charge, which represents the total value of the capacitance in the system when the resistance is held constant (i.e., one capacitor in series with a resistor will take a certain time to charge, but two identical capacitors in series with each other and with the same resistor will have half the total capacitance, so the charging time should be less).

Be ready for the endless stream of questions that will arise during the execution of an open-ended lab investigation such as this one. Students will want to know if the circuit they have created is correct. You can have them explain how they connected it and point out any flaws in their reasoning. They will want to know if they are getting accurate results. Encourage them to double check their measurements. You can ask them to explain why they think their measurements may be incorrect. Depending on their answers, guide their reasoning with more questions. The Socratic method works best — constantly asking students questions that require the reasoning behind their choices — but it also requires you to think quickly and respond to unanticipated questions. Be prepared for improper use of meters to show up at some point, which will cause false or misleading results. Also, be prepared for some bulbs or capacitors to show some nonuniformity during these investigations. The bulbs (depending on the manufacturer) can vary in resistance/brightness. The bulbs also are non-ohmic. Keep this in mind in case some small measuring issues show up at some lab stations.

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This lab will surely challenge your students. Be prepared for them to make mistakes and to meet those mistakes with good questions and discussion that yield critical thinking. You should operate from the standpoint that if students take their time and make careful and deliberate measurements and make decent observations, then they will discover the fundamental nature of the capacitor's behavior in these RC circuits.

Analyzing Results

It is left to your discretion as to whether you want to stop after each part of the lab and discuss that part individually with the class, or wait until all students complete all three parts. Alternatively, you could assess each group individually to make sure they have gained the proper understanding before continuing. The following should be part of the discussion of each part.

Part I:
Each group will put their results on a large piece of butcher paper or on a large whiteboard. Ask students to answer the following guiding questions:

- What procedure was followed?
- How was the circuit setup?
- What measurements/observations were taken?
- What was observed?
- What conclusions can be drawn from these observations?

Students should start by sharing their findings with the class. They should have created a circuit with at least one bulb in series with the capacitor or capacitors. They can then use the bulb lighting time and brightness as a measure of how much charge had flowed onto the capacitor. They should conclude that the longer lighting time implies more charge had flowed onto the capacitor. They should have observed that when two capacitors are connected in series, the lighting time is shorter, and when two bulbs are connected in parallel, the lighting time is longer. Thus, the parallel arrangement holds more charge and thus has a greater capacitance. If a 100,000-microfarad capacitor is charged using a three D-cell battery pack, through a #40 bulb, the bulb will be lit (indicating charging time) for roughly 4 seconds. When two capacitors are in parallel, the charging time will approximately double. When the two capacitors are in series, the charging time will decrease by a factor of two, indicating that two capacitors in series act as a smaller capacitor.

Part II:
If students grasp the concept from Part I that the bulb lighting is a sign of how much charge is on a capacitor, then they should be able to use that to gauge the amount of charge on the capacitor. If they have not grasped that yet, they should be shown demonstrations to help them come to this understanding before they continue. Again, have the students answer the following guiding questions in their sharing with the class:

- What procedure was followed to determine the effect of resistance on total capacitor charge?

- What was observed?
- What conclusions can be drawn from these observations?

They should have charged the capacitor with different resistors, and discharged the capacitor always through the same set of bulbs. They should have observed that the discharging behavior is the same for all of the resistors implying that the size of the resistor used for charging does not affect the charge on the capacitor. When a 100,000-microfarad capacitor is charged using a three D-cell battery pack, it should discharge through a #40 bulb, lighting the bulb for approximately 4 seconds. This should be independent of which resistor or combination of resistors was used to charge it.

Part III:
Ask students to summarize their observations and any patterns they observed to share with the class in a whole-class discussion. Have them consider the following questions:

- How does the potential across the capacitor change during charging? During discharging?
- How does the potential difference across the resistor change during charging? During discharging?
- Compare the potential differences across the capacitor and resistor during charging. What do you notice?
- Compare the potential differences across the capacitor and resistor during charging to the battery potential difference. What do you notice?

What students report from this part of the lab will vary from group to group. Lead them to focus on the potential difference just after the switch is closed and a long time after the switch is closed (i.e., the steady state). Once each group has shared its results, you can lead the group to focus on patterns that they see in the data. How much leading they need will depend on the group of students in the room. Some may immediately notice that the resistor has a large potential difference initially, and then at the end has a zero potential difference (in the simple circuit). You may then lead them to look at the capacitor's potential difference at these times in light of Kirchhoff's loop rule (i.e., the rule for potential differences in series).

Assessing Student Understanding

After completing this investigation, students should be able to:

- Cite evidence that the system of identical capacitors in parallel has more capacitance than a single identical capacitor;
- Cite evidence that the system of identical capacitors in series has less capacitance than a single identical capacitor;
- Cite evidence that the resistor in a simple RC circuit does not control how much charge is on a fully charged capacitor (there is a dependence when a resistor is placed in series with a resistor and capacitor in parallel);

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- Demonstrate an understanding of the potential difference across a resistor in a simple RC circuit is equal to the battery potential difference initially and finally it is zero; and
- Demonstrate an understanding that the potential difference across a charging capacitor in a simple RC circuit is initially zero but equal to the battery potential difference in the steady state.

Assessing the Science Practices

Science Practice 1.4 The student can *use representations and models* to analyze situations or solve problems qualitatively and quantitatively.

Proficient	Connects a circuit according to a circuit diagram containing a capacitor and one or more resistors in series and/or parallel. Draws a circuit diagram when presented with a physical circuit consisting of batteries, capacitors, wires, and bulbs or resistors.
Nearly Proficient	Connects a simple series circuit containing a resistor and a capacitor and a battery based on circuit diagram. Draws an accurate circuit diagram for a circuit containing a resistor and a capacitor in series.
On the Path to Proficiency	Requires significant assistance in drawing an accurate circuit diagram.
An Attempt	Identifies a battery, bulb, resistor, and capacitor in a circuit diagram.

Science Practice 2.2 The student can *apply mathematical routines* to quantities that describe natural phenomena.

Proficient	Applies Kirchhoff's loop rule appropriately to multiloop circuits containing resistors and capacitors.
Nearly Proficient	Applies Kirchhoff's loop rule to a single circuit loop with a resistor and a capacitor.
On the Path to Proficiency	Requires significant assistance in applying Kirchhoff's loop rule to a simple circuit.
An Attempt	Identifies objects in a single loop, but does not accurately identify their potential differences to apply Kirchhoff's loop rule.

Science Practice 4.2 The student can *design a plan* for collecting data to answer a particular scientific question.

Proficient	Designs a valid plan for determining how capacitance is affected by adding capacitors in series and parallel. Designs an effective plan for determining how the resistance in circuit affects the charge stored on a capacitor.
Nearly Proficient	Designs a mostly complete plan but fails to account for one factor.
On the Path to Proficiency	Requires significant assistance in designing a plan, designs a plan with major flaws, or fails to account for multiple factors.
An Attempt	Designs a plan that is not valid for answering the questions posed in the inquiry.

Science Practice 4.3 The student can *collect data* to answer a particular scientific question.

Proficient	Makes accurate observations of bulb lighting and voltage measurements around a circuit containing bulbs and/or resistors, capacitors, and a battery.
Nearly Proficient	Makes mostly accurate measurements with a minor flaw or mistake in observation or measurement.
On the Path to Proficiency	Makes measurement mistakes or inaccurate observations of bulb lighting.
An Attempt	Makes major measurement mistakes or largely inaccurate observations of bulb lighting.

Science Practice 5.1 The student can *analyze data* to identify patterns or relationships.

Proficient	Correctly identifies patterns of bulb lighting to determine the effect of the resistance in a circuit on the charge stored in a capacitor.
Nearly Proficient	Uses the result but cannot articulate why bulb lighting time is related to the total charge on the capacitor.
On the Path to Proficiency	Cannot demonstrate that bulb lighting time is indicative of charge stored on the capacitor.
An Attempt	Unable to correctly discharge the capacitor through the same bulbs each time.

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Science Practice 6.1 The student can *justify claims with evidence.*

Proficient	Justifies the claim that two capacitors in parallel have a greater capacitance than one alone with accurate reference to observations made in the lab. Justifies that capacitors in series have less capacitance than a single capacitor alone.
Nearly Proficient	Makes correct observations, but needs significant assistance in justifying the relationships between capacitors in series and parallel.
On the Path to Proficiency	Demonstrates that bulb lighting is related to total capacitance, but cannot accurately articulate the relationship or use it to justify a claim.
An Attempt	Unable to support a claim with evidence; instead merely restates the claim in different words as justification.

Science Practice 6.4 The student can *make claims and predictions about natural phenomena* based on scientific theories and models.

Proficient	Predicts the potential differences around the circuit based on Kirchhoff's loop rule, relating a zero sum of potential difference to conservation of energy.
Nearly Proficient	Predicts the potential differences around the circuit based on Kirchhoff's loop rule without being able to thoroughly articulate the basis for the rule.
On the Path to Proficiency	Makes minor errors in claims about the potential difference around a loop.
An Attempt	Requires extensive assistance in making claims about potential differences around a loop using Kirchhoff's loop rule.

Supplemental Resources

"Capacitor Lab." PhET. University of Colorado Boulder. Accessed September 1, 2014. http://phet.colorado.edu/en/simulation/capacitor-lab. [*Provides an excellent simulation with plenty of teacher resources attached. It does an excellent job of simulating visually a few properties of the capacitor (charging, geometrical changes, and energy storage). Students can measure capacitance (with meters on tutorial) and then plot capacitance vs. area. The effect of the dielectric constant on capacitance can also be explored in a similar way.*]

"CASTLE Kit." PASCO. Accessed September 1, 2014. http://www.pasco.com/prodCatalog/EM/EM-8624_castle-kit/.

Chabay, Ruth, and Bruce Sherwood. *Electric & Magnetic Interactions*. 3rd ed. Hoboken, NJ: John Wiley & Sons, Inc., 2007. [*An excellent resource for topics in Electricity and Magnetism. There is plenty in this textbook to help a physics teacher supplement his or her course.*]

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"Curriculum Resources." Arizona State University. Accessed September 1, 2014. http://modeling.asu.edu/Curriculum.html.

McDermott, Lillian C., Peter S. Shaffer, and the Physics Education Group at the University of Washington. *Tutorials in Introductory Physics*. Upper Saddle River, NJ: Pearson, 2012. [*An outstanding supplementary resource for activities and teaching approaches for electrostatics and circuits topics. Many of the tutorials are completely in the "inquiry" mode.*]

Rosenthal, Alvin S., and Charles Henderson. "Teaching about Circuits at the Introductory Level: An Emphasis on Potential Difference." *American Journal of Physics* 74, no. 4 (2006): 324–328. [*This paper reinforces the idea that using the idea of "potential difference" or the voltmeter reading is a much better approach to teach circuit ideas to introductory students. The authors believe this approach discourages the memorization of combination rules and Kirchhoff's rules and gives the student a better footing of the fundamental concepts involving capacitors in circuits that have always challenged physics students.*]

Steinberg, Melvin S., and Camille L. Wainright. "Using Models to Teach Electricity — the CASTLE Project." *The Physics Teacher* 31, no. 6 (1993): 353–357. [*This paper provides an introduction to the CASTLE curriculum.*]

Young, Douglas. "Exploring Series and Parallel Combinations of Capacitors by Inquiry." *The Physics Teacher* 44, no. 6 (2006): 366–368. [*This paper gives some details of the general approach outlined in this investigation. It is a good read for teachers who have not had much exposure to capacitors in circuits. The paper also points out many of the details a teacher will need to obtain and use the proper electronic materials for their classroom.*]

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AP Physics 2 Investigation 5: Magnetism

How can we investigate magnetic fields?

Central Challenge

This investigation encourages students to explore the magnitude and direction of the magnetic field of magnets, current-carrying wires, and Earth, both qualitatively and quantitatively, using magnets, compasses, iron filings, and (optional) magnetic field probes.

Background

This investigation can be used to either generate or enhance students' knowledge of a vector or field-line representation of the magnetic field based on qualitative measurements. Once the vector nature of the field has been explored qualitatively, students can make quantitative measurements of the magnitude of the magnetic field. The investigation also explores the concept of superposition, as applied to the magnetic field. While there is not significant numerical analysis, vector fields is an area of quantitative difficulty for students, and this will help in developing semiquantitative representations.

Representations of these fields are important to the skills that students need to develop in the course. This activity develops the pattern of magnetic field vectors tangent to concentric circles around a current-carrying wire; the dipole pattern of field vectors around a bar magnet are needed representations. It also helps develop the needed representations of magnetic materials as containing magnetic domains that are themselves little magnets.

Real-World Application

Magnets are used in many real-life applications, such as burglar alarms, doorbells, loudspeakers, and electromagnetic motors and generators. Since the invention of the compass, the Earth's magnetic field has long been used for navigation and finding direction (even birds and magnetotactic bacteria use Earth's magnetic field to navigate or orient themselves). Archeomagnetic dating is also used to study the past history of the earth's magnetic field. Other applications include MAGLEV, or magnetically levitated trains, and MRI machines. Investigations into magnetic fields will help connect students to these real-life applications.

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Inquiry Overview

This guided-inquiry based investigation allows students to explore the magnetic fields all around us, such as that of Earth, and the fields of permanent magnets and current-carrying wires. With proper guidance, this investigation can be implemented before students study magnetism in class. However, the investigation is also quite useful after the concept and vector nature of magnetic fields (along with some mathematical representations) have been introduced, in order to give a laboratory experience with qualitative observations and the design of procedures for measuring a magnetic field. The investigation also enhances the understanding of superposition of fields.

Part I is a qualitative investigation of magnetic fields. [**NOTE:** This can take between 50 and 70 minutes of instructional time if it is used as a basic introduction, as students explore new concepts. If this is a first lesson on magnetism, Parts I and II should be separated by a lesson developing a quantitative model for the magnetic field of a wire.]

- In Parts I (A) and (B), students explore some basic properties of magnetic fields. The instructor wanders among the students, asking questions to help them focus on relevant phenomena.
- In Part I(C), students are asked to design experiments to explore an area of common misconception: the cause of magnetic attraction. The questions guiding the inquiry are not directly student generated, but the methods to answer those questions are significantly under students' control. Some particular techniques, such as the use of a Faraday cage, must either be directly suggested or can be motivated by a recent review of properties of electric fields.
- In Part I(D), students receive more guidance to allow them to develop a representation of the magnetic field inside a magnet. In Part I(E), students are prompted to make the observations needed to allow them to create a representation of the magnetic field due to a current-carrying wire. In order to ensure sufficient teacher guidance in subpart (D) while keeping subpart (E) mostly student guided, it may be best to provide two separate stations with setups for each activity so that students can rotate between them. Alternatively, subpart (D) can be carried out as a demonstration with the class. If this is a first exploration, and the concepts are new, whole-class demonstration is recommended.

In between the implementation of Parts I and II, students should have a lesson in developing quantitative models for magnetic fields, or a reading and homework, if Part I was students' introduction to magnetic fields. If class time is an issue, planning the investigation for Part II can be given as homework. Students could then make brief whiteboard presentations to the group discussing their chosen methods.

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In Part II, students carry out a quantitative exploration of magnetic fields. [**NOTE:** This can take 50–90 minutes, depending on the extensions pursued and the equipment available to students.] Again, the major guiding questions are given to them, but students must figure out exactly how to do this activity, what measurements need to be made, and how the data should be analyzed. The activity develops scientific practices beyond those associated with the learning objectives related to magnetic fields.

Connections to the AP Physics 2 Curriculum Framework

Big Idea 2 Fields existing in space can be used to explain interactions.

Enduring Understanding	Learning Objectives
2.D: A magnetic field is caused by a magnet or a moving electrically charged object. Magnetic fields observed in nature always seem to be produced either by moving charged objects or by magnetic dipoles or combinations of dipoles and never by single poles.	**2.D.2.1** The student is able to create a verbal or visual representation of a magnetic field around a long straight wire or a pair of parallel wires. (Science Practice 1.1) **2.D.3.1** The student is able to describe the orientation of a magnetic dipole placed in a magnetic field in general and the particular cases of a compass in the magnetic field of the Earth and iron filings surrounding a bar magnet. (Science Practice 1.2) **2.D.4.1** The student is able to use the representation of magnetic domains to qualitatively analyze the magnetic behavior of a bar magnet composed of ferromagnetic material. (Science Practice 1.4)

[**NOTE:** In addition to those listed in the learning objectives above, Science Practices 2.2, 4.2, 4.3, 5.1, and 7.1 are also addressed in this investigation.]

Skills and Practices Taught/Emphasized in This Investigation

Science Practices	Activities
1.1 The student can *create representations and models* of natural or man-made phenomena and systems in the domain.	In Part I of the investigation, students learn to create representations for a variety of magnetic fields.
1.2 The student can *describe representations and models* of natural or man-made phenomena and systems in the domain.	In Parts I and II, students identify relevant information about their representations and models and use it to formulate descriptions of each.

Science Practices	Activities
1.4 The student can *use representations and models* to analyze situations or solve problems qualitatively and quantitatively.	In Part II, students create representations for the fields and use these representations to determine what measurements are necessary.
2.2 The student can *apply mathematical routines* to quantities that describe natural phenomena.	The extension is quantitative and requires a number of calculations.
4.2 The student can *design a plan* for collecting data to answer a particular scientific question.	In Part I (B) and (C) and Part II, students design plans to collect data, although in Part I, the data are qualitative.
4.3 The student can *collect data* to answer a particular scientific question.	In each part of the activities, students make observations that allow them to answer scientific questions.
5.1 The student can *analyze data* to identify patterns or relationships.	In Part II, students carry out a semiquantitative analysis to understand addition of vector fields. They must analyze data to determine magnetic field in the extension.
7.1 The student can *connect phenomena and models* across spatial and temporal scales.	In Part I(D), students relate the field of a set of magnets to the behavior of magnetic domains in order to understand the field inside a magnet.

[**NOTE:** Students should be keeping artifacts (lab notebook, portfolio, etc.) that may be used as evidence when trying to get lab credit at some institutions.]

Equipment and Materials

Per lab group:

- 6–10 small compasses
- Three or more bar and horseshoe magnets of various sizes and shapes
- 6–10 1 3/8-inch unmarked bar magnets (it is very important to have small magnets available)
- Container, such as a salt shaker, filled with iron filings
- Sheet of paper, transparency, plastic zipper bag, or sheet protector
- Pith ball or paper clip hung on an insulated string
- Rubber rod or PVC pipe (for making charged rods)
- Glass or acrylic rod
- Rabbit fur or other material (for negatively charging the rods or PVC pipe)
- Silk or equivalent material (for positively charging the glass or acrylic rods)
- Styrofoam cup

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- Piece of aluminum foil (large enough to cover a Styrofoam cup)
- Battery holders
- 10–15 copper wires (16–18 gauge) with alligator clips
- Switch
- 6–10 pieces of string to hang magnets
- Magnaprobe (optional: a small alnico bar magnet mounted in a gimbal that rotates in 3D so that all *x*, *y*, and *z* coordinates can be mapped)
- Magnetic field probe
- (Optional) 3 × 3-inch sheet of magnetically sensitive film (for increased variety of observations)

For the extension:

- Linear variable resistor
- Ammeter

For Part I(D) and 1(E) of the investigation (each setup as a single station, students may bring the compasses from one to the other):

- 20 ceramic bar magnets (1 7/8 × 7/8 × 3/8 inches) to form a large rectangular magnet
- Flat piece of wood or cardboard (approximately 6 × 6 inches) with a hole in the middle for wire to pass through to serve as a platform for compasses
- Clamp
- Rod stand to support clamps to hold the cardboard or wood
- 8–10 1.5-volt D-cell batteries or another power supply
- Two 22-gauge wires to attach power supply to wire (optional)

Timing and Length of Investigation

- **Teacher Preparation/Set-up:** 15–20 minutes

  This is the time needed to set up the equipment, assuming everything is available at your school.

- **Student Investigation:** 130–160 minutes
- **Part I: Qualitative Investigations** 50–70 minutes (depending on if this part of the investigation serves as an introduction to magnetic fields)
- **Part II: Quantitative Investigations** 80–90 minutes (depending on if the design of the data collection procedures is assigned as homework)

  Design of procedure: 30 minutes

  Data collection: 30 minutes

Data analysis: 20–30 minutes (depending on if they have access to magnetic probes or must measure Earth's magnetic field relative to the field of a long straight wire)

- **Postlab Discussion:** 30 minutes
- **Total Time:** approximately 3–3.5 hours

Safety

Warn students that it is possible to get pinched as they manipulate magnets; the ceramic magnets in particular are quite strong and the force of attraction increases rapidly as the separation decreases. Otherwise, there are no specific safety concerns for this lab. General lab safety guidelines should always be observed.

Preparation and Prelab

While students are generally very familiar with magnets, they will often hold some common beliefs that are not consistent with the scientific view. You could have an introductory discussion with students to address these misconceptions, but if time allows, it is probably more productive to allow students to do the qualitative Part I of this activity as an exploration in order to gain a more scientific view. This requires careful observation and questioning as students investigate.

If you wish to pretest students on their understanding, you will find several good questions to pose in "Magnetism," "Magnetic Field of a Current-Carrying Wire," "Surveying Students' Conceptual Knowledge of Electricity and Magnetism," and "Arkansas Conceptual Electricity and Magnetism Conceptual Inventory Construction and Self-Testing Site" (see Supplemental Resources). "PHYS 1404" has two examples of what homework questions might look like early in students' study of magnetic fields, and "Magnet and Compass" and "Magnets and Electromagnets" provide simulations students can use to explore magnetic fields further (see Supplemental Resources for all).

The Investigation

Part I: Qualitative Investigations of Magnetic Fields

Start by hanging 1 3/8-inch unmarked bar magnets from the ceiling and letting them come to equilibrium. Discuss the meaning of *north-seeking* pole and *south-seeking* pole in terms of a magnet, and ask the students to label the poles. Then give them a compass and discuss which is the "north" end and which is the "south" end. This should lead to a discussion of the difference between magnetic and geographic north. Once students are clear about how the poles of a compass are labeled, they can use the compass to explore the magnetic behavior of other systems, including bar magnets and current-carrying wires.

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Part I(A): Exploring the magnetic field of Earth

In this part, students explore qualitatively Earth's magnetic field. One way to do this activity is to use the small magnets hung from the ceiling at various points in the room. Magnaprobes, which allow rotation in three dimensions, can also be used. The magnets hanging from the ceiling or the Magnaprobes should all point in the same direction. One note of caution: in some buildings, the steel in the walls is an iron alloy that will attract a magnet. Magnets near the steel will not point in the direction of Earth's field, which should either be avoided or used for discussion. The concept of superposition of the field of permanent magnets and Earth's field can also be explored here. To do this, ask students to explore the following questions:

- How does the direction of a small magnet suspended in Earth's field change when a permanent magnet is brought nearby?
- How close does it have to be to see the superposition? When does one field dominate over the other?

Part I(B): Exploring the magnetic field of a magnet

In this next activity you may choose to have students explore the behavior of compasses placed at different locations and distances away from a bar (or other shaped) magnet. Ask students if they have information on the existence of a magnetic field and its strength or direction based on their observations. Be sure to ask them to observe how quickly or slowly the compasses "lock" into position and if that gives them any information on the strength of the field.

Students can also use iron filings or magnetically sensitive film to explore the magnetic field (see "Magnetic Field Viewing Cards" and the two YouTube videos in Supplemental Resources for examples). The iron filings will not help with the direction of the magnetic field, but they will help with the strength of the field and the concept of field lines (if introduced). If students use iron filings, make sure they place any magnet to be studied either under a sheet of paper or transparency or inside a sheet protector or plastic zip-lock bag, so that the filings do not come into direct contact with the magnet (the sheet of paper might create the best contrast to see the pattern created by the filings). Magnetically sensitive film can help with the direction of the field that is perpendicular or parallel to the film. Have students draw a picture using field vectors at six to ten points in space around the magnet to indicate the strength of the field at various locations around the magnet. It is useful to explore the fields of different shaped magnets and combinations of magnets (e.g., between like and unlike poles).

Part I(C): Exploring magnetic poles

In this activity, students develop experiments to determine if magnetic poles are (or behave as) positive and negative charges. [**NOTE:** If students need a refresher on electrostatics before beginning this section, you might choose to either review the operational definition of charge and the behavior of objects in the presence of a charge, or provide guidance to each group as they explore various options.]

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Ask the students to design experiments that would either support or prove incorrect the hypothesis that the north and south ends of a magnet act as a cluster of positive or negative charges. They should write out their predictions for positive and negative charges and for north and south poles in simple situations. For example, they could predict the attraction or repulsion of a charged (or uncharged) pith ball by a charged rod, and predict the behavior of the pith ball (charged or uncharged) near a north or south magnetic pole. Once they have completed the predictions, they carry out the experiments. A charged pith ball will attract equally to both ends of a metallic magnet, so make sure students using such magnets test their predictions at both poles.

Direct students to explore the interaction of a charged pith ball or paper clip hung inside a Faraday cage with a charged rod outside the Faraday cage; then with a magnetic pole outside the Faraday cage. A simple example of a Faraday cage is a paper cup covered in foil. The students should observe the behavior of both north and south magnetic poles and positively and negatively charged rods. They can also observe the behavior with just the Styrofoam cup (not coated with foil). They should perform enough experiments so that they consistently observe (and come to understand) that north and south poles are not a cluster of positive and negative charges. It is imperative that they make sure the pith ball or paper clip is not too close to the upper edge of the Faraday cage, or the shielding will not be effective. If you did not cover the Faraday cage in your discussions on electric fields and conductors, this is a good time to ask students what they predict the electric field to be inside a conductor, and develop an explanation of the behavior of the Faraday cage.

Part I(D): Determining magnitude and direction of a magnetic field (superposition of magnetic fields)

In this activity, students are challenged to determine the magnitude and direction of the field inside a magnet. [**NOTE:** Students might need significant direction with this activity in order to relate the phenomena they observe with the superposition of fields due to multiple magnets. This may be most effectively done as a demonstration with strong student involvement followed by a class discussion. Another option would be to have each group cycle through this station (in parallel with students at the current-carrying wire station (see Part I(E)), which will not need as much teacher direction) and have each group create a whiteboard representation of the results of their exploration. This way you can check that they have developed the correct understanding while they are still in the activity.]

One way to approach this part of the investigation is to use the concept of superposition of magnetic fields. First, ask students to observe the superposition of fields outside of magnets by observing the change in direction of compass needles near a magnet when a second or third magnet is brought close to the first. If done as a whole group, use a compass that is clear on both sides and place it on an overhead along with the magnets, or use any compass with an Elmo-like projection system. Then, have students stack the twenty ceramic bar magnets to simulate a bar magnet that can be “broken” so as to enable them to “see” the field inside. The direction of the field inside can be determined by the following process:

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1. With the stack of magnets in the shape of a bar magnet lying horizontally on a table, remove half of the stack and replace it with a small compass at the end of the remaining stack.
2. Observe the direction of the field caused by the half of the magnet on the table.
3. Remove the compass and replace the half that was removed.
4. Remove the other half and replace it with the compass at the end of the stack now on the table. (This will allow an observation of the direction of the magnetic field at approximately the same position caused by the second half of the stack.)
5. By superposition, the field inside the total stack, with both halves together, must be the sum of the fields caused by each stack independently.
6. To help students see that the field is stronger when both halves are together, be sure students observe that the field varies from straight out from the end of the bar magnet more quickly (for magnets placed perpendicular to Earth's field) as they weaken the magnet, and relate this to the net force on the needle.

Students determine that the field inside of a magnet points south to north and that it should be quite strong. This type of analysis is often presented in textbooks in a discussion of magnetism and magnetic domains at the microscopic level, but this activity provides a way to analyze magnetic fields macroscopically. More details and pictures of a setup can be found in a number of available teaching materials, such as the "Magnetism" and "PHYS 1404" (see Supplemental Resources).

Part I(E): Exploring the magnetic field of a current-carrying wire

In this final part of Part I, students observe qualitatively the field of a current-carrying wire using compasses.

[**NOTE:** Since this activity might require a relatively high current and use a significant amount of table space, it is beneficial to setup a separate station for it and let the student groups rotate through as they complete the previous activities.]

Setup a circuit with the D-cell batteries or a power supply, a switch, and low resistance. Pass the wire vertically through a hole in a piece of cardboard or a piece of wood, supported by clamps and a stand, which the compasses can rest on. For images of a setup, see "PHYS 1404" and "Magnetic Field of a Current-Carrying Wire" in Supplemental Resources. The switch should only be closed for brief periods as observations are being made.

Students should note the direction of the magnetic field when there is no current through the wire. When the switch is closed, the compasses will point in the direction of the magnetic field of the wire (the current must be large enough to make Earth's field irrelevant). The switch should be held down just long enough to see the effect and then opened, or the batteries will run down quite quickly. The batteries or power supply can then be turned around in order to observe the direction of the magnetic field when the direction of the current is reversed. Encourage students to draw a picture of the wire using magnetic field vectors at various points around the wire to indicate the strength of the field, which as previously observed is qualitatively determined by how quickly the compass aligns with the field. Ask students to draw both side and end views of the wire. [**NOTE:** With fewer batteries, or as batteries run down, the field may not be strong enough to make qualitative observations of the decrease in the magnetic field with distance from the wire.]

Part II: Quantitative Investigations of Magnetic Fields

In this part of the investigation, students measure the magnitude of the magnetic fields of various combinations of magnets, using a magnetic field sensor. This should be done at various distances from the magnets. Different sensors work in different ways, so it is important that you properly instruct students in their use.

While measuring the magnitude of a magnet's magnetic field, it is necessary to take Earth's field into account, in order to obtain just the field of the magnet. Before students design their data-collection procedures (either as homework or at the beginning of this activity), ask them to figure out different ways to do this. One option is to take measurements on both sides of the magnet: the first on the side where the direction of the Earth's field is opposite that of the magnet's field, and the second on the side where the direction of the Earth's field is in the same direction as the magnet's field. The Earth's field will cancel out when the two measurements are added together, yielding just twice the magnitude of the desired field. Other ways to obtain the field of the magnet include zeroing the sensor far from the magnet, fixing the sensor in one location and zeroing it, and then moving the magnet closer to it at different locations; or taping the sensor to the table, and then moving the bar magnet's N end to various distances along a line that is normal to the sensor surface.

[**NOTE:** Before proceeding, ask for volunteers to present their draft procedures to the class, and solicit feedback from the various groups.]

During the investigation, the students should take measurements in the following manner and sequence:

1. Take measurements surrounding one bar magnet.
2. Repeat the same measurements for a different bar magnet.
3. Place the first magnet on top of the second and repeat the same measurements.
4. Reverse the orientation of the top magnet and repeat the measurements.

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Instruct students to create a vector field representation to semiquantitatively justify the results of their measurements. Additional magnets can be stacked to further vary the field. Ask students to make predictions about what they expect to measure before taking additional measurements.

Extension

Once students have studied the form of the equation for the magnetic field of a straight wire in class, as an extension they can measure the horizontal component of Earth's field using a current-carrying wire as a reference (finding the magnetic field from the equation for a long current-carrying wire at a given distance and current) by determining the direction of Earth's magnetic field, placing the wire such that the horizontal field produced by the wire is perpendicular to Earth's field, and varying the current (measured by an ammeter) in the wire until a compass placed next to the center of the wire (so the long wire approximation is most valid) is deflected to an angle 45 degrees from Earth's field. Students can plan the details of this measurement and for other angles. This is just an example: at 45 degrees the magnitude of the two fields are equal so the analysis is simplified greatly.

One possible source of error here is that the surface on which the experiment is carried out may contain materials that can become magnetized. Discussing the impact this would have on their measurements helps students achieve the AP Physics 2: Algebra-Based goal of being able to discuss what happens when some parameter of a physical situation is changed. It always helps to ask students to remember to make careful observations and consider their control variables.

Common Student Challenges

This lab is useful because by carrying out the qualitative and quantitative observations and measurements, students, through their observations, overcome the challenges many still face after instruction. Magnetism is a subject in which some prior beliefs exist that should be altered, and this exploration also serves as a place to help students in several areas that they often have difficulty: dealing with superposition of vectors, graphing data, and interpreting graphs.

In particular, Part I(C) provides students the opportunity to demonstrate that magnetic poles are not and do not behave as clusters of positive and negative charges. Throughout all of Part I, students who have not yet studied magnetism have the opportunity to observe magnetic behavior and begin to build mental models of the magnetic field for the first time. They will develop a better understanding of the strength and the direction of the field inside a magnet, how a measured field is the superposition of the existing fields, and the strength and direction of Earth's magnetic field. If they have studied magnetism, they will come to understand some of the challenges of the material through the laboratory observations.

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Magnetic field explorations are often very interesting to students, as the macroscopic observations of the noncontact magnetic force are often exciting. You may find that students want to just play and observe magnetic effects — a little of this is encouraged before serious observations and data taking begins. After working through the lab, students should have a better understanding of magnetic fields and have improved their measurement skills. They should also be able to answer the guiding questions found in the Analyzing Results section of this investigation.

If students are still struggling with particular challenges during the lab, such as the field inside a magnet or the superposition of fields, ask probing questions at that point. In the later parts of the lab, ask students explicitly about what they are graphing and why, and what information can be gained from the graph.

Analyzing Results

The qualitative results can be analyzed by having students draw magnetic field vectors or magnetic field lines for the various magnets or current-carrying wires they observe. They can also discuss the strength and direction of the field at various points away from a magnet or current-carrying wire. It is particularly useful to ask students explicit questions about challenging topics. The guiding questions for this lab include:

- Based on qualitative laboratory explorations, what evidence do we have of the existence of magnetic fields, and how can the magnitude and direction of the fields be represented by either field lines or vectors?
- Are magnetic poles the same as positive and negative charges, and what experiments could be done to demonstrate this?
- How can we use the concept of superposition of fields to determine the strength of the magnetic field inside a magnet?
- How can we qualitatively and quantitatively measure the strength and determine the direction of Earth's magnetic field?
- Are permanent magnets the only way to create a magnetic field?
- Do stationary or moving charges give rise to a magnetic field?
- Do we have to account for Earth's magnetic field when measuring the magnetic field of magnets or current-carrying wires?
- How could we use the magnetic field of a current-carrying wire to measure the magnetic field of the earth?

Assessing Student Understanding

At the end of the investigation, students answer the questions that guide the investigation. They should understand and be able to articulate how to determine the magnitude and direction of the magnetic field at a point in space qualitatively, and how to make a quantitative measurement of the strength of the magnetic field.

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They should understand and/or be able to:

- Articulate that magnetic poles are not clusters of positive or negative charge;
- Visualize the magnitude and direction of the magnetic field due to a magnet and due to a current-carrying wire at various points in space and be able to draw pictures using magnetic field vectors to represent the field at those points in space;
- Conceptualize the magnitude and direction of Earth's magnetic field at various points in space and be able to experimentally measure the strength of the field and determine its direction;
- Articulate the superposition of magnetic fields; and
- Use the superposition of magnetic fields to design an experiment to measure an unknown field, given a reference field.

[**NOTE:** Further questions useful for assessment can be found in various references listed in Supplemental Resources.]

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Assessing the Science Practices

Science Practice 1.1 The student can *create representations and models* of natural or man-made phenomena and systems in the domain.

Proficient	Creates a representation that: (1) is appropriate and correct for representing the magnetic field of a current-carrying wire (magnetic field vectors form concentric circles and have no component toward the wire); (2) includes features representing all essential aspects of the phenomenon (magnitude of the magnetic field is proportional to the magnitude of the current, strength of the field decreases as the distance from the wire increases, and the direction of the field is determined by a right-hand rule); (3) contains an appropriate level of detail (indicating the subtraction or addition of field's different sources); is accurate; (4) accurately labels the elements of the representation (the current and the magnetic field, and indicating how the magnitude of the field decreases with distance from the wire or wires); (5) expresses a causal explanation (i.e. that the magnetic field is caused by the current and decreases as the distance from the current increases); and (6) correctly superposes the magnetic fields to find the direction of net field.
Nearly Proficient	Creates a representation but does not show all of the main features of the phenomenon; or the representation is missing one necessary detail; or the representation accounts for the most important features of the phenomenon and is experimentally testable, but the limitations and assumptions are not mentioned.
On the Path to Proficiency	Creates a representation but more than one main feature of the phenomenon is not represented; or the representation is missing more than one necessary detail; or the representation does not account for the most important features of the phenomenon and is not experimentally testable.
An Attempt	Creates a representation but many of the main features of the phenomenon are not represented; or the representation is missing many of the necessary details; or the representation does not account for the most important features of the phenomenon and is not experimentally testable.

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Science Practice 1.2 The student can *describe representations and models* of natural or man-made phenomena and systems in the domain.

Proficient	Includes the following in describing the model: (1) a compass needle is a permanent magnetic dipole with north and south polarity; (2) iron filings in a magnetic field become induced magnetic dipoles; (3) dipole orientation is antiparallel with field direction; and (4) the magnetic north of a compass needle points toward the magnetic south of Earth, which is near Earth's geographic north pole.
Nearly Proficient	Extracts relevant information from the representation but one important piece of information is missing, or while describing a model, makes one important omission.
On the Path to Proficiency	Extracts information from the representation but focuses on irrelevant features, or while describing a model, omits multiple important aspects.
An Attempt	Extracts incomplete information from the representation and focuses on irrelevant features, and while attempting to describe a model, omits multiple important aspects.

Science Practice 1.4 The student can *use representations and models* to analyze situations or solve problems qualitatively and quantitatively.

Proficient	Uses the model correctly, including: (1) accurate representation of the domain model of ferromagnetic materials; (2) accurate translation between the structure of a bar magnet and the magnetic domain representation; (3) accurate predictions based on the magnetic domain representation (if a bar magnet is broken in half, both halves are magnetic dipoles and have magnetic fields; no magnetic north pole has ever been isolated from a south pole).
Nearly Proficient	Utilizes a domain model and uses it to predict how the bar magnets together act like a single magnet, but a feature of the prediction is missing or inaccurate.
On the Path to Proficiency	Utilizes a domain model to predict how the fields of two bar magnets add or how bar magnets in the same direction together act like a single magnet, but has trouble considering multiple magnets or cases when all domains are not aligned.
An Attempt	Utilizes a domain model but cannot accurately use it to predict how the bar magnets together act like a single magnet.

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Science Practice 2.2 The student can *apply mathematical routines* to quantities that describe natural phenomena (extension).

Proficient	Substitutes values into the developed equation and correctly solves by manipulating the variables in the equation.
Nearly Proficient	Creates an accurate mathematical representation of the form of the appropriate magnetic field for the method chosen and the vector relationship between the reference field and the field under study.
On the Path to Proficiency	Uses appropriate terms or symbols in developing an equation (mathematical representation), but there is an error in the form of the equation.
An Attempt	Attempts to create a mathematical representation, but significant errors in the use of symbols, equations, etc. are present.

Science Practice 4.2 The student can *design a plan* for collecting data to answer a particular scientific question.

Proficient	Part II: Designs an appropriate collection plan to answer the question with appropriate qualitative measures and adequate justification for the selection of equipment.
Nearly Proficient	Part II: Designs an appropriate collection plan to answer the question with appropriate qualitative measures.
On the Path to Proficiency	Comes up with some aspects of a plan to collect data in response to the question, but the plan is not adequate to answer the question.
An Attempt	Attempts to design a plan to collect data, but the plan is not aligned to the question or problem being investigated and it contains significant misalignments and errors.

Science Practice 4.3 The student can *collect data* to answer a particular scientific question.

Proficient	Makes observations that allow for answering scientific questions in each part of the activity.
Nearly Proficient	Implements all parts of the plan to collect data but without sufficient care, or collects data with appropriate care but not for all parts of the planned collection. The insufficiencies in data collection weaken the conclusion that can be drawn from the data.
On the Path to Proficiency	Implements some aspects of his or her plan to collect data, but the data was not taken with sufficient care or contained some mistakes or unnecessary inaccuracies.
An Attempt	Attempts the collection of data, but the data is not appropriate for the investigation and the collection strategies contain major errors.

Science Practice 5.1 The student can *analyze data* to identify patterns or relationships.

Proficient	Identifies the patterns in the data relevant to the question of the relationships between individual fields and a resultant field or between the direction of field of interest and direction of reference field in the extension. Uses appropriate terminology to describe the relationships.
Nearly Proficient	Identifies the directions from the compass and applies the vector representation to add fields of individual magnets, but makes some errors.
On the Path to Proficiency	Uses the direction of the compass to identify the direction of a field of a single magnet, but cannot use a vector representation to show addition of vectors.
An Attempt	Attempts to use data, but fails to identify any patterns and illustrates many errors when representing vectors.

Science Practice 7.1 The student can *connect phenomena and models* across spatial and temporal scales.

Proficient	Explains the relationship of the external observed field to the internal structure of the magnet as being made of multiple magnetic domains, relating this to the addition of many magnets and that the magnetic field represents the direction of the net field, so that all the individual magnets do not have to be aligned.
Nearly Proficient	Explains a relationship between the external observed field and the internal structure of the magnet as being multiple magnetic domains but in terms of all of the domains being aligned within the magnet.
On the Path to Proficiency	Describes that magnets can be made up of smaller magnets but in terms of physical magnets, not at the scale of magnetic domains.
An Attempt	Attempts to describe the relationship between the external magnetic field to the concept of internal magnetic domains, but many errors are present.

Supplemental Resources

"Arkansas Conceptual Electricity and Magnetism Conceptual Inventory Construction and Self-Testing Site." University of Arkansas. Accessed September 1, 2014. http://physinfo.uark.edu/inventory.

Giordano, Nicholas J. "Applications: DNA Fingerprinting." In *College Physics: Reasoning and Relationships*. Belmont, California: Brooks/Cole, 2010.

Kanim, Stephen, and John R. Thompson. "Magnetic Field Viewing Cards." *The Physics Teacher* 43, no. 6 (2005): 355-359.

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"Magnet and Compass." PhET. University of Colorado Boulder. Accessed September 1, 2014. http://phet.colorado.edu/en/simulation/magnet-and-compass.

"Magnetic Viewing Film, Looking at Zen Magnets." YouTube. Video, 3:53. Accessed September 1, 2014. http://www.youtube.com/watch?v=AMggixEu0q0.

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Maloney, David P. "Charged poles?" *Physics Education* 20, no. 6 (1985) 310–316.

Maloney, D.P., T.L. O'Kuma, C.J. Hieggelke, and A. Van Heuvelen. "Surveying Students' Conceptual Knowledge of Electricity and Magnetism." *Physics Education Research Supplement to the American Journal of Physics* 69, no. 7 (2001): S12–S23.

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McDermott, Lillian C., and the Physics Education Group at the University of Washington. "Magnetic Field of a Current-Carrying Wire." In *Physics by Inquiry: Volume II*. Hoboken, NJ: John Wiley and Sons, Inc., 1996.

"See Invisible Magnetic Fields with Magne-View Film." YouTube. Video, 2:22. Accessed September 1, 2014. http://www.youtube.com/watch?v=uLHl9mnRSbc.

Stewart, Gay B. "Measuring Earth's Magnetic Field Simply." *The Physics Teache*r, 38, no. 2 (2000):113–114.

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AP Physics 2 Investigation 6: Electromagnetic Induction

What factors relate to the production of an emf in the interaction of a magnetic field and a coil of wire?

Central Challenge

This investigation introduces the topic of electromagnetic induction and provides a solid experiential background for the discussion of magnetic flux, Faraday's law of induction, and Lenz's law. Students are asked to design experiments to determine the variables that determine the emf that can be induced in a coil by a permanent magnet.

Background

Magnetic flux is the scalar product of the magnetic field ($\vec{B}$) through a loop and the area vector associated with that loop. The area vector ($\vec{A}$) is a vector that is perpendicular to the plane of the loop and has a magnitude that is equal to the area of the loop. This can be expressed mathematically as $\Phi = \vec{B} \cdot \vec{A}$. If a uniform magnetic field is perpendicular to the plane of the loop, then the flux through the loop is simply the product of the magnitude of the magnetic field and the area of the loop ($\Phi = BA$). When the magnetic flux through a loop is changing, a current is induced in the loop. Even if the loop is not closed (i.e., has a small gap in it), an induced emf is created in the loop. This induced emf is proportional to the rate of change of the flux. If a coil consists of several loops, the induced emf in the coil is proportional not only to the rate of change of flux through each loop, but also to the number of loops. This is Faraday's law. In equation form, it is $\mathcal{E} = -N\frac{\Delta \Phi}{\Delta t}$, where $\mathcal{E}$ is the induced emf and N is the number of loops in the coil.

Real-World Application

There are so many applications of the principles studied in this investigation that a short list should invoke many others. Electric generators and transformers in the power grid are perhaps the most relevant applications. Back emf is also a very important factor in the design and operation of all electric motors. Many of the new designs for efficient hybrid and electric automobiles include the use of the braking system to transfer the mechanical energy of the moving vehicle into electrical energy that can recharge the batteries. The principle of induction is used in some types of microphones and in the magnetic coil pickups of electric guitars.

Inquiry Overview

This is a guided-inquiry investigation in which students construct small coils and use a voltmeter to measure the induced emf when small neodymium magnets are moved in and through the coil. Students need to come up with a list of variables (e.g., coil size, magnet strength, speed) that may affect the emf induced in a small coil when a magnet is moved near the coil. Then they need to design and execute experiments to vary those factors and determine if and how they affect the emf generated.

Connections to the AP Physics 2 Curriculum

Big Idea 2 Fields existing in space can be used to explain interactions.

Enduring Understanding	Learning Objectives
2.D A magnetic field is caused by a magnet or a moving electrically charged object. Magnetic fields observed in nature always seem to be produced either by moving charged objects or by magnetic dipoles or combinations of dipoles and never by single poles.	**2.D.3.1** The student is able to describe the orientation of a magnetic dipole placed in a magnetic field in general and the particular cases of a compass in the magnetic field of the Earth and iron filings surrounding a bar magnet. (Science Practice 1.2)

Big Idea 3 The interactions of an object with other objects can be described by forces.

Enduring Understanding	Learning Objectives
3.A All forces share certain common characteristics when considered by observers in inertial reference frames.	**3.A.1.3** The student is able to analyze experimental data describing the motion of an object and is able to express the results of the analysis using narrative, mathematical, and graphical representations. (Science Practice 5.1)

Big Idea 4 Interactions between systems can result in changes in those systems.

Enduring Understanding	Learning Objectives
4.E The electric and magnetic properties of a system can change in response to the presence of, or changes in, other objects or systems.	**4.E.2.1** The student is able to construct an explanation of the function of a simple electromagnetic device in which an induced emf is produced by a changing magnetic flux through an area defined by a current loop (i.e., a simple microphone or generator) or of the effect on behavior of a device in which an induced emf is produced by a constant magnetic field through a changing area. (Science Practice 6.4)

[**NOTE:** In addition to those listed in the learning objectives above, Science Practice 4.3 is also addressed in this investigation.]

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Skills and Practices Taught/Emphasized in This Investigation

Skills Practices	Activities
1.2 The student can *describe representations and models* of natural or man-made phenomena and systems in the domain.	Students describe a magnetic field around a neodymium magnet.
4.3 The student can *collect data* to answer a particular scientific question.	Students collect data on the effect of moving a magnet through a coil and inducing an emf. Students use a voltmeter to measure induced emf.
5.1 The student can *analyze data* to identify patterns or relationships.	Students analyze the data to identify patterns and relationships between the motion of a magnet and the emf induced in a coil.
6.4 The student can *evaluate alternative scientific explanations.*	Students examine alternate explanations for how an emf might be induced in a coil using a small neodymium magnet.

[**NOTE:** Students should be keeping artifacts (lab notebook, portfolio, etc.) that may be used as evidence when trying to get lab credit at some institutions.]

Equipment and Materials

Per lab group (two students):

- 5–6 meters of enameled magnet wire (100 meters can be purchased online or at most electronics retail stores for less than $10)
- Plastic/cardboard tube to act as a base for winding the coil of wire, such as PVC pipe or very wide drinking straw (approximately 1/2-inch inside diameter and 2 inches is long enough)
- Plastic tube of larger diameter to wind a larger coil, such as a small prescription bottle (1-inch diameter and about 2 inches is long enough)
- Four or more neodymium axially polarized nickel-plated disc magnets (5/16 × 1/4 inches) The exact number of magnets and their dimensions are not important so long as there is a sufficient quantity to separate them into at least two piles, so that the strength of each magnet group can be adjusted, and the diameters are narrow enough to be moved easily back and forth through the diameter of the smaller coil (such magnets are widely available on the web for about $0.50 each).
- Digital multimeter (DMM) with a setting that will indicate to the tenths of a millivolt [**NOTE:** As an alternative, analog meters or a galvanometer may be used.]
- Pair of connecting wires, preferably with alligator clip connectors
- Electrical tape to secure leads

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- Sandpaper to sand off ends of coated wire
- String to suspend magnets
- Masking tape
- Compasses
- (Optional) Two eightpenny or tenpenny nails

Extension:

- Two coils, one that fits inside the other
- Demonstration transformer
- Old AC to DC wall transformer to disassemble
- Digital multimeter with AC voltmeter capability or dedicated AC voltmeter

Timing and Length of Investigation

- **Teacher Preparation/Set-up:** 5–10 minutes

  After the materials are ordered/gathered, the teacher preparation time is minimal.

- **Prelab:** 10–20 minutes

  The prelab should be a short discussion or review of the fact that a current passing through a coil produces a magnetic field and that its sense can be determined by the right-hand rule. Emphasize that there will be no external source of emf used in this investigation. The purpose of this initial discussion is to start students thinking in terms of current direction, magnetic poles, and interactions. You might want to have the class identify a list of possible variables as part of the prelab. That could add another 10 minutes. If there is some concern about the time available for the actual investigation, some coils could be assembled during an extended prelab.

- **Student Investigation:** 60–70 minutes

  This time will vary depending on the number of variables students identify and, more importantly, their skill winding a coil of wire. As a postlab you might want to bring the research together to discuss common findings and pose additional questions (e.g., “Would a different gauge wire produce different results?”). This would require another 15–20 minutes and could lead to an extension of the study as time allows.

- **Postlab Discussion:** 15–20 minutes
- **Total Time:** 1.5–2 hours

Safety

While there are no great dangers associated with this lab, students should be cautioned that these magnets are very strong and will accelerate toward each other or other metal objects very quickly and possibly cause unexpected and significant pinches to any skin that happens to be in the way.

The nickel coating on the magnets might chip and flake off if they are struck or dropped on hard surfaces or one another. The underlying magnetic material is then exposed to breaking up, so the magnets should be handled with care. Not a lot is known about the toxicity of these materials so it is a good idea to wash hands if exposed to disintegrating magnetic material. Individual sets of magnets are easily stored in small plastic prescription bottles to keep them separated.

Preparation and Prelab

Prior to beginning the investigation, students should establish a list of possible variables that determine the emf that can be induced in a coil by a permanent magnet, and go on to design coils and procedures to test those variables with the supplied materials. You might have to show students how to separate the magnets by sliding them apart and reconnecting them by sliding them back together. (See "Separating Neo Magnets" in Supplemental Resources.)

The Investigation

To begin, tell the students to find a way to identify and label the polarity of their magnet by reference to Earth's magnetic field in the room. [**NOTE:** If they are given compasses, have students keep them at a relatively large distance from the magnets because the strong magnets used in this lab can ruin them. The compass will still work at a greater distance and help to identify the polarity. Because of the strength of the magnets, simply placing them on a smooth surface or suspending them horizontally from a thread will allow them to quickly line up with the Earth's field.] Some smartphone apps can help identify the polarity of the magnet, but students should be careful not to bring the strong magnets too near their phones to prevent damaging them.

Next, with the appropriate materials, students construct simple coils to carry out their experiments. Depending on the color of the enamel on the wire, it might appear to students to be bare, so you might have to tell them to scrape or sand the coating off the end sections to connect the voltmeter. Students should design experiments to discover that the motion of the magnet through the coil along its axis will generate an emf. If a particular group doesn't discover this, you may have to guide them in this direction.

Students may need some assistance using the digital multimeter. Suggest a setting of approximately 200 millivolts on the DMM to detect the emf. Also tell students that the sampling rate of the meter limits its response. Typically this rate might be five samples per second, so therefore the average speed with which the magnet moves through the coil should not be too fast for the meter. [**NOTE:** Some digital meters do not have the sensitivity to provide good readings. If this is the case, analog meters or a galvanometer may need to be used, but they have their limitations. It may be more difficult to gather data as the moment of inertia of the rotating coil in the meter can interfere, thus making the motion of the needle not as responsive to changes in current as a DMM may be. The traditional use of a galvanometer makes the connection of current and the change in flux. While this can be done, there are two factors that might make this a less effective lesson: 1) Changing the length of the coil will change the resistance so the current will respond to two variables, increased number of turns and increased resistance. This would be a poor example of experimental design even though the results of this semi-quantitative investigation are not likely to be significantly changed and 2) Faraday's law relates induced emf to rate of change of flux. Current is another step removed from the fundamental idea.]

One method of controlling the speed of the magnet through the coil is to have the coil propped up at a small angle as illustrated in Figure 1:

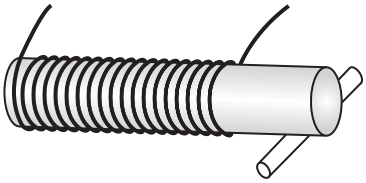

Figure 1

The angle should be such that the magnet, starting at the beginning of the coils, will just slide down the tube where it can be stopped before exiting by a finger placed over the lower end. This can be achieved by placing a small object under one end of the tube as shown. Depending on the material of the tube this could be something like a matchstick or a pencil. By doing five or six runs, a good indication of the relative size of the induced emf can be determined. Then the angle can be increased slightly and the procedure repeated.

An alternate method of moving the magnet slowly down the tube would be to hold a nail in one end of the horizontal coil and allow the force it exerts on the magnet to pull the magnet along the tube. Moving the nail closer to the magnet would produce a higher speed, and thus the speed could be varied to see the effect on the emf.

One end of the set of magnets can be placed on the head of the nail to provide a convenient axial handle, as illustrated in Figure 2:

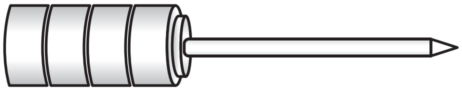

Figure 2

As students perform their experiments, remind them to keep careful records of all observations and keep track of the N-S orientation of the magnet, the direction of its motion, and the direction of the induced current as well as some rough numerical data for the range of emfs induced in each case.

Tell students that in some individual trials the sampling rate will be such that a measurement is missed and the meter will read zero. Regardless of the method used to move the magnets through the coil, students will probably notice that the magnet is actually accelerating, but it is the average speed that is important in enabling them to make qualitative conclusions. They should be told that they are not gathering exact quantitative data but merely using the average numerical readings of several trials to judge the effect of each variable they consider.

It is expected that they will be able to come up with their own list of variables, but if you have chosen not to make this part of the prelab discussion, you might need to guide them to identify and investigate some missed variables: strength of the magnetic field (number of discs in "series"), number of turns of wire in the coil, diameter of the coil, spacing of the windings — spread out or tight — more than one layer of windings, the effect of the speed of the motion of the magnet into/out of the coil, and the sign of the induced voltage for each motion. Students may inadvertently vary two quantities at the same time if they are not careful in winding the coils. For example, when comparing two different diameter tubes, students need to make sure that each tube has the same number of turns, the same spacing, etc.

Even though this is a qualitative lab, and does not lend itself to a quantitative error analysis, you should lead a discussion with students about methods of improving their observations to be certain that they are observing true effects. If they obtain an unexpected result, they should repeat the observation. If they still are not convinced they are observing accurately, they should ask you for confirmation, or to make sure their equipment is connected correctly.

Extension

Once students have an understanding of how a changing magnetic field can induce a current in a coil, the concept can be extended to a primary and secondary coil. Depending on student experience, this may work best as a teacher demonstration projected onto a larger screen for students to observe.

Alternating current is supplied to a larger coil. When a smaller coil with an AC voltmeter attached to it is inserted inside the larger coil, students should see a reading on the voltmeter. The changing magnetic field here is due to the changing direction of current from the alternating supply. From this, you can also demonstrate other related concepts, such as how inserting the secondary coil only halfway into the larger coil affects the reading.

You can also take apart a wall charger to show students how the input coil and output are not directly connected to each other, but that the output current is induced in the output line by a changing current (and changing magnetic field strength and direction) in the input coil.

Even though AC currents are not part of the curriculum, this extension has very common application in students' lives and fairly simply shows how current is induced in transformers.

Common Student Challenges

It is likely since this is an investigation to introduce the topic, students will still be having difficulties with the abstraction of the magnetic field. This provides some of the mystery and wonder of forces that do not appear to be anything like the attractions or repulsions that students have studied in other parts of the course. Their previous experiences combined with common intuitive understandings of how forces behave can make it difficult for students to understand what is happening. Spending a few minutes reviewing what they know about magnetic fields might help alleviate this. For example, have them draw the field around a bar magnet as a review or introduction to this lab. And have them review how to determine the polarity of a magnet, both by suspending it and allowing it to align with Earth's field and using a compass.

The two-fold idea that the changing magnetic field can exert a force on the charges in the conductors of the coil, and that the force is not in the direction of the field or even in the direction of its motion, can be overwhelming. Confusion about the magnitude of the flux as opposed to the rate of change of flux is also a common difficulty. Encourage students to see the cause and the effect, even though at this stage they cannot formulate it into something that seems to make sense. It can help to build intuition by starting with the example of a square loop of wire being moved into, through, and out of a uniform, constant magnetic field. Students can use the right-hand rule to see that there will be a net movement of charge (leading to current in a closed loop, or a potential difference in a loop with a gap) when one edge of the loop is outside of the field. There is no current when the loop is completely within the field because there is no net movement of charge. It is also important to show them that the same forces, and therefore currents and potential differences, are created if the loop remains constant and the magnetic field "moves."

Analyzing Results

For this type of open-ended investigation, there are many ways of recording/reporting the results. One way would be an organized individual report that might include the following:

- A preliminary list of the variables that might determine the emf generated in the coil (e.g., number of coil loops per given length, speed of the magnet through the coil, cross-sectional area of the coil, strength of the magnet).
- A description of the method to be used to investigate the effect of changing each variable. (Encourage students to use diagrams to clarify their descriptions/explanations.)
- A sentence or two describing the reasons, including estimated typical potential differences, as supporting evidence for the groups' conclusions regarding each variable.

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- A final concluding essay making overall claims about the determining factors in terms of size and direction of the emf.

A postlab class discussion of the observations can serve to reinforce the conclusions. When a consensus is reached, with your guidance, each student should compile his or her individual reports of the results of the class investigation. Students might then begin to develop the relationship $\mathcal{E} = -N\frac{\Delta\Phi}{\Delta t}$.

Assessing Student Understanding

After completing this activity, students should be able to:

- Generate an emf using a magnet and a coil;
- Design an experiment to determine the factors that affect the magnitude of the emf in the coil;
- Make predictions about the direction of the induced emf in a coil based on the motion of a magnet through the coil;
- Explain that the induced emf depends on the number of turns of wire through which the magnetic field passes and the rate at which this occurs; and
- Discuss the effect of a difference in cross-sectional area on the rate at which the magnetic flux changes.

Assessing the Science Practices

Science Practice 1.2 The student can *describe representations and models* of natural or man-made phenomena and systems in the domain.

Proficient	Describes the magnetic field and the change in flux and how it relates to the induced emf in the coil.
Nearly Proficient	Describes the magnetic field of a magnet, and understands the difference between flux and field, but does not distinguish between flux and rate of change of flux.
On the Path to Proficiency	Describes the magnetic field of a magnet but has difficulty distinguishing between flux and field.
An Attempt	Creates an inaccurate description or representation of the magnetic field around a magnet.

Science Practice 4.3 The student can *collect data* to answer a particular scientific question.

Proficient	Creates an emf in a coil by moving a magnet. Makes accurate and complete observations of the induced emf for various motions of the magnet and various sizes of coils (including both relative strength of the emf and the direction).
Nearly Proficient	Makes mostly accurate measurements with a minor flaw or mistake in observation or measurement.
On the Path to Proficiency	Makes major measurement mistakes or inaccurate observations of the emf induced, such as failing to record the sign, or in the orientation of the magnet.
An Attempt	Cannot make any relevant or appropriate observations, and connects the voltmeter incorrectly.

Science Practice 5.1 The student can *analyze data* to identify patterns or relationships.

Proficient	Explains patterns of voltmeter readings to infer the relationships between the emf induced in different trials of varying complexity.
Nearly Proficient	Describes patterns of voltmeter reading, but cannot connect the voltmeter reading to the emf induced in complex trials.
On the Path to Proficiency	Identifies only the simplest patterns from the voltmeter readings due to induced emf and cannot draw conclusions from these patterns.
An Attempt	Cannot correctly organize the data to allow identification of any patterns.

Science Practice 6.4 The student can *make claims and predictions about natural phenomena* based on scientific theories and models.

Proficient	Makes claims and/or predictions regarding the magnitude and direction of the induced emf due to a moving magnet in a coil. Explains what factors affect the induced emf in the coil.
Nearly Proficient	Makes claims or predictions regarding the rate of change of flux and the emf induced with an occasional or minor error.
On the Path to Proficiency	Describes changing flux and induced emf without correct reasoning or justification and with some errors.
An Attempt	Identifies and articulates the results of induced emf with significant errors.

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Supplemental Resources

“Faraday’s Electromagnetic Lab.” PhET. University of Colorado Boulder. Accessed September 1, 2014. http://phet.colorado.edu/en/simulation/faraday. [*This interactive Java simulation provides a good postlab discussion and enables the students to “see” what is going on in this application of Faraday’s Law. It represents the magnetic field not as a series of lines but as a field of vectors/arrows, thereby reinforcing the use of such representations in AP Physics 1 and 2*.]

“Faraday’s Magnetic Field Induction Experiment.” Michael W. Davidson. The Florida State University. Accessed September 1, 2014. http://micro.magnet.fsu.edu/electromag/java/faraday2/index.html. [*Not as elaborate or interactive as the PhET simulation, this simple simulation could be used very effectively in a postlab discussion. It might best be used before the PhET simulation because it “shows it like it is” without visible fields or visible charge particles, and is a good transition from the student experiment to the model represented in the PhET simulation*.]

John Belcher’s videos. YouTube. http://www.youtube.com/user/ElectromagnetismAnim.

“Separating Neo Magnets.” YouTube. Video, 9:27. Accessed September 1, 2014. http://www.youtube.com/watch?v=GYiTjdKhRWg. [*This video about using neodymium magnets can help your students manipulate these strong magnets without damaging the magnets or pinching their fingers.*]

“Visualizing Electricity and Magnetism at MIT.” Accessed September 1, 2014. http://web.mit.edu/8.02t/www/802TEAL3D/teal_tour.htm. [*John Belcher’s field visualizations are really well done and several of them might be useful as part of the postlab discussion*.]

AP Physics 2 Investigation 7: Geometric Optics

How do we find the focal length of a thin lens?

Central Challenge

In this investigation, students are given a converging lens and asked to make measurements that can be graphed to find the focal length of that lens.

Background

Lenses are formed by shaping glass or plastic into one of two general shapes. Lenses that are convex on both sides, and thus thicker in the middle than on the edges, take rays that are incident on the lens parallel to each other and converge them to a single point, called the *focal point.* These are called *converging lenses*. Lenses that are concave on both sides, and thus thinner in the middle and thicker on the edges, take rays that are incident on the lens parallel to each other and diverge them as if they came from a single focal point. These are called *diverging lenses* (in fact, in air, any lens that is thicker in the middle, for example a plano-convex lens, is a converging lens; and any lens that is thinner in the middle, for example a plano-concave lens, is a diverging lens).

Snell's law governs the way that rays are refracted through each lens. The path of light is reversible, so light rays that are incident through the focal point will be refracted parallel to the *principal axis* — a line that passes through the lens, perpendicular to the plane of the lens. We can use these facts to trace specific rays from an object through the lens to determine, qualitatively, where the image is formed.

We define the following terms:

- *image distance* — the distance from the image to the center of the lens
- *object distance* — the distance from the object to the center of the lens
- *focal length* — the distance from the focal point to the center of the lens

We can use the geometry of similar triangles to derive an equation that relates the image distance, object distance, and focal length (Equation 1). This is known as the lens equation.

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$$

Equation 1: Lens Equation

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Real-World Application

Both diverging and converging lenses are used to correct vision, and many students are curious about how their glasses (or contact lenses) work. During the unit on optics, you can include a discussion about how the two types of lenses are used to correct vision, and how the focal length of each lens relates to how good (or poor) an individual's vision is. In particular, you could show how converging lenses are used for reading glasses. The focal length is long enough that the object distance is always less than the focal length, and the image produced is virtual, enlarged, and upright. Students could use ray boxes (which create parallel beams of light) to examine how their own glasses refract the light and determine for themselves whether they are diverging or converging.

You could also discuss how diverging lenses, which are used to correct near-sightedness, make virtual images that are always closer to the lens than the object. If you want to expand upon this activity, you could look into multi-lens systems. Multi-lens systems have many applications, including telescopes, binoculars, microscopes, and other optical instruments.

Inquiry Overview

This lab is designed to provide students a guided-inquiry experience: you give them the question to answer, and they design the experimental procedure. After students have gained experience using the lens equation, present them with a converging lens of unknown focal length. Students then design an experiment to determine the focal length of the converging lens, make a graph, and use the graph to determine the focal length.

If some students complete the above task in significantly less time than their peers, you can provide them with a diverging lens and ask them to find its focal length. As the diverging lens does not form real images, they will have to use the converging lens in a system with the diverging lens to create a real image and measure image and object distances. Alternatively, you could require all students to find the focal length of both lenses.

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Connections to the AP Physics 2 Curriculum Framework

Big Idea 6 Waves can transfer energy and momentum from one location to another without the permanent transfer of mass and serve as a mathematical model for the description of other phenomena.

Enduring Understanding	Learning Objectives
6.E The direction of propagation of a wave such as light may be changed when the wave encounters an interface between two media.	**6.E.5.1** The student is able to use quantitative and qualitative representations and models to analyze situations and solve problems about image formation occurring due to the refraction of light through thin lenses. (Science Practices 1.4 and 2.2) **6.E.5.2** The student is able to plan data collection strategies, perform data analysis and evaluation of evidence, and refine scientific questions about the formation of images due to refraction for thin lenses. (Science Practices 4.1, 5.1, and 5.2)

[**NOTE:** In addition to those listed in the learning objectives above, Science Practice 4.3 is also addressed in this investigation.]

Skills and Practices Taught/Emphasized in This Investigation

Science Practices	Activities
1.4 The student can *use representations and models* to analyze situations or solve problems qualitatively and quantitatively.	Students use a ray diagram to locate the image for both a converging and diverging lens, indicating the correct refraction for each ray and accurately locating the image and identifying as real or virtual.
2.2 The student can *apply mathematical routines* to quantities that describe natural phenomena.	Students draw a best-fit line to the data and extrapolate to find the intercept, the reciprocal of which is the focal length. Students use the lens equation and a graph of their choosing to calculate the focal length of the converging lens.

Science Practices	Activities
4.1 The student can *justify the selection of the kind of data* needed to answer a particular scientific question.	Students justify the selection of the kind of data needed by: ◆ comparing the lens equation to their collected data to determine what to plot to get a straight line that can be used to determine the focal length ◆ explaining why the distance from the object to the lens and from the image to the lens are measured. ◆ explaining how the data collected will be plotted to answer the question being investigated
4.3 The student can *collect data* to answer a particular scientific question.	Students accurately measure the image and object distances from the lens for real images formed by the converging lens. (Optional) Students use a converging lens in conjunction with a diverging lens to form a real image, and accurately measure the image and object distances for both lenses.
5.1 The student can *analyze data* to identify patterns or relationships.	Students relate the information from the best-fit line to the graph and use that information to calculate the focal length of the converging lens.
5.2 The student can *refine observations and measurements* based on data analysis.	Students adjust the object position (independent variable) based on observations of the corresponding image positions in order to obtain data over as wide a range of positions as possible.

[**NOTE:** Students should be keeping artifacts (lab notebook, portfolio, etc.) that may be used as evidence when trying to get lab credit at some institutions.]

Equipment and Materials

Per lab group (two to four students):

- Light source such as a clear lamp with a filament or a candle (either wax or battery operated)
- Converging lenses, focal length 15–25 cm
- Lens holders
- Metersticks
- Index cards for screen (5 × 7 inches or larger)
- (Optional) Diverging lens

Timing and Length of Investigation

- **Teacher Preparation/Set-up:** 15 minutes

  The set-up time depends on how long it will take you to retrieve the equipment from storage and make it available to the students.

- **Student Investigation:** 90 minutes

  This time is for an introductory presentation of the question, and for your students to design the experiment and collect the data.

- **Extension (Diverging Lenses):** 60 minutes

  This time is for students to work through a double-lens system to create a real image. They will need this much time to design their setup, make their measurements, and calculate the focal length of the diverging lens.

- **Postlab Discussion:** 45 minutes

  The following day, you can engage students in a discussion of the precision and accuracy of their results. In particular, you can tell them the focal length specifications for each group's lenses, have them compute a percent difference, and then engage in a discussion about the sources of uncertainty.

- **Total Time:** 3.5 hours

Safety

If wax candles are used as a light source, be aware of the danger of students' hair or clothing catching fire. Students with long hair should pull it back and fasten it so it doesn't get near the candle. Students should secure any loose clothing (shawl, sweater, etc.) so it doesn't hang near the flame. Other than that, there are no safety concerns with this lab. General lab safety guidelines should always be observed.

Preparation and Prelab

This investigation should follow treatment of Snell's law and refraction. You can transition to convex lenses by having students consider how a ray is refracted at each surface of a converging lens. Demonstrate for students how a ray parallel to the principal axis is converged through the focal point. Inexpensive laser levels (or other laser pointers) can be used to create a bright ray that students can see as the ray is refracted through a piece of Plexiglas shaped like a converging lens. Several of these rays will converge to a focal point on the far side of the Plexiglas. Alternatively, if available, a Black Board Optics kit may be used or ray boxes that produce parallel rays, although these produce smaller effects and need to be used in smaller groups.

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Give students instruction in ray tracing. Demonstrate ray tracing for students for a converging lens forming both a real image and a virtual image and for a diverging lens; then provide them the chance to practice on their own. You can refer them to “Lenses” (see Supplemental Resources), where they can move the object and see the corresponding location of the image for both kinds of lenses.

Next, either derive the lens equation for them from geometric principles and a ray diagram, assign them to read it from the text, or just simply present them with the equation and the conditions for its application. Students should be given several practice problems for homework in the application of the lens equation.

Once students are familiar with ray tracing and the lens equation and its use in problem solving, you can then present them with the question and the converging lens.

The Investigation

Each group designs an experiment to determine the focal length of a converging lens by taking measurements and creating a graph. Lenses with focal lengths between 15 cm and 25 cm work well. You can either provide students the equipment they need, or have them decide what they will need and ask for it. If they get stuck at this initial stage, ask them which equations they know that relate to focal length, and prompt them to explain that equation and all the terms in it. This should help them zero in on what they want to measure.

Students will hopefully decide to form real images with a light source and screen, and then measure the image distance and object distance. Ask them to identify what the independent variable and dependent variable are. Once they have done this, ask them to consider how to make multiple measurements of the dependent variable for each chosen value of the independent variable. You may need to suggest to them that each member of their team should independently locate the image and record the image distance for each object position. That way they will have several independent values of the dependent variable. They can then take the average of these values, and use them to determine the uncertainty in their measurements. The average of the image distance should be used for the graph. Students should make measurements of the dependent variable (image distance) for at least five different values of the independent variable.

Once students have completed the data collection portion of this lab, they should consider how to plot the data in order to determine the focal length. As discussed in the section on common student challenges, this is usually difficult for students and they may need significant guidance.

One approach is to graph the inverse of the image distance on the vertical axis and the inverse of the object distance on the horizontal axis. This should produce a straight line with a slope equal to –1. The focal length can then be found as the inverse of the *y*-intercept. However, since the lens equation is symmetric with respect to object and image distance, students can reverse which quantities are plotted on each axis, and the inverse of the focal length will still be the intercept. In fact, it will be both the *x*-intercept and the *y*-intercept.

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Another approach is to algebraically rearrange the lens equation and plot the product of the image distance and the object distance on the vertical axis, and the sum of the two on the horizontal axis. When plotted this way, the focal length is the slope of the graph.

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$$

$$\frac{1}{f} = \frac{d_i + d_o}{d_i d_o}$$

$$d_i d_o = f(d_i + d_o)$$

A third approach would be to have students plot the image distance versus the object distance and use the asymptotes to find the focal length. When the data is plotted this way, the graph is asymptotic both to the vertical line at $d_o = f$ and to the horizontal line $d_i = f$. If data of a great enough range is taken, these asymptotes will be easier to determine. However, this is not the most precise way to find the focal length, and students, after attempting this method, should be encouraged to consider one of the two previous methods.

Extension

Often there is one group of students that finishes far ahead of the other groups. You can ask this group to find the focal length of a diverging lens, which, by itself, only produces virtual images. Alternatively, you could ask all the students to consider how they could find the focal length of a diverging lens. To produce a real image, a system of two lenses, one converging and one diverging, is needed. Each group of students could determine the exact details of the how they would arrange two lenses to create a real image; the location and size of which can be measured and used to calculate the focal length of the diverging lens.

Typically, the simplest arrangement of lenses for this task is to place the object next to the diverging lens, a converging lens on the other side of the diverging lens, and the screen (where the real image will be focused) on the far side of the converging lens, as shown in Figure 1. The focal points of the diverging lens are labeled *f*, and the focal points of the converging lens are labeled *f'*. The virtual image formed by the diverging lens acts as the object for the converging lens. The final image distance is measured from the screen to the converging lens. Given the focal length of the converging lens, students can then calculate the location of the intermediate image formed by the diverging lens. This intermediate image is serving as the object for the converging lens.

Using the object distance (for the converging lens) and the distance between the lenses, the students can calculate the location of the intermediate image. Once the students know the location of the intermediate image formed by the diverging lens, they can use that, and the original object distance from the diverging lens, to calculate the focal length of the diverging lens.

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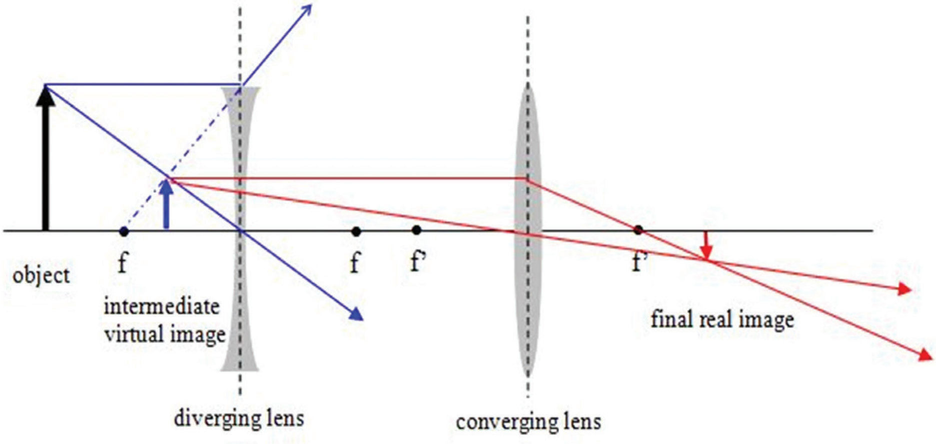

Figure 1

In drawing a ray diagram for a double-lens system, it is typical to simply address the image formation process by one lens at a time. In Figure 1, the rays shown in blue are refracted first by the diverging lens to form the intermediate virtual image. The rays that were traced to form this image are not useful for finding the final, real image. So the intermediate virtual image is used as the object for the converging lens. The rays shown in red, parallel to the principal axis and through the center of the converging lens, are drawn from the intermediate image to the converging lens, and shown as NOT refracted at the diverging lens. This is a convention used for the purpose of using the ray-tracing process with two lenses. In fact, those rays originate with the object (path not shown), strike the diverging lens at the location shown, and then follow the parallel or center paths (between the lenses) toward the converging lens.

It probably is not practical to repeat the process of drawing the graph, but students could be asked to move the lenses or the object until a different real image is formed and repeat the calculation to verify their first result.

Common Student Challenges

Students will probably easily understand that they need to use the lens to create an image, and then measure the object distance and the image distance. They may need to be reminded that when using the lens equation, these distances are measured from the lens, not from each other. They may also need to be encouraged to make sure that their image is well focused and sharp.

Students probably will also need to be reminded to measure the image distance several times and take an average. Students are not very good at taking multiple independent measurements, and thus will need to be reminded of techniques they can use to do this, such as having each group member take their own measurement of the image distance.

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They might also have some difficulty determining what to plot on each axis of their graph and how to use it to determine the focal length. Each group should be allowed to struggle with this a bit, and you should refrain from having a whole-group discussion before the groups get a chance to plot their graphs. If they struggle, encourage each group to write out the equation relating image, distance, object distance, and focal length (i.e., the lens equation) and to write out the generic form of the equation for a straight line ($y = mx + b$). They should then be encouraged to consider what the dependent variable is, what the independent variable is, and make comparisons between the two equations.

Analyzing Results

If you decide to give students lenses all with the same focal length, you might want the groups to post their results on whiteboards to share in a class discussion. Or you can task each group (or individual) to prepare a formal written report of the procedure, diagrams (both of the physical set up used and ray tracing diagrams), data, and conclusions. The use of multiple representations (mathematical, graphical, and ray diagrams) helps reinforce the concepts that students are studying in this lab. Perhaps you will choose to have a class discussion and then have each group turn in a formal report with their results.

Students should be asked to compare their measured value of the focal length to the actual focal length of the lens (as specified by the manufacturer). If all students are given lenses of the same focal length, then they should be asked to compare their results to other groups' results and comment on which group's result was closest to the actual value. This comparison of the groups' various values should lead to a comparative discussion of the techniques used in measurements. Students should discuss what measurement techniques led to more precise and accurate measurements.

Next, lead them in a discussion of the uncertainty in their image distance measurement. Some guiding questions you could ask include:

- What was the measured uncertainty in the image distance?
- How much variation was there between the measurements made by each member of the group?
- What is the percent uncertainty in the image distance measurement?
- What is the percent uncertainty in the object distance measurement? (They may have to estimate this if they only measured it once.)
- Is the true value of their focal length (as specified by the manufacturer) within the range indicated by their uncertainties? In other words, if their image distance measurement has an uncertainty of 5 percent, is their measured value within 5 percent of the true value?
- What could be done to improve the precision of their measurements?
- How do the measurement uncertainties affect the value of the focal length? For example, if the image distances measured are all too small, what effect would that have on the calculation of the focal length?

If you want to further explore the behavior of lenses, you could introduce "Lens-Maker's Formula" (see Supplemental Resources) and discuss how the manufacturer creates a lens of a specified focal length, and what the uncertainties are in this focal length.

Assessing Student Understanding

After completing this investigation, students should be able to:

- Articulate the difference between a virtual and a real image;
- Use a converging lens to form a real image;
- Articulate why they plotted the inverse of the image distance as a function of the inverse of the object distance, referring to the lens equation;
- Justify why the focal length is the inverse of the intercept; and
- Estimate the uncertainty in all measurements that were made several times.

Assessing the Science Practices

Some of the practices below need to be observed in a formative manner, as students are performing the measurements. For example, Science Practices 4.1 and 5.2 are best assessed while students are in the lab in a formative way rather than as summative assessments.

Science Practice 1.4 The student can *use representations and models* to analyze situations or solve problems qualitatively and quantitatively.

Proficient	Accurately draws a ray diagram to locate the image for both a converging and diverging lens, indicating the correct refraction for each ray and accurately locating the image; identifies when an image is real or virtual.
Nearly Proficient	Accurately draws a ray diagram for a converging lens but has difficulty with a diverging lens; accurately locates the image if it is real, but has difficulty ray tracing to find virtual images.
On the Path to Proficiency	Accurately draws the rays for a converging lens but does not accurately locate the image.
An Attempt	Draws an incomplete or incorrect ray diagram.

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Science Practice 2.2 The student can *apply mathematical routines* to quantities that describe natural phenomena.

Proficient	Uses the lens equation to determine which quantities should be plotted to determine the focal length, and accurately calculates the focal length as the inverse of the intercept.
Nearly Proficient	Plots the inverses of the image and object distances, but uses the slope to calculate the focal length.
On the Path to Proficiency	Plots the image distance and the object distance, and uses a data point to calculate the focal length.
An Attempt	Plots an incomplete or incorrect graph.

Science Practice 4.1 The student can *justify the selection of the kind of data* needed to answer a particular scientific question.

Proficient	Explains why she/he is measuring the distance from the object to the lens and the image to the lens, and the justification is clear, easy to follow, and correct in terms of the lens equation; explains how the data will be plotted to answer the question and provides mathematical models that are justified; includes a discussion of assumptions inherent in each model and what to do to validate them.
Nearly Proficient	Explains why the inverse of the image and object distances are plotted, and knows to calculate the inverse of the slope, but there is no discussion of the assumptions inherent in the model.
On the Path to Proficiency	Describes that the image distance and object distance need to be measured, but does not articulate how to use them to calculate the focal length.
An Attempt	Cannot accurately measure the image or object distances.

Science Practice 4.3 The student can *collect data* to answer a particular scientific question.

Proficient	Accurately measures object distance and image distance for the converging lens; data spans an adequate range for both the independent and dependent variables; the student makes multiple measurements of each image distance and averages the values.
Nearly Proficient	Accurately measures image and object distance; the range of the data is too small or there are too few data points collected; or the student only made one measurement of each image distance.
On the Path to Proficiency	Measurements of the image distance and object distance are imprecise or inaccurate.
An Attempt	Cannot accurately measure the image distance from the screen to the lens.

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Science Practice 5.1 The student can *analyze data* to identify patterns or relationships.

Proficient	Accurately plots the inverse of the image distance vs. the inverse of the object distance; draws a best-fit line to the data, and extrapolates the best-fit line until it intersects the axis; calculates the inverse of the intercept and reports this as the focal length.
Nearly Proficient	Accurately plots the inverse of the image distance vs. the inverse of the object distance, and draws a best-fit line; uses the slope to calculate the focal length.
On the Path to Proficiency	Plots the dependent variable vs. the independent variable, and draws a best-fit line to the data (which is not straight), but does not know how to calculate the focal length
An Attempt	Cannot use the image-distance and object-distance data to get a focal length.

Science Practice 5.2 The student can *refine observations and measurements* based on data analysis.

Proficient	Observes if data points seem to be inconsistent with the rest of the data set or do not lie near a best-fit line through the rest of the data; repeats any observations that seem to be inaccurate; accurately locates a clearly formed image; estimates the uncertainty in the object and image distance measurements, and explains how they impact the calculation of the focal length.
Nearly Proficient	Doesn't address wildly deviant data points in the drawing of the best-fit line; some of the images formed are slightly out of focus; estimates the uncertainties in the image and object distances, but is not clear on how they affect the focal length calculation.
On the Path to Proficiency	Cannot explain how to determine if data needs to be retaken or observations need to be refined; many of the images are slightly out of focus; cannot estimate the uncertainty in the image or object distance measurements.
An Attempt	Cannot adjust the position of the screen until a clear, crisp image is seen.

Supplemental Resources

“Geometric Optics 2.05.” PhET. University of Colorado Boulder. Accessed September 1, 2014. http://phet.colorado.edu/sims/geometric-optics/geometric-optics_en.html. [*This simulation allows the students to move the object and show where the image location is. In addition, it allows them to change the refractive index of the material in the lens, and change the radius of curvature of the lens as well as the position of the object above or below the principal axis of the lens.*]

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"Lens-Maker's Formula." HyperPhysics. Georgia State University. Accessed September 1, 2014. http://hyperphysics.phy-astr.gsu.edu/hbase/hph.html.

"Lenses." Northwestern University. Accessed September 1, 2014. http://groups.physics.northwestern.edu/vpl/optics/lenses.html. [*This Java applet allows the students to move the lens and the object and show the primary rays refracted through the lens (either converging or diverging) to form the image. Real images are shown in blue, and virtual images are shown in green*.]

Maley, Tim, Will Stoll, and Kadir Demir. "Seeing an Old Lab in a New Light: Transforming a Traditional Optics Lab into Full Guided Inquiry." *The Physics Teacher* 51, no. 6 (2013): 368–371.

"The Physics of Optics." Annenberg Learner. Video, 54:32. Accessed September 1, 2014. http://www.learner.org/resources/series126.html.

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AP Physics 2 Investigation 8:
The Particle Model of Light

What interactions of light and matter require a photon model of light for explanation?

Central Challenge

In this investigation, students use circuits with light-emitting diodes to gather evidence that supports the particle perspective of the interactions between light and matter. Students gain understanding of the necessity of a discrete model of the interaction between matter and radiation, the phenomena from which that necessity arose, and the experimental evidence supporting the model.

Background

Some properties of interactions between light and matter are better explained when light is described as a wave, and some properties of these interactions are better explained when light is described as a particle. Light-emitting diodes (LEDs) have become popular for experiments in which the photovoltaic effect can be used to develop evidence for the particle model of light (see "A Laboratory Investigation of Light-Emitting Diodes" in Supplemental Resources). The historical approach was to determine the maximum kinetic energy of an electron that was emitted from a conducting material upon the absorption of a photon by obtaining the measurement of the potential energy [as the product of] the fundamental charge of the electron and the electrical potential threshold at which current vanishes. Equivalent evidence supporting the particle model can be obtained by measuring the minimum potential at which photons are emitted from a semiconducting material and current appears. From both perspectives, explanation of the process requires a model of discrete events whose energies are proportional to the frequency of light absorbed or emitted ($E_{photon} = hf$).

Photons are individual energy packets of electromagnetic waves, with $E_{photon} = hf$, where h is Planck's constant and f is the frequency of the photon. Evidence for discrete energy packets is provided by a frequency threshold for electron emission due to the photoelectric effect. The threshold frequency (f_o) provides the minimum amount of energy necessary to free electrons from a solid to a point immediately outside the solid. This minimum amount of energy is called the work function (ϕ). When a material with work function (ϕ) absorbs photons of energy (hf), the energy above this threshold ($hf - \phi$) is the maximum kinetic energy the emitted electrons can have (K_{max}). Above the threshold, emission (i.e., the number of electrons freed) increases with the intensity of the incident radiation (since intensity is proportional to the number of photons). The photoelectric effect is described with energy conservation during the process of photon absorption ($E_{photon} = hf = \phi + K_{max}$).

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Since ϕ is the minimum amount of energy necessary to free an electron, $E_{photon} - \phi$ represents the most kinetic energy an electron can gain. If more energy is required to free it, it would have less kinetic energy. At the threshold frequency (f_0), the lowest frequency that can result in electron emission, the photon energy is equal to the work function ($E_{photon} = hf_o = \phi$).

Electric potential energies in semiconductors have a gap between a ground state referred to as the valence band and a higher energy state called the conduction band. The work done by an external electric potential can excite electrons over this energy gap (E_g). In the photovoltaic effect, when an electron returns to the ground state, radiant energy is emitted ($E_{photon} = hf_o = \phi_g$).

The common use of conservation of energy and the types of energy involved in the photoelectric and photovoltaic effects are used in this set of activities to explore interactions between light and matter and develop an understanding of the photoelectric effect. These comparisons are somewhat oversimplified but reasonable at an introductory level.

Real-World Applications

The photoelectric effect occurs in situations where radiation striking materials causes those materials to emit electrons, leaving the materials with a net positive charge. Spacecraft exposed to intense solar radiation may develop a net positive charge on their solar side by the same mechanism. The opposite side of the craft may develop a negative charge due to exposure to plasma, leading to the possibility of an electrical discharge from one side to the other — or through the craft — potentially causing damage to delicate instruments inside the spacecraft (see Shu T. Lai's book in Supplemental Resources).

Night vision goggles use the photoelectric effect to amplify images in dim light. As photons hit a semiconducting material, electrons are emitted and directed by an electric field into a photoamplifier.

Photovoltaic devices are benefiting human society and promise even greater improvements in the future. Solar panels apply the photovoltaic effect to generate power, helping to reduce atmospheric carbon pollutants. Local power generation and storage using photovoltaics may someday help relieve high energy demand in urban centers and provide a sustainable technology for personal transportation.

Inquiry Overview

In this two-part investigation, inquiry-based instruction takes place through the integration of group discussions with student-run data collection and analysis activities. In Part I, an engagement phase is used to demonstrate the particle perspective based on observations of the photoelectric effect. Then, in Part II, a student-directed investigation involving LEDs provides an opportunity for data collection using the photovoltaic effect that leads students to an understanding of the relationship between photon energy and frequency. The suggested extension provides a more advanced activity that increases the precision of the measurement of the threshold electric potential.

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In Part I of the investigation, a series of interactive demonstrations enables you to engage students in a conversation that will help them develop concepts related to the photoelectric effect. Students then work in small groups to process this information and use it to develop a plan for the investigation. In Part II, groups of students build a circuit using inexpensive components to observe, gather data, and graph the data relating energy of electrons to frequency of light. Analysis of the data will lead students to develop their written and verbal discussions that support the particle model of light.

The students' role in data collection depends on the choice of instrumentation. Using simple methods and common lab equipment (as described in the Investigation section), students can setup their own circuit to measure the threshold potential difference across the LEDs. Students can then observe the threshold potential by adjusting the current supplied by a battery using a potentiometer (or by a variable DC power supply) while measuring the potential difference across the LED with a multimeter. Using an expensive commercial apparatus (such as a photovoltaic measurement kit) will likely produce better results, but will involve students less in the design of the circuit to setup the experiment. The advantage of having students setup their own circuit is that it provides an opportunity for students to review what they have learned about circuit components and circuit connections, and to take a larger role in deciding how to make those connections (e.g., remembering to connect the voltmeter in parallel across each LED).

Connection to the AP Physics 2 Curriculum Framework

Big Idea 5 Changes that occur as a result of interactions are constrained by conservation laws.

Enduring Understanding	Learning Objectives
5.B The energy of a system is conserved.	**5.B.8.1** The student is able to describe emission or absorption spectra associated with electronic or nuclear transitions as transitions between allowed energy states of the atom in terms of the principle of energy conservation, including characterization of the frequency of radiation emitted or absorbed. (Science Practice 1.2)

Big Idea 6 Waves can transfer energy and momentum from one location to another without the permanent transfer of mass and serve as a mathematical model for the description of other phenomena.

Enduring Understanding	Learning Objectives
6.F Electromagnetic radiation can be modeled as waves or as fundamental particles.	**6.F.3.1** The student is able to support the photon model of radiant energy with evidence provided by the photoelectric effect. (Science Practice 6.4) **6.F.4.1** The student is able to select a model of radiant energy that is appropriate to the spatial or temporal scale of an interaction with matter. (Science Practices 6.4 and 7.1)

[**NOTE:** In addition to those listed in the learning objectives above, Science Practices 1.4, 3.2, 4.2, 4.3, 5.1, and 6.2 are also addressed in this investigation.]

Skills and Practices Taught/Emphasized in This Investigation

Science Practices	Activities
1.1 The student can *create representations and models* of natural or man-made phenomena and systems in the domain.	Students reason from observations to construct a model of the interaction between light and a conductor in contact with an electroscope, which they will discuss in their lab report and in the postlab presentations. They construct energy diagrams to describe the electron energies during the formative assessment.
1.2 The student can *describe representations and models* of natural or man-made phenomena and systems in the domain.	Students describe the interaction between light and the electrons in a metal in contact with the electroscope using energy diagrams. They also use a circuit schematic diagram to setup a circuit and describe how potential difference will be measured for the LEDs.
1.4 The student can *use representations and models* to analyze situations or solve problems qualitatively and quantitatively.	Students use energy diagrams to analyze the behavior of photoelectric and photovoltaic processes.
3.2 The student can *refine scientific questions.*	Students refine a broad question concerning the interaction of light with electrons by considering the difference between the behaviors of positively and negatively charged electroscopes illuminated by mercury-vapor lamps.
4.2 The student can *design a plan* for collecting data to answer a particular scientific question.	Students design a test of their claim that a positively charged electroscope will not behave in a manner that is consistent with the removal of electrons.
4.3 The student can *collect data* to answer a particular scientific question.	Students construct a circuit and use that circuit to collect data on the threshold potential difference of LEDs.

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Science Practices	Activities
5.1 The student can *analyze data* to identify patterns or relationships.	Students identify a pattern of increasing threshold potentials with increasing wavelength, both by observation and by graphical analysis of data.
6.2 The student can *construct explanations of phenomena based on evidence* produced through scientific practices.	Students construct a particle-based model of light that incorporates the particle and use it to explain their observations.
6.4 The student can *make claims and predictions about natural phenomena* based on scientific theories and models.	Students develop a model based on data and the application of the principle of energy conservation. Students then apply a particle model of light to predict system behavior and support their prediction using evidence obtained from a demonstration in which an electric current is generated by the interaction between light and a conductor.
7.1 The student can *connect phenomena and models* across spatial and temporal scales.	Students differentiate the models of light used to explain diffraction and the photoelectric effect, summarizing the essential evidence supporting each model.

[**NOTE:** Students should be keeping artifacts (lab notebook, portfolio, etc.) that may be used as evidence when trying to get lab credit at some institutions.]

Equipment and Materials

Part I: Demonstration of Photoelectric Effect

- Electroscope (with a conducting surface on which metals can be placed)
- Plastic rod or cylinder, such as a disposable pen or a piece of PVC pipe
- Glass rod
- Metal plates (zinc, copper, and steel) and steel wool
- Mercury-vapor lamp or ultraviolet light source (The lamp must emit UV-light below 280 nm, the threshold set by the work function of copper metal. Some UV lamps may emit only UV-b and UV-a wavelengths and so cannot be used. These include fluorescent tubes used in tanning beds and black lights. Lamps that are described as *germicidal* will work.)
- Emery cloth
- Fur, felt, or wool cloth to transfer negative charge to the plastic rod or cylinder
- Silk or equivalent material to transfer negative charge from the glass rod

Part II: Student Investigation of Photovoltaic Effect

For prelab discussion:

- Power supply with variable potential difference or battery with potentiometer
- Small incandescent bulb with base

Per lab group (three to four students):

- Light-emitting diodes (red, green, and blue), available from science supply catalogs or electronics supply stores
- 2–6 volt variable DC power supply or a battery pack with three or four C or D cells
- Alligator clips and jumper wires
- Potentiometer or trimpot (to use with batteries to vary potential difference if a variable DC power supply is not used)
- Multimeter
- (Optional) Breadboard (connections can be made directly without it, if preferred)
- (Optional) Cardboard tube from an empty paper towel roll to reduce ambient light while taking readings for LED
- (Optional) Several small resistors (330–660 ohms), as needed, to reduce current in the circuit when a power supply is used
- (Optional) Photovoltaic measurement kits, available from many online sources

Timing and Length of Investigation

- **Teacher Preparation/Set-up:** 15–20 minutes

  This time is needed to prepare the demos and set out whiteboards and equipment.

- **Demonstration (Part I):** 45–60 minutes

  Engagement phase: 15–20 minutes

  Model building, small and large group discussions: 30–40 minutes

- **Student Investigation (Part II):** 45–60 minutes
- **Postlab Discussion:** 30 minutes
- **Total Time:** approximately 2.5–3 hours

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Safety

In Part I, a mercury-vapor lamp (or other UV source) is used, if available, for the demonstration. The lamp must be uncoated so that ultraviolet light is emitted. Great caution should be taken when using the mercury-vapor lamp (or any ultraviolet source) to minimize students' exposure. Students should wear safety goggles that can protect against UV damage to the eyes, and be advised to avoid looking directly into the UV source.

If variable DC power supplies are used in Part II, students should have experience with these or be provided with instruction in safe and proper use. There is little safety concern if batteries are used instead.

There are no specific safety concerns for this lab beyond what has been articulated above. General lab safety guidelines should always be observed.

Preparation and Prelab

A formative assessment should be given to students to assess readiness, and to provide you information that you can subsequently refer to in the development of a particle model of light. The entire formative assessment and small-group discussion may require one class period to complete.

The first two questions of the formative assessment address the essential features of the potential difference in terms of familiar representations of the photoelectric effect and energy conservation. Recent research on the use of multiple representations of energy is applied in many simulations on the PhET website (see Supplementary Resources for two examples).

The third question reminds students that a wave model of light is necessary to describe diffraction, and the last three questions address the essential features from which students will be asked to construct the particle model of light and contrast it with the wave model.

Formative Assessment

Ask students to respond to the following prompts:

1. Describe in terms of an energy diagram the forms of energy of a charged particle in an electric field.
2. Use a diagram to describe the energy of an electron injected into the space between two charged plates.
3. Represent with a sketch how, due to wave interference, light from a laser striking a pair of slits leads to the formation of maxima and minima in the intensity of light on a distant screen.

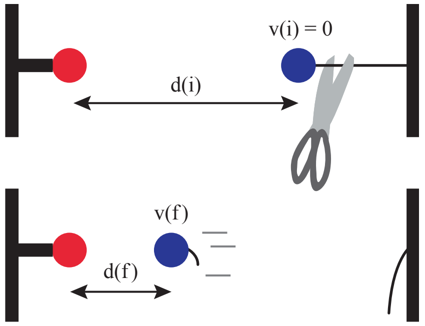

Figure 1

4. In Figure 1 a small ball with a positive charge is strongly attracted to another ball with a negative charge and a mass (*m*). The positively charged ball is fixed on the end of an insulating rod. The negatively charged ball is attached to an insulating thread. The thread is cut. Describe changes in electric potential energy, kinetic energy, and total energy when the thread is cut using an energy diagram with at least three states: before the thread is cut, after the thread is cut but before the balls collide, and just before the balls collide.

5. An electric potential difference ΔV is used to accelerate an electron through a small opening in a positively charged plate. The electron is shot through the opening into a space between this plate and a second, parallel plate that is oppositely charged. Draw a picture of the system. Using two energy diagrams, describe changes in potential, kinetic, and total energy of the electron when (a) $K_{initial} < e\Delta V$ and (b) when $K_{initial} > e\Delta V$. As one state in your energy diagram, consider the moment when the electron just emerges from the small opening. If there is a time when the kinetic energy of the electron vanishes, include that state in your energy diagram. If the electron arrives at the second, negatively charged plate, include that moment as a state in your energy diagram.

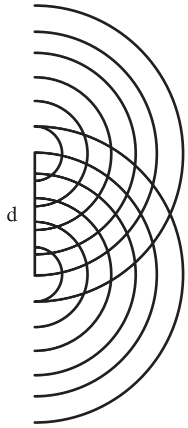

Figure 2

6. Light from a laser strikes two slits separated by a distance that is small but larger than the wavelength of the light. Using the diagram in Figure 2, identify interactions leading to at least five bright spots that appear on a screen across the room.

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The Investigation

Part I: Demonstration of Photoelectric Effect

The teacher demonstration performed in this section is used to develop concepts in readiness for students to design an experiment in Part II. In a large class an overhead projector or USB camera should be used to allow all students to participate. It's best if you take a Socratic approach — asking questions at key points and allowing students to think about their responses before answering. Occasionally, give students 2 minutes or so to discuss their answers with a partner; then randomly select students to give their answers.

Stroke a plastic cylinder with fur, wool cloth, or felt and use it to charge an electroscope with negative charge, causing the leaves to spread apart. Direct light from a mercury-vapor lamp onto a plate contacting the leaves so the spread of the leaves decays slowly. Place a sheet of zinc on the plate to repeat the charging process. When the lamp shines on the zinc sheet, the repulsion between the leaves decays completely in just a few seconds. Students have likely observed repulsion in the electroscope during their study of electrostatics and with a few prompts are able to recall that this repulsion can be explained in terms of the distribution and interactions of electrons. [**NOTE:** If students have not observed the charging of an electroscope, this should be a prerequisite activity or should be researched in a textbook or online.]

The ability to construct an explanation is a skill this lesson is intended to support. Elicit the following initiating question from the class: "Why does the light cause the rearrangement of electrons in the electroscope?" When the question has been posed by a student, involve others in the evaluation of the quality of the question. If students do not pose this question, pause to review earlier experiences with charged objects and electroscopes. It is important that students feel ownership of the development of the model toward which this instructional sequence is directed. [**NOTE:** It is *not* the intention here that students actually use the ultraviolet light or mercury-vapor lamp to do the follow-up investigations. At all times, you perform the actual operations proposed by students, with appropriate caution for use of the ultraviolet light or mercury-vapor lamp.]

Students will shortly come to a consensus that the energy of the light is somehow responsible. This is an opportunity to replace *energy* with *intensity*. A rationale for identifying intensity as opposed to energy as an initial composing element of the model is that their model should be based on observable evidence. Confirm their thinking by grounding the electroscope and decreasing the intensity by raising the lamp a few centimeters. The decay is much slower.

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The lamp has only a small effect when held about 20 cm above the plate. By lowering and then raising the lamp, students will be able to observe oscillations in the leaves that are induced by changes in the distance between the lamp and the plate. The intensity of light from the mercury-vapor lamp is confirmed as a factor. However, shining an intense halogen flashlight on the zinc plate can then be shown to have no effect.

Substitute other metals for zinc and repeat the process. The work function for zinc is small. When zinc is replaced with sheets of steel and copper, the decay of the repulsion between the leaves is much slower. In each case, an emery cloth should be used to remove any corrosion on the metal that might limit conduction at the surfaces of the metals. You might want to ask students to consider how the contact between the metal surfaces affects the experiment. Further investigation into the role of contact between the metal plate and the electroscope surface can be pursued by using a wad of coarse steel wool instead of a metal plate on top of the electroscope. The decay of the repulsion between the leaves is slower when the shape of a wad of steel wool is varied to change the area of contacting surfaces between the electroscope and the steel wool. When the wool is flattened on the surface, the decay of the repulsion occurs more quickly. Guide students towards the conjecture that electrons are being removed from the electroscope through a process involving the interaction of light and metal.

Now have students work in small groups to revise the vague question, “How can we know if the light is having an effect on electrons?” The goal is to have students refine this question into the testable question, “How can evidence be obtained supporting the conclusion that what you are seeing when light from the mercury-vapor lamp shines on the metal involves electrons being removed from the electroscope?”

Visit the groups and gradually, as necessary, remind them that electroscopes can also be positively charged by using a glass rod that has been rubbed with a silk cloth. Each group should make a claim for which they design a method of testing how the electroscope could be positively charged. When the groups have arrived at a testable question, their task is to develop a strategy to evaluate the question. Share with them the rubrics for this task (see the rubric for SP 3.2 and 4.2 in the Assessing the Science Practices section).

Questioning can guide students toward the idea that if the leaves of a positively charged electroscope become less repulsive or are not affected, evidence is provided that electrons are being removed from the metal. Positive charging is often more difficult to produce than negative charging. [**NOTE:** If this activity is done when humidity is high, a hair dryer can be used to gently dry out the apparatus prior to the demonstration.] Perform the experiment for positive charging as a demonstration for the class, using methods suggested by student groups or using what you have readily available, and verbally reinforcing other successful methods suggested by groups.

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This is a good point to begin the development of a model as a whole-class discussion. Ask students to recall the sequence of questions on the formative assessment. Ask them to describe a situation in which an electron that is released into the space between two oppositely charged plates stops moving towards the repulsive plate in terms of the energy diagram. Guide them to define the initial state where the electron emerges from the hole and the final state as the point at which the electron stops. Help them to recall that as the electron is slowed by the field between the plates, work is done on the electron by the field. The sign of the work is negative because energy is being removed from the electron. Ask them to confirm the sign of the quantity $e\Delta V$ by setting the electric potential at the positively charged plate equal to zero and by identifying the direction of the electric field.

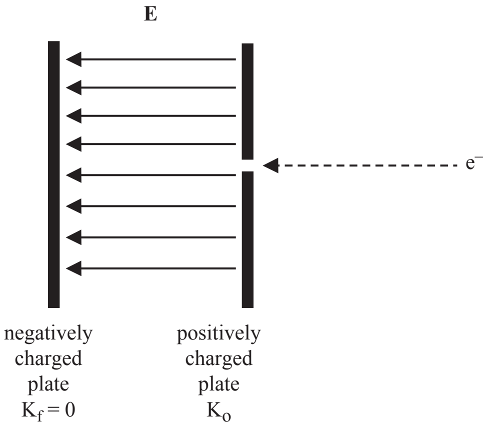

Figure 3

In this way the discussion is guided toward the role of light from the mercury-vapor lamp in doing work on electrons in the zinc plate. Guide discussion with the following questions: "If electrons are being removed from the electroscope, is the work done on the electrons positive or negative? If the electrons in the metal are initially at rest, what change in the system does work produce?"

Have students work in small groups to construct an energy diagram to answer the question, using whiteboards to construct the representation. (See the rubric for SP 6.4 in the Assessing the Science Practices section.)

Part II: Student Investigation of the Relationships between Electron Energy and Light Frequency in a Circuit

The focus now shifts towards the dependence of electron energy on frequency. The learning objective is the ability to construct explanations, so it is your responsibility to support students' construction. A good place to start this segment of the model development is to return to the demonstration apparatus and compare the effects of the bright halogen flashlight and the not very bright mercury-vapor lamp. To segue to data collection, use the idea that the energy of electrons removed from the zinc sheet can't be measured with the demonstration device and that a better detection device than the electroscope is needed. You might want to demonstrate first that increasing the potential difference in a simple circuit with an incandescent light bulb will cause the bulb to shine at a certain point; below a certain potential difference, the bulb won't shine.

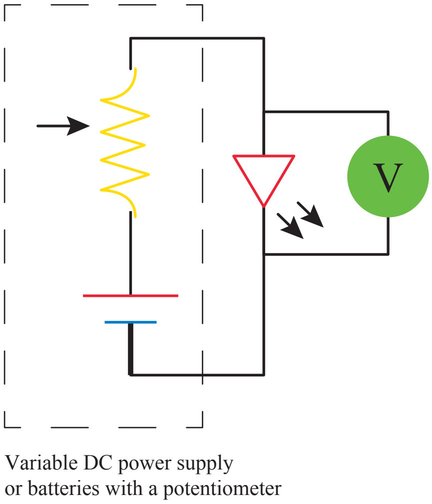

Figure 4: Schematic LED circuit

Data Collection

Sketch the circuit on the board as shown in Figure 4, consisting of a battery, a potentiometer (variable resistor), and an LED. The circuit could also consist of a variable DC power supply, a small resistor to reduce current, and an LED. [**NOTE:** If the power supply goes to zero, which some do, the resistor isn't even necessary except to keep the current low enough to prevent burning out the LEDs.] Whatever is used, the idea is to be able to provide a potential difference to each LED so that the LED can be turned off and on without burning it out.

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Remind students that LEDs have polarity with the shorter lead closest to ground and the longer lead connected to the higher potential side of the battery or power supply. [**NOTE:** Though semiconductors are not part of the curriculum, students are often curious about how LEDs work, since they are common now in everyday usage. You might want to encourage students to pursue their own research on LEDs — either to append to their laboratory analysis or to satisfy their own curiosity. The mechanism of how LEDs work, however, is not part of this investigation.]

By adjusting the electric potential difference across the LED using either a variable voltage power supply or a potentiometer, students can use the reading on the voltmeter to record the minimum electric potential difference at which the LED emits light. A section of PVC pipe or cardboard tube placed over the LED in a darkened room during the determination of when the bulb is at its shutoff point will eliminate bias due to the ambient light. Encourage the groups to achieve precision with at least three measurements made by different group members.

Provide information about the wavelength of each LED color to students. (This can be acquired by referring to markings on the package for the LED or by doing an online search for "LED wavelengths.") Although the method of measurement is prescribed, students can play a role in the experimental design by constructing a data table in which the entire class may record their observations. In this way, each group has access to a large data set for analysis.

To compare the reproducibility of the measured threshold potentials among the three LED colors, students should record their data and compare with the class data. The red light will probably show the greatest variability and the blue light the least. [**NOTE:** The package markings for LED wavelengths are probably accurate, but many LEDs have a small bandwidth around the average. This introduces a systematic error into the investigation.]

Students may choose to do a cross-check of LED wavelengths by performing a diffraction experiment with each LED to determine wavelength. This will depend upon the motivation and skill of each group. One such procedure is described in the University of Wisconsin-Madison's LED diffraction lesson (see Supplemental Resources).

After a group has completed their measurements, ask them to contrast their observations with the process observed for the mercury-vapor lamp. Guiding questions such as the following can be provided to motivate further analysis by students:

- In which of these systems is light absorbed and in which is light emitted?
- What work is done on the electrons in the electroscope by the mercury-vapor lamp, and what are the consequences?
- What work is done on the electrons in the circuit by the power supply, and what are the consequences?

Ask each group to whiteboard their description and support it with an energy diagram. You can then confirm the accuracy and thoroughness of the representations, using the rubric provided (see the rubric for SP 1.4 in the Assessing the Science Practices section). When the group has identified the measurement of the potential difference at which the LED shines, it is referred to as the *threshold potential*.

Extensions

If students have had previous experience in recording measurements with uncertainties or graphing with error bars, the experiment in Part II lends itself well to that statistical examination. However, it is not included as part of the main experiment description as it is assumed that the experiment in Part II has enough challenges for most students; introducing uncertainties at this point may be overwhelming for many.

Students can extend the work in Part II and significantly increase the precision of their measurements of Planck's constant by constructing a spectrometer with materials that cost just a few dollars. Several open source projects for this extension are listed in the Supplementary Resources (see, for example, "Spectrometer"). With some inexpensive components, an op-amp circuit can be used to boost the sensitivity of a photodiode circuit that captures the output of the LED. This extension is also described at a link found in Supplementary Resources: "DIY Science: Measuring Light with a Photodiode II."

As an additional challenge, if time permits, ask students to review the article by Planinšič and Etkina that's listed in the Supplemental Resources. Challenge them to try one of the simple activities described there, using their LED circuit from Part II. For example, they can use the circuit to show that shining a laser directly onto an LED at the threshold potential difference produces a small current in the circuit. The challenge would be for students to make a claim about the phenomenon, try it, and then support the claim with their observations and research from the article.

Common Student Challenges

Typically, until now most students have constructed and analyzed circuits consisting only of capacitors and resistors. The following properties of an LED should be presented to students:

- They are directional and need to be connected with the long post to the positive.
- They easily burn out if used in a circuit with little or no resistance.
- They convert electrical potential energy into radiant energy.

Many students only slowly develop the ability to translate a schematic into a physical circuit. It is important that students who struggle are in groups that are small enough that they are active participants. Components and contacts can fail. Give students troubleshooting skills such as the continuity and diode tester on their multimeter, but don't solve their problem for them. It is a good idea to have a clone of the circuit they are building in the room that they can refer to as an example.

It may be difficult for students to grasp the significance of the photoelectric effect and its unexpected dependence on the intensity of light. Frequent questioning regarding the accumulation of energy below the threshold frequency can help them to understand just how surprising these discoveries were. (Several references in Supplemental Resources, including the one by Knight, can provide further information regarding energy below the threshold frequency, if needed.)

There are two common practical challenges in this inquiry. First, many students cannot manipulate exponents in scientific notation and the larger the exponent value the greater the difficulty. Second, if students perceive measurement as the determination of points rather than intervals they may conclude either that the measurements did not work or that Planck's constant is undefined. Both of these technical challenges are best addressed by a consistent expectation for the use of scientific notation and an understanding that measurements have accompanying uncertainty.

Analyzing Results

Ask the students to look for a pattern in their measured potential differences for the LED, and report that pattern with a sketch of a graph using whiteboard, large poster paper, or projection from a computer (see the rubric for SP 5.1 in the Assessing the Science Practices section.) The graphs will show that as the wavelength decreases the potential difference of the LED for LEDs of that wavelength increases. When this dependence is revealed, ask students in their lab groups to consider the question, "Why does the brightness of the LED increase as the electric potential difference increases beyond the threshold potential?"

The goal of this question is to arrive at an apparent contradiction — light intensity continuously increases with potential difference above the threshold frequency but not below. If intensity could accumulate, wouldn't enough light below the threshold lead to some current? The student groups should report out their ideas using whiteboards. A rubric for this assessment is provided (see the rubric for SP 6.2 in the Assessing the Science Practices section.).

The next point to reach for can be expressed with the question, "Why does the LED have a threshold potential?" It is useful to redirect the discussion to photoelectron emission from a metal and the demonstration of Part I.

When groups have reported on their representations using whiteboards, introduce the term *stopping potential*. The goal is to lead them to the idea that there is current only when the kinetic energy (K) of an electron emitted from the metal surface is greater than the potential energy difference ($e\Delta V$) between the plates.

If the model does not emerge among students, the PhET "Photoelectric Effect" simulation (see Supplemental Resources) would be an appropriate activity here to support development of students' conceptual understanding. In a class with sufficient computer resources, students can be guided to explore and rediscover the wavelength dependence already observed in their earlier measurements. Ask, "What properties of the system initiate the emission of electrons from the zinc plate?" If computer resources are not available, the app can be projected for a whole-class discussion.

The representation of the photoelectric effect is again contrasted with the threshold potential observed in this experiment with LEDs (always emphasizing to students that this is not the photoelectric effect but a model that supports similar basic concepts — a "reverse photoelectric" of sorts.) In the photoelectric effect in Part I, the work function can be thought of as the charge of the electron times the potential difference between the bound state and the free state for the electron. Using $e\Delta V$ instead of ϕ to represent this amount of energy, the maximum kinetic energy of the electron freed by the energy provided by the radiant energy of a light source is:

$$K = E_{light} - e\Delta V \qquad \text{photoelectric effect}$$

In the lab with LEDs in Part II, the potential difference over the light source causing the emission of radiant energy is:

$$e\Delta V = E_{light} - eV_{threshold} \qquad \text{photovoltaic effect}$$

Suggest this relationship when visiting the small groups. They should now be able to plot the threshold electric potential difference against the frequencies of the light emitted by the LEDs. The slopes of these graphs will be h/e, where h is the constant of proportionality in their model of the energy of light. These slopes typically differ by about 20 percent but encompass an accepted value.

Characteristic student measurements illustrate the quality of data that can be expected from measurements of the threshold potential as described here. In the figure at the top of the next page, a commercially available device gives an acceptably accurate result. The regression line gives a value of Planck's constant of 6.1×10^{-34} J-s.

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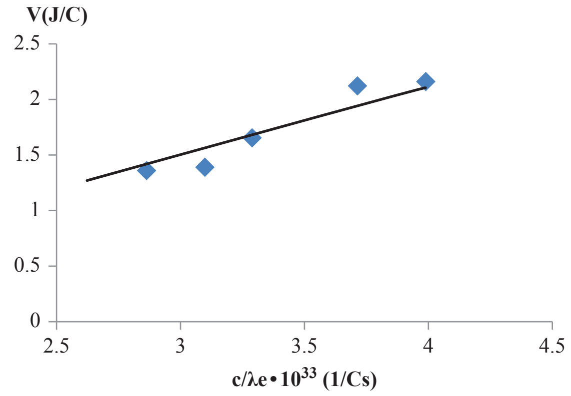

Figure 5

Students may also choose to plot energy as a function of frequency. The energy is calculated by multiplying the measured threshold potential difference across the LED by the charge of each electron, (eV, in joules). The LED frequency is calculated using an average reported value for LED wavelength (c/λ, in hertz; see Figure 6). The LEDs used for the graph below were purchased from a local electronics store, and plotted data values are from readings of threshold potential differences for four different LED colors supplied by 26 different students. In this more direct method of graphing, the experimental value for Planck's constant is simply the slope of the graph, determined here using a spreadsheet graphing program to be 6.0×10^{-34} J-s. Using more LED colors (in this case red, orange, yellow, and green, respectively) and accumulating data from the entire class gives students a better idea of the spread of data and the sense of using larger numbers of data points. [**NOTE:** The values for wavelengths used for this data were 535 nm for green, 580 nm for yellow, 600 nm for orange, and 685 nm for red.]

Caution students to be careful about reaching conclusions from mere observation of the sample graph here (obtained from actual student data), since many of the data points cluster on top of each other. This sample was derived from 92 data points, using the Excel graphing program to calculate the slope uses all the data points. Averaging from separate calculation of the 92 data points, using $E_{photon} = hf$ is much less desirable.

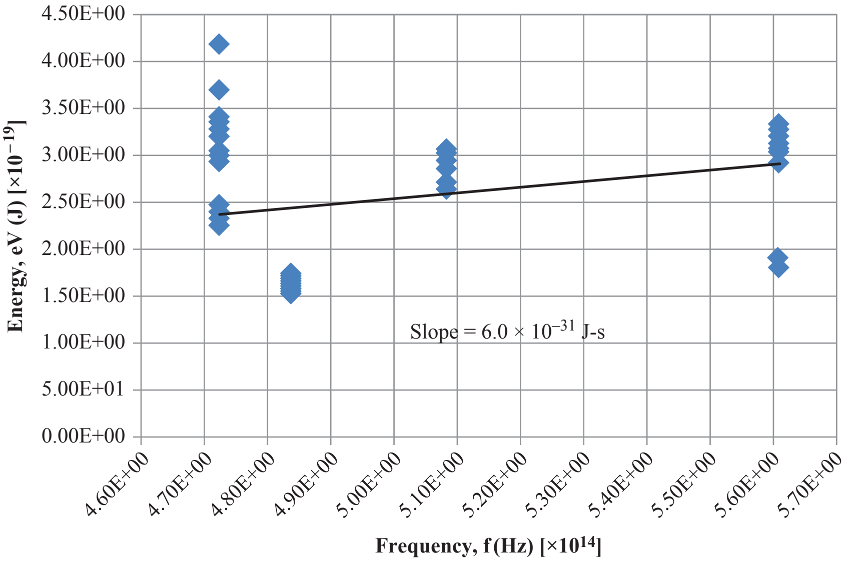

Figure 6: Sample Student Graph

[**NOTE:** It may be pointed out to your more advanced students that extension of the graph line by rescaling the *x*-axis so the line extends to the intercepts on both axes (shown below) provides interesting results. The graph line should go through a positive frequency intercept that represents the threshold frequency, assuming a common value for these LEDs. The line will also have a negative *y*-axis or energy intercept representing the work function, assuming a common value for these LEDs. It is often the case that the various colors of LEDs used in the experiment are made of the same metal base, hence they all can be assumed to have the same work function. The color variations come from "doping" the metal. Further research on this topic might be interesting to both you and your students.]

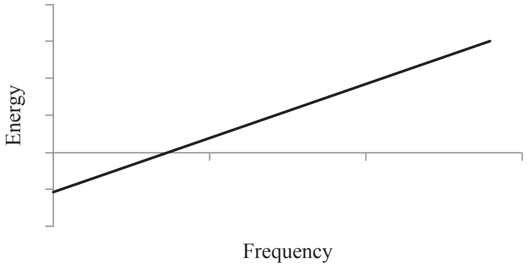

Figure 7

Post-Inquiry Discussion and Assessment

The observations made during the first demonstrations (Part I) should now be revisited and each explained by student groups using whiteboards and an explicit rubric (see the rubric for SP 6.4 in the Assessing the Science Practices section). Ask students the following questions:

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- Why did the halogen flashlight, which was apparently more intense than the mercury-vapor lamp, not affect the electroscope leaves?
- Why did increasing the distance of the metal surface from the mercury-vapor lamp slow down the decay of the repulsion between the leaves?
- Why do different metals show different rates of decay?
- Why does rate of decay depend on the shape of the steel wool?

Now ask students to return to the formative assessment (see the rubric for SP 7.1 in the Assessing the Science Practices section). They should briefly describe:

- What are the features of the model of light that explain diffraction? Contrast these with features of the model of light that explain the photoelectric effect.
- What are the features of the inappropriate model that are inconsistent with the behavior of light (a) when diffracted and (b) when causing the emission of an electron from a conducting surface?

Assessing Student Understanding

After completing this investigation, students should be able to:

- Articulate the features of the photoelectric effect that necessitate a particle model of light;
- Explain the similarities of the photoelectric and photovoltaic effects in terms of the conservation of energy; and
- Express the relationship between the energy and frequency of a photon and the evidence supporting the validity of that model.

Assessing the Science Practices

Science Practice 1.1 The student can *create representations and models* of natural or man-made phenomena and systems in the domain.

Proficient	Draws energy diagrams with annotations; kinetic and potential energy are shown correctly; and connections made to arrangement of electrons and to the question.
Nearly Proficient	Draws energy diagrams that include annotations with kinetic and potential energies and a connection made to the arrangement of electrons.
On the Path to Proficiency	Draws energy diagrams that include annotations with connections made to the arrangement of electrons, but kinetic and potential energy are shown incorrectly.
An Attempt	Draws energy diagrams with no annotations; the kinetic and potential energy are shown incorrectly; and no connections are made to the arrangement of electrons.

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Science Practice 1.4 The student can *use representations and models* to analyze situations or solve problems qualitatively and quantitatively.

Proficient	Draws energy diagrams with clear connections between both processes with essential features of both processes and clear, accurate representations of forms of energy. Describes differences between the behaviors of the electroscope and LED systems. Uses descriptions and qualitative representations (both calculations and graphs) correctly to show the relationship between photon energy and frequency/wavelength.
Nearly Proficient	Draws energy diagrams with clear connections between both processes with essential features of both processes and clear, accurate representations of forms of energy; the qualitative/quantitative work relating photon energy to frequency contains several errors.
On the Path to Proficiency	Draws energy diagrams with many essential features of both processes and connections between the two processes, but representations of forms of energy are not clear or contain errors; attempts qualitative/quantitative representations that are not correct.
An Attempt	Draws energy diagrams with some essential features of both processes but connections between the two processes are not made clear; representations of forms of energy are incorrectly shown; does not include qualitative representations from Part II of the investigation.

Science Practice 1.5 The student can *re-express key elements of natural phenomena across multiple representations* in the domain.

Proficient	Presents energy drawings that are complete and correctly identify the processes and energy. Constructs a graph of data from Part II of the investigation that shows the relationship between frequency and photo energy.
Nearly Proficient	Presents energy drawings and a graph from Part II of the investigation but does not correctly identify all of the processes or energy states.
On the Path to Proficiency	Presents energy drawings but does not correctly identify the processes or energy states. Attempts a graph for Part II of the investigation but does not correctly show energy and frequency relationships for the LEDs.
An Attempt	Presents energy drawings but does not identify the processes or energy states; unable to construct a graph from Part II of the investigation.

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Science Practice 3.2 The student can *refine scientific questions*.

Proficient	Restates the initial question from Part I of the investigation by incorporating relevant features of charged particles and the properties of the electroscope, and states a refined question that identifies features that can be manipulated, including a method of testing the question.
Nearly Proficient	Restates the initial question from Part I of the investigation by incorporating relevant features of charged particles and the properties of the electroscope, and states a refined question that identifies features that can be manipulated.
On the Path to Proficiency	Restates the initial question from Part I of the investigation by incorporating relevant features of charged particles and the properties of the electroscope.
An Attempt	Restates the initial question from Part I of the investigation without identifying relevant features of electrons and the electroscope.

Science Practice 4.2 The student can *design a plan* for collecting data to answer a particular scientific question.

Proficient	Clearly restates the question, and proposes measurements to be taken to evaluate how frequency/wavelength and energy are related. Makes clear connections between evidence obtained from measurement and the question.
Nearly Proficient	Clearly restates the question, and proposes measurements to be taken to evaluate how frequency/wavelength and energy are related, but does not make connections between evidence obtained from measurement and the question.
On the Path to Proficiency	Clearly restates the question, but does not propose measurements to be taken to evaluate the question or make connections between evidence obtained from measurement and the question.
An Attempt	Unable to clearly restate the question, and unable to propose measurements to be taken to evaluate the question or make connections between evidence obtained from measurement and the question.

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Science Practice 4.3 The student can *collect data* to answer a particular scientific question.

Proficient	Collects complete, clearly presented and identified data, with labels.
Nearly Proficient	Collects complete, clearly presented data for the LEDs, but the data is not identified and labeled.
On the Path to Proficiency	Collects partial data for the LEDs, or the data is not clearly presented, identified, or labeled.
An Attempt	Collects partial data for the LEDs; none of the data is clearly presented, identified, and labeled.

Science Practice 5.1 The student can *analyze data* to identify patterns or relationships.

Proficient	Includes a mathematical representation (including a graph) of the model describing the relationship of energy and frequency with the meaning of each variable identified and expressed in a form consistent with the observed data. There is a reasonable discussion of precision of data and possible sources of uncertainty.
Nearly Proficient	Includes a mathematical representation or graph of the model describing the relationship of energy and frequency with the meaning of each variable identified, but it is not expressed in a form consistent with the observed data. The data analysis does not include a clear discussion of precision or possible sources of uncertainty.
On the Path to Proficiency	Includes a mathematical representation of the model with the meaning of each variable identified, but it is not expressed in a form consistent with the observed data or that describes the relationship of energy and frequency. The data analysis does not include any discussion of precision or possible sources of uncertainty.
An Attempt	Includes a mathematical representation of the model, but the meaning of each variable is neither identified nor expressed in a form consistent with the observed data or that shows the relationship between energy and frequency. The data analysis does not include any discussion of precision or uncertainty.

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Science Practice 6.2 The student can *construct explanations of phenomena based on evidence* produced through scientific practices.

Proficient	Clearly restates the interaction of energy and matter, and the explanations of the effect of changes in intensity and the concept of threshold frequency are clear and concise.
Nearly Proficient	Clearly restates the interaction of energy and matter, but either the explanation of the effect of changes in intensity or the concept of threshold frequency contains flaws.
On the Path to Proficiency	Does not clearly restate the interaction of energy and matter, and the explanations of the effect of changes in intensity and the concept of threshold frequency contain major flaws.
An Attempt	Does not clearly restate the interaction of energy and matter, nor does the explanation include the effect of changes in intensity or the concept of threshold frequency.

Science Practice 6.4 The student can *make claims and predictions about natural phenomena* based on scientific theories and models.

Proficient	Prelab summary: Makes clear predictions about the differences between the flashlight and the mercury-vapor lamp and about the dependence on distance to photon source, metal type, and area of contact.
Nearly Proficient	Prelab summary: Describes the difference between the flashlight and the mercury-vapor lamp but explains the dependence on only one of the variants — either distance to photon source, metal type, or area of contact.
On the Path to Proficiency	Prelab summary: Includes discussion of variation due to metal type, but does not explain the difference between the flashlight and the mercury-vapor lamp, dependence on distance to photon source, dependence on metal type, or dependence on area of contact.
An Attempt	Prelab summary: Does not articulate the difference between the flashlight and the mercury-vapor lamp, dependence on distance to photon source, dependence on metal type, or dependence on area of contact.

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Science Practice 7.1 The student *can connect phenomena and models* across spatial and temporal scales.

Proficient	Correctly identifies both the features of the wave model of light that are and are not consistent with diffraction and the features of the particle model of light that are and are not consistent with the photoelectric effect.
Nearly Proficient	Correctly identifies either the features of the wave model of light that are and are not consistent with diffraction or the features of the particle model of light that are and are not consistent with the photoelectric effect.
On the Path to Proficiency	Incorrectly identifies some aspects of the wave model of light that are and are not consistent with diffraction, and incorrectly identifies the features of the particle model of light that are and are not consistent with the photoelectric effect.
An Attempt	Does not correctly identify any features of the wave model of light that are and are not consistent with diffraction, and does not identify features of the particle model of light that are and are not consistent with the photoelectric effect.

Supplemental Resources

"DIY Science: Measuring Light with a Photodiode II." Outside Science. Accessed September 1, 2014. http://outsidescience.wordpress.com/2012/11/03/diy-science-measuring-light-with-a-photodiode-ii/.

"Energy Skate Park." PhET. University of Colorado Boulder. Accessed September 1, 2014. https://phet.colorado.edu/en/simulation/energy-skate-park. [*This simulation can be set with an "energy bar graph" similar to what students might construct for the energy of a charged particle in an electric field.*]

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von Lenard, Phillipp E. A. “On Cathode Rays.” Nobelprize.org. Accessed September 1, 2014. http://www.nobelprize.org/nobel_prizes/physics/laureates/1905/lenard-lecture.pdf.

Appendix A:
Science Practices for AP Physics 1 and 2

Science Practice 1: The student can use representations and models to communicate scientific phenomena and solve scientific problems.

The real world is extremely complex. When physicists describe and explain phenomena, they try to simplify real objects, systems, and processes to make the analysis manageable. These simplifications or models are used to predict how new phenomena will occur. A simple model may treat a system as an object, neglecting the system's internal structure and behavior. More complex models are models of a system of objects, such as an ideal gas. A process can be simplified, too; free fall is an example of a simplified process, when we consider only the interaction of the object with the Earth. Models can be both conceptual and mathematical. Ohm's law is an example of a mathematical model, while the model of a current as a steady flow of charged particles is a conceptual model (the charged particles move randomly with some net motion [drift] of particles in a particular direction.) Basically, to make a good model, one needs to identify a set of the most important characteristics of a phenomenon or system that may simplify analysis. Inherent in the construction of models that physicists invent is the use of representations. Examples of representations used to model introductory physics are pictures, motion diagrams, force diagrams, graphs, energy bar charts, and ray diagrams. Mathematical representations such as equations are another example. Representations help in analyzing phenomena, making predictions, and communicating ideas. An example here is using a motion diagram and a force diagram to develop the mathematical expression of Newton's second law in component form to solve a dynamics problem.

1.1 The student can *create representations and models* of natural or man-made phenomena and systems in the domain.

1.2 The student can *describe representations and models* of natural or man-made phenomena and systems in the domain.

1.3 The student can *refine representations and models* of natural or man-made phenomena and systems in the domain.

1.4 The student can *use representations and models* to analyze situations or solve problems qualitatively and quantitatively.

1.5 The student can *re-express key elements of natural phenomena across multiple representations* in the domain.

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Science Practice 2: The student can use mathematics appropriately.

Physicists commonly use mathematical representations to describe and explain phenomena as well as to solve problems. When students work with these representations, we want them to understand the connections between the mathematical description, physical phenomena, and the concepts represented in the mathematical description. When using equations or mathematical representations, students need to be able to justify why using a particular equation to analyze a particular situation is useful, as well as to be aware of the conditions under which the equations/mathematical representations can be used. Students tend to rely too much on mathematical representations. When solving a problem, they need to be able to describe the problem situation in multiple ways, including picture representations, force diagrams, and so on, and then choose an appropriate mathematical representation, instead of first choosing a formula whose variables match the givens in the problem. In addition, students should be able to work with the algebraic form of the equation before they substitute values. They also should be able to evaluate the equation(s) and the answer obtained in terms of units and limiting case analysis: Does the equation lead to results that can be predicted qualitatively if one of the quantities in the problem is zero or infinity? They should be able to translate between functional relations in equations (proportionalities, inverse proportionalities, etc.) and cause-and-effect relations in the physical world. They should also be able to evaluate the numerical result in terms of whether it makes sense. For example, obtaining 35 $\mathrm{m/s^2}$ for the acceleration of a bus — about four times the acceleration of a freely falling object — should raise flags in students' minds. In many physics situations, simple mathematical routines may be needed to arrive at a result even though they are not the focus of a learning objective.

2.1 The student can *justify the selection of a mathematical routine* to solve problems.

2.2 The student can *apply mathematical routines* to quantities that describe natural phenomena.

2.3 The student can *estimate numerically quantities* that describe natural phenomena.

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Science Practice 3: The student can engage in scientific questioning to extend thinking or to guide investigations within the context of the AP course.

Research scientists pose and answer meaningful questions. Students may easily miss this point since, depending on how a science class is taught, it may seem that science is about compiling and passing down a large body of known facts (e.g., the acceleration of free-falling objects is 9.8 m/s^2; $\vec{a}=\frac{\Sigma\vec{F}}{m}$). At the opposite end of the spectrum, some students may believe that science can solve every important societal problem. Thus, helping students learn how to pose, refine, and evaluate scientific questions is an important instructional and cognitive goal, albeit a difficult skill to learn. Even within a simple physics topic, posing a scientific question can be difficult. When asked what they might want to find out about a simple pendulum, some students may ask, "How high does it swing?" Although this is a starting point from which a teacher may build, students need to be guided toward refining "fuzzy" questions and relating questions to relevant models and theories. As a first step to refining this question, students might first consider in what ways one can measure physical quantities relevant to the pendulum's motion, leading to a discussion of time, angle (amplitude), and mass. Follow-up discussions can lead to how one goes about evaluating questions such as, "Upon what does the period of a simple pendulum depend?" by designing and carrying out experiments, and then evaluating data and findings.

3.1 The student can *pose scientific questions.*

3.2 The student can *refine scientific questions.*

3.3 The student can *evaluate scientific questions.*

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Science Practice 4: The student can plan and implement data-collection strategies in relation to a particular scientific question.

[**NOTE:** Data can be collected from many different sources, e.g., investigations, scientific observations, the findings of others, historic reconstruction, and/or archived data.]

Scientific questions can range in scope from broad to narrow, as well as in specificity, from determining influencing factors and/or causes to determining mechanism. The question posed will determine the type of data to be collected and will influence the plan for collecting data. An example of a broad question is, "What caused the extinction of the dinosaurs?" whereas a narrow one is, "Upon what does the period of a simple pendulum depend?" Both questions ask for influencing factors and/or causes; an answer to the former might be "An asteroid collision with Earth caused the extinction of the dinosaurs," whereas an answer to the latter might be "The period depends on the mass and length of the pendulum." To test the cause of the pendulum's period, an experimental plan might vary mass and length to ascertain if these factors indeed influence the period of a pendulum, taking care to control variables so as to determine whether one factor, the other, or both influence the period. A question could be posed to ask about mechanism, e.g., "How did the dinosaurs become extinct?" or "How does the period of a simple pendulum depend on the mass and length?" In the second question, the object is to determine a mathematical relationship between period, mass, and length of a pendulum. Designing and improving experimental designs and/or data collection strategies is a learned skill. A class discussion among students in a pendulum experiment might find some who measured the time for a single round-trip, while others timed 10 round-trips and divided by 10. Such discussions can reveal issues of measurement uncertainty and assumptions about the motion. Students need to understand that the result of collecting and using data to determine a numerical answer to a question is best thought of as an interval, not a single number. This interval, the experimental uncertainty, is due to a combination of uncertainty in the instruments used and the process of taking the measurement. Although detailed error analysis is not necessary to convey this pivotal idea, it is important that students make some reasoned estimate of the interval within which they know the value of a measured data point and express their results in a way that makes this clear.

4.1 The student can *justify the selection of the kind of data* needed to answer a particular scientific question.

4.2 The student can *design a plan* for collecting data to answer a particular scientific question.

4.3 The student can *collect data* to answer a particular scientific question.

4.4 The student can *evaluate sources of data* to answer a particular scientific question.

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Science Practice 5: The student can perform data analysis and evaluation of evidence.

Students often think that to make a graph they need to connect the data points or that the best-fit function is always linear. Thus, it is important that they can construct a best-fit curve even for data that do not fit a linear relationship (such as quadratic or exponential functions). Students should be able to represent data points as intervals whose size depends on the experimental uncertainty. After students find a pattern in the data, they need to ask why this pattern is present and try to explain it using the knowledge that they have. When dealing with a new phenomenon, they should be able to devise a testable explanation of the pattern if possible (see Science Practice 6.4). It is important that students understand that instruments do not produce exact measurements and learn what steps they can take to decrease the uncertainty. Students should be able to design a second experiment to determine the same quantity and then check for consistency across the two measurements, comparing two results by writing them both as intervals and not as single, absolute numbers. Finally, students should be able to revise their reasoning based on the new data, data that for some may appear anomalous.

5.1 The student can *analyze data* to identify patterns or relationships.

5.2 The student can *refine observations and measurements* based on data analysis.

5.3 The student can *evaluate the evidence provided by data sets* in relation to a particular scientific question.

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Science Practice 6: The student can work with scientific explanations and theories.

Scientific explanations may specify a cause-and-effect relationship between variables or describe a mechanism through which a particular phenomenon occurs. Newton's second law, expressed as $\vec{a} = \frac{\Sigma\vec{F}}{m}$, gives the acceleration observed when a given combination of forces is exerted on an object with a certain mass. Liquids dry up because randomly moving molecules can leave liquids if their kinetic energy is higher than the negative potential energy of interaction between them and the liquid. A scientific explanation, accounting for an observed phenomenon, needs to be experimentally testable. One should be able to use it to make predictions about a new phenomenon. A theory uses a unified approach to account for a large set of phenomena and gives accounts that are consistent with multiple experimental outcomes within the range of applicability of the theory. Examples of theories in physics include kinetic molecular theory, quantum theory, atomic theory, etc. Students should understand the difference between explanations and theories. In the AP Physics 1 and 2 Curriculum Framework the word "claim" means any answer that a student provides except those that constitute direct and simple observational evidence. To say that all objects fall down is not a claim, but to say that all objects fall with the same acceleration is a claim, as one would need to back it up with evidence and a chain of reasoning. Students should be prepared to offer evidence, to construct reasoned arguments for their claim from the evidence, and to use the claim or explanation to make predictions. A prediction states the expected outcome of a particular experimental design based on an explanation or a claim under scrutiny. Consider the claim that current is directly proportional to potential difference across conductors based on data from an experiment varying voltage across a resistor and measuring current through it. The claim can be tested by connecting other resistors or lightbulbs in the circuit, measuring the voltage, using the linear relationship to predict the current, and comparing the predicted and measured current. This procedure tests the claim. Students should be able to design experiments to test alternative explanations of phenomena by comparing predicted outcomes. For example, students may think that liquids dry because air absorbs moisture. To test the claim they can design an experiment in which the same liquid dries in two conditions: in open air and in a vacuum jar. If the claim is correct, the liquid should dry faster in air. If the outcome does not match the prediction, the explanation is likely to be false. By contrast, if the outcome confirms the prediction, it only means that this experiment does not disprove the explanation; alternate explanations of the given outcome can always be formulated. Looking for experiments that can reject explanations and claims is at the heart of science.

6.1 The student can *justify claims with evidence.*

6.2 The student can *construct explanations of phenomena based on evidence* produced through scientific practices.

6.3 The student can *articulate the reasons that scientific explanations and theories are refined or replaced.*

6.4 The student can *make claims and predictions about natural phenomena* based on scientific theories and models.

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6.5 The student can *evaluate alternative scientific explanations*.

Science Practice 7: The student is able to connect and relate knowledge across various scales, concepts, and representations in and across the domains.

Students should have the opportunity to transfer their learning across disciplinary boundaries so that they are able to link, synthesize, and apply the ideas they learn across the sciences and mathematics. Research on how people learn indicates that providing multiple contexts to which major ideas apply facilitates transfer; this allows students to bundle knowledge in memory together with the multiple contexts to which it applies. Students should also be able to recognize seemingly appropriate contexts to which major concepts and ideas do not apply. After learning various conservation laws in the context of mechanics, students should be able to describe what the concept of conservation means in physics and extend the idea to other contexts. For example, what might conservation of energy mean at high-energy scales with particle collisions, where Einstein's mass–energy equivalence plays a major role? What does conservation of energy mean when constructing or evaluating arguments about global warming? Another context in which students may apply ideas from physics across vast spatial and time scales is the origin of human life on Earth coupled with the notion of extraterrestrial intelligent life. If one views the age of the Earth in analogy to a year of time (see Ritger & Cummins, 1991) with the Earth formed on January 1, then life began on Earth around April 5; multicellular organisms appeared on November 6; mammals appeared on December 23. Perhaps most amazingly, humans appeared on December 31 just 28 minutes before midnight. What are the implications of this for seeking intelligent life outside our solar system? What is a reasonable estimate of the probability of finding intelligent life on an earthlike planet that scientists might discover through astronomical observations, and how does one go about making those estimates? Although students are not expected to answer these very complex questions after a single AP science course, they should be able to talk intelligently about them using the concepts they learned.

7.1 The student can *connect phenomena and models* across spatial and temporal scales.

7.2 The student can *connect concepts* in and across domain(s) to generalize or extrapolate in and/or across enduring understandings and/or big ideas.

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Appendix B:

Rubrics for Science Practices in AP Physics 1 and 2 Investigations

Science Practice 1.1. The student can *create representations and models* of natural or man-made phenomena and systems in the domain.

Proficient	Creates accurate and appropriate representations/models to represent novel phenomena. The models account for the most important features of the phenomena and are experimentally testable, and limitations of the models (or inherent assumptions) are clearly explained.
Nearly Proficient	Creates accurate and appropriate representations/models to represent familiar phenomena. The models account for the most important features of the phenomena and are experimentally testable, but the limitations and assumptions are not explained.
On the Path to Proficiency	Creates representations/models that generally represent familiar phenomena. The models may not fully reflect all aspects of phenomena. The models are not experimentally testable (either involve some artificial explanations or the experiments are difficult to perform).
An Attempt	Creates flawed or incomplete representations/models to represent familiar phenomena.

Science Practice 1.2. The student can *describe representations and models* of natural or man-made phenomena and systems in the domain.

Proficient	Accurately articulates most links among representations/models and the natural phenomena or mechanisms they represent. Uses accurate language and definitions in descriptions associated with elements of the models/representations. Addresses relevancy of the description to the goal of the representation/model. Extracts all relevant information from the representations/models.
Nearly Proficient	Accurately articulates some links among representations/models and the natural phenomena or mechanisms they represent. Uses accurate language and/or definitions in descriptions associated with elements of the representations/models. Extracts relevant information from the representations/models.
On the Path to Proficiency	Accurately articulates a few links among representations/models and the natural phenomena or mechanisms they represent. Includes some inaccuracies in language and definitions within descriptions associated with elements of the representations/models.
An Attempt	Includes errors when articulating links among representations/models and the natural phenomena or mechanisms they represent. Uses generally inaccurate language and definitions within descriptions associated with elements of the representations/models.

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Science Practice 1.3. The student can *refine representations and models* of natural or man-made phenomena and systems in the domain.

Proficient	Effectively refines representations/models of phenomena, using accurate and precise definitions and language. Comprehensively identifies deficiencies of given representations/models and explains how the revised representations/models address these deficiencies.
Nearly Proficient	Correctly refines, with occasional or minor errors on complex problems, representations/models of phenomena, using accurate definitions and language. Accurately identifies nearly all deficiencies of given representations/models and explains how the revised representations/models address these deficiencies.
On the Path to Proficiency	Refines representations/models of phenomena with some errors and inaccuracies in definitions and language.
An Attempt	Makes significant errors in refining representations/models of phenomena. Uses incomplete definitions and/or language.

Science Practice 1.4. The student can *use representations and models* to analyze situations or solve problems qualitatively and quantitatively.

Proficient	Accurately uses representations and models to analyze situations or solve complex problems qualitatively and quantitatively without any errors. May manipulate a representation/model as an alternative to manipulation of equations and/or numerical data.
Nearly Proficient	Accurately uses representations and models to analyze situations or solve complex problems qualitatively and quantitatively with occasional or minor errors in analysis or problem solving.
On the Path to Proficiency	Uses representations and models to analyze situations or solve problems qualitatively and quantitatively, but with some significant errors and inaccuracies, either in analysis or problem solving.
An Attempt	Uses representations and models to analyze situations or solve problems qualitatively and quantitatively, but analysis and/or problem solving strategies contain multiple inaccuracies and errors.

Science Practice 1.5. The student can *re-express key elements of natural phenomena across multiple representations* in the domain.

Proficient	Re-expresses key elements of natural phenomena across multiple representations in the domain, and appropriately links elements of one representation with another familiar representation without any errors. The representations address different (but important) aspects of the phenomenon.
Nearly Proficient	Re-expresses key elements of natural phenomena across multiple representations in the domain, and links elements of one representation with another familiar representation with occasional or minor errors.
On the Path to Proficiency	Re-expresses key elements of natural phenomena across multiple representations in the domain, and links elements of one representation with another familiar representation with some significant errors and inaccuracies.
An Attempt	Re-expresses a very limited number of elements of natural phenomena across multiple representations in the domain with many errors and inaccuracies.

Science Practice 2.1. The student *can justify the selection of a mathematical routine* to solve problems.

Proficient	Provides relevant and detailed justification for the selection of mathematical routines. Uses accurate and precise language and terminology.
Nearly Proficient	Provides relevant justification for the selection of mathematical routines, but precise detail is lacking. Uses accurate language and terminology.
On the Path to Proficiency	Provides justification for the selection of mathematical routines that lacks some evidence, reasoning, and/or key factors. Uses language and terminology that includes a few errors and inaccuracies.
An Attempt	Provides justification for the selection of mathematical routines that may bear little relevance to the routines. Uses language and terminology with major errors and inaccuracies.

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Science Practice 2.2. The student can *apply mathematical routines* to quantities that describe natural phenomena.

Proficient	Appropriately and accurately selects and applies mathematical routines in new contexts and in simple to complex problems.
Nearly Proficient	Selects and applies appropriate mathematical routines in new contexts but with occasional minor errors on complex problems.
On the Path to Proficiency	Selects and applies appropriate mathematical routines in new contexts but with some inconsistency and/or errors.
An Attempt	Selects and applies mathematical routines, but the selections are inappropriate or the applications contain major errors.

Science Practice 2.3. The student can *estimate numerically quantities* that describe natural phenomena.

Proficient	Correctly estimates quantities that describe phenomena through the use of appropriate mathematical routines on complex problems.
Nearly Proficient	Estimates quantities that describe phenomena through the use of appropriate mathematical routines; the estimates contain occasional minor errors on complex problems.
On the Path to Proficiency	Estimates quantities that describe phenomena through the use of appropriate mathematical routines on familiar and/or simple problems.
An Attempt	Estimates quantities that describe phenomena through the use of mathematical routines; the estimates contain some errors or are not always relevant to the description.

Science Practice 3.1. The student can *pose scientific questions.*

Proficient	Poses scientific questions using precise language and terminology. Links the questions to existing knowledge and purpose with clarity and detail. Poses scientific questions which extend thinking about a concept, relationships between concepts, causal mechanism, and/or phenomena.
Nearly Proficient	Poses scientific questions using appropriate language and terminology with occasional minor errors. Links the questions to existing knowledge and purpose.
On the Path to Proficiency	Poses scientific questions using appropriate language and terminology with some inconsistency, errors, and/or inaccuracies. Identifies essential aspects of the phenomenon being queried.
An Attempt	Poses scientific questions using incorrect or imprecise language and terminology, resulting in a lack of clarity in linking a question to its purpose. Incorrectly identifies many of the aspects of the phenomena being queried.

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Science Practice 3.2. The student can *refine scientific questions.*

Proficient	Removes ambiguity, fully clarifies, and/or limits focus in refining scientific questions. Provides appropriate justification for refining questions, including appropriate reasoning and evidence.
Nearly Proficient	Removes ambiguity and/or clarifies focus in refining scientific questions. Provides justification for refining the question, and justification includes some reasoning and/or evidence.
On the Path to Proficiency	Reduces ambiguity and/or improves focus in refining scientific questions. Provides simple justification for refining questions; however, justification lacks reasoning and use of complete evidence.
An Attempt	Modifies scientific questions but with little positive effect in removing ambiguity or clarifying focus. Provides some justification for refining questions; however, justification lacks reasoning and/or evidence and includes major inaccuracies.

Science Practice 3.3. The student can *evaluate scientific questions.*

Proficient	Identifies evaluation criteria, explains the relevance of selected evaluation criteria, and evaluates, with convincing justification, scientific questions for inclusion in the scope of an investigation and domain.
Nearly Proficient	Identifies basic evaluation criteria, explains the relevance of the selected criteria with only a few errors, and evaluates and justifies scientific questions for inclusion in the scope of an investigation using some evidence and/or appropriate reasoning.
On the Path to Proficiency	Evaluates scientific questions for inclusion in the scope of either an investigation or domain, but justification is unclear. May use incomplete and/or inaccurate evidence and/or faulty reasoning.
An Attempt	Provides an evaluation of scientific questions that lacks justification for inclusion of the scientific questions in the scope of an investigation or domain, and the evaluation includes inaccuracies.

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Science Practice 4.1. The student can *justify the selection of the kind of data* needed to answer a particular scientific question.

Proficient	Provides accurate and detailed justification for data selection, explaining relevance of the selected variables and the data.
Nearly Proficient	Provides accurate justification for data selection with only an occasional or minor error.
On the Path to Proficiency	Provides justification for data selection with occasional and/or minor errors; justification may be correct but lacks completeness and/or reference to relevance.
An Attempt	Provides generally weak justification for data selection; justification includes minimal reasoning and evidence.

Science Practice 4.2. The student can *design a plan* for collecting data to answer a particular scientific question.

Proficient	Designs an effective data collection plan to answer a particular scientific question via well-selected qualitative and quantitative measures, providing rationale for all choices. Accurately evaluates and explains sources of error. Effectively explains tool selection for acquiring data and conditions for their use. Accurately identifies and explains independent, dependent, and controlling variables, and justifies choices.
Nearly Proficient	Designs an appropriate data collection plan to answer a particular scientific question via qualitative and/or quantitative measures; measures may lack complete details. Identifies the selected observation schedule, units of measurement, and tools. Identifies appropriate data sources and sources of error. Accurately identifies and describes independent, dependent, and controlling variables.
On the Path to Proficiency	Designs a data collection plan to answer a particular scientific question via qualitative or quantitative measures; measures may not be clearly defined or articulated. Acknowledges need to consider sources of error. Accurately identifies independent, dependent, and controlling variables with few errors and/or misuse of language or scientific terminology.
An Attempt	Presents an incomplete data collection plan to answer a particular scientific question; includes appropriate data sources but makes insufficient distinction between qualitative and quantitative measures. Makes errors in identifying the variables (independent, dependent, and controlling), and/or misuses language or scientific terminology.

Science Practice 4.3. The student can *collect data* to answer a particular scientific question.

Proficient	Collects appropriate data to fully answer a particular scientific question with precision of observations, accuracy of records, and accurate use of scientific tools and conditions. Accurately applies mathematical routines, and appropriately uses measurement strategies for the question and data sources.
Nearly Proficient	Collects appropriate and adequate data to answer some aspects of a particular scientific question with only minor errors in the precision of observation, record keeping, and use of tools and conditions. Selects appropriate mathematical routines, and provides measurements with only a few minor errors.
On the Path to Proficiency	Collects appropriate data to answer a particular scientific question. Provides observation logs and record keeping that contain several errors; however, the use of tools and conditions is adequate and appropriate for the most part. Selects appropriate mathematical routines and provides measurements with a few errors or a significant error.
An Attempt	Collects relevant but significantly inadequate data to answer a particular scientific question. Provides observations and/or record keeping that are incomplete and/or inadequate for answering a particular question. Selects inappropriate mathematical routines; measurements contain many errors.

Science Practice 4.4. The student can *evaluate sources of data* to answer a particular scientific question.

Proficient	Identifies fully legitimate sources of data to answer a particular scientific question, and justifies the completeness of the data set. Selects, justifies, and/or applies appropriate mathematical routines. Evaluates, explains, and estimates the percent uncertainty based on the largest source of uncertainty (instrumental or random).
Nearly Proficient	Identifies fully legitimate data sources to answer a particular scientific question, and appropriately identifies the data set as complete. Selects and applies appropriate mathematical routines. Estimates the percent uncertainty based on the largest source of uncertainty (instrumental or random), and articulates efforts to minimize uncertainty.
On the Path to Proficiency	Makes only minor errors in identifying legitimate data sources to answer a particular scientific question, and/or fails to recognize that some selected data sets are incomplete or inappropriate. Selects and justifies appropriate mathematical routines for answering the question. Evaluates uncertainty in the data.
An Attempt	Inconsistently identifies legitimate data sources to answer a particular scientific question, frequently failing to recognize the incomplete or inappropriate nature of the data. Identifies and selects appropriate mathematical routines for answering the question but selections lack justification.

Science Practice 5.1. The student can *analyze data* to identify patterns or relationships.

Proficient	Comprehensively describes the patterns and relationships within data, relative to a scientific question being asked. Accurately applies appropriate mathematical routines.
Nearly Proficient	Identifies most patterns within data, relative to a scientific question being asked, with only an occasional, minor error. Selects appropriate mathematical routines and applies them with only minor errors.
On the Path to Proficiency	Identifies the most obvious patterns within data, relative to a scientific question being asked, with some errors and inaccuracies. Selects appropriate mathematical routines but makes some application errors.
An Attempt	Identifies one or more legitimate patterns in data, though these may be irrelevant to a scientific question being asked. Identifies some mathematical routines that are appropriate.

Science Practice 5.2. The student can *refine observations and measurements* based on data analysis.

Proficient	Accurately identifies relevant data, and makes appropriate and comprehensive refinements to the observations and/or measurements. Appropriately selects and accurately applies mathematical routines.
Nearly Proficient	Accurately identifies relevant data, and makes appropriate refinements to the observations and/or measurements. Appropriately selects mathematical routines; application includes only minor errors.
On the Path to Proficiency	Accurately identifies relevant data, but selects incomplete and/or inappropriate refinements to observations and/or measurements. Selects and applies mathematical routines with at least one significant error.
An Attempt	Identifies relevant data, but applies mathematical routines that fail to improve the data collection plan.

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Science Practice 5.3. The student can *evaluate the evidence provided by data sets* in relation to a particular scientific question.

Proficient	Evaluates evidence provided by data sets by effectively justifying the appropriateness of the data as they relate to a particular scientific question and the data collection methods used. Appropriately selects and accurately applies mathematical routines.
Nearly Proficient	Evaluates evidence provided by data sets by justifying the appropriateness of the data, but provides only basic insight into how data relate to a particular scientific question and the data collection methods used. Appropriately selects mathematical routines; application includes only minor errors.
On the Path to Proficiency	Relates evidence provided by data sets to a particular scientific question, but fails to fully align the evidence to a question or to the data collection methods used. Selects and applies mathematical routines with at least one significant error.
An Attempt	Makes some judgment errors about how data are used as evidence in relation to a particular scientific question and about appropriate data collection methods. Selects and applies mathematical routines with some significant errors.

Science Practice 6.1. The student can *justify claims with evidence.*

Proficient	Accurately identifies and aligns a comprehensive array of evidence with claims the evidence supports. Provides a substantive justification for the selection and/or exclusion of evidence. Considers data from multiple sources, and provides appropriate rationales for the selection and exclusion of evidence.
Nearly Proficient	Accurately identifies and aligns most but not all available and relevant evidence with claims it supports. Provides a clear justification for the selection and/or exclusion of evidence. Considers data from multiple sources.
On the Path to Proficiency	Identifies and aligns evidence with claims; but some evidence is inappropriate or fails to support the claims. Accurately differentiates between a claim and the evidence that supports it.
An Attempt	Identifies some appropriate evidence in support of claims, but connections drawn between evidence and claims are generally weak.

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Science Practice 6.2. The student can *construct explanations of phenomena based on evidence* produced through scientific practices.

Proficient	Provides comprehensive explanations of phenomena based on evidence produced through scientific practices, presents a logical argument that links evidence and claims, and uses scientific language precisely and accurately.
Nearly Proficient	Provides explanations of phenomena based on evidence produced through scientific practices, links evidence and claims, and uses precise and accurate scientific language with only minor or occasional errors.
On the Path to Proficiency	Provides explanations of phenomena based on some evidence produced through science practices using basic logic. Identifies some links between evidence and claims, and uses scientific language with few significant errors.
An Attempt	Describes phenomena with limited reference to evidence produced through scientific practice. Uses flawed language and terminology.

Science Practice 6.3. The student can *articulate the reasons that scientific explanations and theories are refined or replaced.*

Proficient	Articulates the reasons why scientific explanations and theories are refined or replaced, and appropriately explains why particular revisions were necessary.
Nearly Proficient	Articulates the appropriate reasons that scientific explanations and theories are refined or replaced, and uses appropriate and accurate scientific language with only minor or occasional errors. Descriptions of why revisions are necessary may be incomplete.
On the Path to Proficiency	Articulates general reasons why scientific explanations and theories are refined or replaced, and uses scientific language and terminology with few errors and inaccuracies. Describes why revisions are necessary but not why they might represent an improvement.
An Attempt	Articulates a limited number of reasons why familiar scientific explanations and theories are refined or replaced, but offers inaccurate or incomplete reasoning or justification for why they were refined or replaced.

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Science Practice 6.4. The student can *make claims and predictions about natural phenomena* based on scientific theories and models.

Proficient	Draws convincingly from scientific theories and models in making claims and predictions about phenomena, with appropriate justification and reasoning. Explains the connections between the model and the claim or prediction and the model and the phenomena.
Nearly Proficient	Draws directly and appropriately from scientific theories and models in making claims and predictions about phenomena, with appropriate justification and reasoning. Explains, with few errors, connections between the model and the claim or prediction and the model and the phenomena.
On the Path to Proficiency	Draws directly from scientific theories and models in making claims and predictions about phenomena, with incomplete justification or reasoning. Describes, with some limitations, connections between the model and the claim or prediction and the model and the phenomena. Accurately describes the scope of the claim or prediction.
An Attempt	Describes scientific theories and models in making claims and predictions about phenomena but without appropriate reasoning and/or justification. Some connections are inappropriate or inaccurate. Articulates the scope of claims or predictions with some errors.

Science Practice 6.5. The student can *evaluate alternative scientific explanations.*

Proficient	Evaluates alternative scientific explanations with consideration of both the alternative's strengths and its weaknesses appropriately based on available evidence.
Nearly Proficient	Evaluates alternative scientific explanations with consideration of both the alternative's strengths and its weaknesses, though the evaluation of either the strengths or weaknesses may be flawed.
On the Path to Proficiency	Evaluates alternative scientific explanations with limited consideration of either the alternative's strengths or its weaknesses based on evidence; evaluation of the strengths or weaknesses may be flawed.
An Attempt	Evaluates the appropriateness or accuracy of contradictory evidence or alternative explanations; evaluation contains some errors.

Science Practice 7.1. The student can *connect phenomena and models* across spatial and temporal scales.

Proficient	Connects phenomena and models across both spatial and temporal scales. Fully explains how a change at one scale affects phenomena or models at another scale with an occasional or minor error.
Nearly Proficient	Connects phenomena and models across spatial and/or temporal scales. Describes the impact on phenomena or models caused by a change at one scale with few or occasional errors.
On the Path to Proficiency	Connects phenomena and models across spatial or temporal scales, relating specific variables across finer or greater spatial and/or temporal scales. Cites relationships among variables that are not fully valid.
An Attempt	Articulates a general account of phenomena described at one scale and some relationships at another scale, but makes some errors in attempts to connect phenomena and models.

Science Practice 7.2. The student can *connect concepts* in and across domain(s) to generalize or extrapolate in and/or across enduring understandings and/or big ideas.

Proficient	Connects concepts in and across domain(s) to accurately generalize or extrapolate in and/or across enduring understandings and/or big ideas. Predicts, with appropriate reasoning and detailed justification, how a change in one phenomenon might affect another.
Nearly Proficient	Connects concepts in and across domain(s) to generalize or extrapolate in and/or across enduring understandings and/or big ideas with only occasional or minor errors. Predicts, with reasoning and justification, how a change in one phenomenon might affect another with only occasional or minor errors.
On the Path to Proficiency	Compares features of phenomena that are related, making connections across concepts and among contexts but not necessarily across big ideas and/or enduring understandings. Predicts, with basic reasoning or justification, how a change in one phenomenon might affect another.
An Attempt	Makes statements linking concepts or phenomena but with some errors and inaccuracies. Makes claims about how a change in one phenomenon might affect another but with an incomplete consideration of evidence.

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Appendix C:

AP Physics 1 and 2 Constants and Equations

Table of Information and Equation Tables for AP Physics 1 and 2 Exams

The accompanying Table of Information and equation tables will be provided to students when they take the AP Physics 1 and 2 Exams. Therefore, students may NOT bring their own copies of these tables to the exam room, although they may use them throughout the year in their classes in order to become familiar with their content. **These tables are current as of the May 2015 exam administration; however it is possible for a revision to occur subsequent to that date. Check the Physics course home pages on AP Central for the latest versions of these tables (apcentral.collegeboard.org).**

The Table of Information and the equation tables are printed near the front cover of both the multiple-choice section and the free-response section. The Table of Information is identical for both exams except for some of the conventions.

The equations in the tables express the relationships that are encountered most frequently in the AP Physics 1 and 2 courses and exams. However, the tables do not include all equations that might possibly be used. For example, they do not include many equations that can be derived by combining other equations in the tables. Nor do they include equations that are simply special cases of any that are in the tables. Students are responsible for understanding the physical principles that underlie each equation and for knowing the conditions for which each equation is applicable.

The equation tables are grouped in sections according to the major content category in which they appear. Within each section, the symbols used for the variables in that section are defined. However, in some cases the same symbol is used to represent different quantities in different tables. It should be noted that there is no uniform convention among textbooks for the symbols used in writing equations. The equation tables follow many common conventions, but in some cases consistency was sacrificed for the sake of clarity.

Some explanations about notation used in the equation tables:

1. The symbols used for physical constants are the same as those in the Table of Information and are defined in the Table of Information rather than in the right-hand columns of the equation tables.
2. Symbols with arrows above them represent vector quantities.
3. Subscripts on symbols in the equations are used to represent special cases of the variables defined in the right-hand columns.
4. The symbol Δ before a variable in an equation specifically indicates a change in the variable (e.g., final value minus initial value).
5. Several different symbols (e.g., d, r, s, h, ℓ) are used for linear dimensions such as length. The particular symbol used in an equation is one that is commonly used for that equation in textbooks.

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ADVANCED PLACEMENT PHYSICS 1 TABLE OF INFORMATION

CONSTANTS AND CONVERSION FACTORS			
Proton mass,	$m_p = 1.67 \times 10^{-27}$ kg	Electron charge magnitude,	$e = 1.60 \times 10^{-19}$ C
Neutron mass,	$m_n = 1.67 \times 10^{-27}$ kg	Coulomb's law constant,	$k = 1/4\pi\varepsilon_0 = 9.0 \times 10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2$
Electron mass,	$m_e = 9.11 \times 10^{-31}$ kg	Universal gravitational constant,	$G = 6.67 \times 10^{-11}\ \text{m}^3/\text{kg}\cdot\text{s}^2$
Speed of light,	$c = 3.00 \times 10^{8}$ m/s	Acceleration due to gravity at Earth's surface,	$g = 9.8\ \text{m/s}^2$

UNIT SYMBOLS	meter,	m	kelvin,	K	watt,	W	degree Celsius,	°C
	kilogram,	kg	hertz,	Hz	coulomb,	C		
	second,	s	newton,	N	volt,	V		
	ampere,	A	joule,	J	ohm,	Ω		

PREFIXES		
Factor	Prefix	Symbol
10^{12}	tera	T
10^{9}	giga	G
10^{6}	mega	M
10^{3}	kilo	k
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p

VALUES OF TRIGONOMETRIC FUNCTIONS FOR COMMON ANGLES							
θ	0°	30°	37°	45°	53°	60°	90°
$\sin\theta$	0	$1/2$	$3/5$	$\sqrt{2}/2$	$4/5$	$\sqrt{3}/2$	1
$\cos\theta$	1	$\sqrt{3}/2$	$4/5$	$\sqrt{2}/2$	$3/5$	$1/2$	0
$\tan\theta$	0	$\sqrt{3}/3$	$3/4$	1	$4/3$	$\sqrt{3}$	∞

The following conventions are used in this exam.

I. The frame of reference of any problem is assumed to be inertial unless otherwise stated.

II. Assume air resistance is negligible unless otherwise stated.

III. In all situations, positive work is defined as work done on a system.

IV. The direction of current is conventional current: the direction in which positive charge would drift.

V. Assume all batteries and meters are ideal unless otherwise stated.

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ADVANCED PLACEMENT PHYSICS 1 EQUATIONS

MECHANICS

$$v_x = v_{x0} + a_x t$$

$$x = x_0 + v_{x0} t + \frac{1}{2} a_x t^2$$

$$v_x^2 = v_{x0}^2 + 2a_x (x - x_0)$$

$$\vec{a} = \frac{\sum \vec{F}}{m} = \frac{\vec{F}_{net}}{m}$$

$$\left|\vec{F}_f\right| \leq \mu \left|\vec{F}_n\right|$$

$$a_c = \frac{v^2}{r}$$

$$\vec{p} = m\vec{v}$$

$$\Delta\vec{p} = \vec{F}\,\Delta t$$

$$K = \frac{1}{2} m v^2$$

$$\Delta E = W = F_{\parallel} d = Fd\cos\theta$$

$$P = \frac{\Delta E}{\Delta t}$$

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\omega = \omega_0 + \alpha t$$

$$x = A\cos(2\pi f t)$$

$$\vec{\alpha} = \frac{\sum \vec{\tau}}{I} = \frac{\vec{\tau}_{net}}{I}$$

$$\tau = r_{\perp} F = rF\sin\theta$$

$$L = I\omega$$

$$\Delta L = \tau\,\Delta t$$

$$K = \frac{1}{2} I \omega^2$$

$$\left|\vec{F}_s\right| = k\left|\vec{x}\right|$$

$$U_s = \frac{1}{2} k x^2$$

$$\rho = \frac{m}{V}$$

a = acceleration
A = amplitude
d = distance
E = energy
f = frequency
F = force
I = rotational inertia
K = kinetic energy
k = spring constant
L = angular momentum
ℓ = length
m = mass
P = power
p = momentum
r = radius or separation
T = period
t = time
U = potential energy
V = volume
v = speed
W = work done on a system
x = position
y = height
α = angular acceleration
μ = coefficient of friction
θ = angle
ρ = density
τ = torque
ω = angular speed

$$\Delta U_g = mg\,\Delta y$$

$$T = \frac{2\pi}{\omega} = \frac{1}{f}$$

$$T_s = 2\pi\sqrt{\frac{m}{k}}$$

$$T_p = 2\pi\sqrt{\frac{\ell}{g}}$$

$$\left|\vec{F}_g\right| = G\frac{m_1 m_2}{r^2}$$

$$\vec{g} = \frac{\vec{F}_g}{m}$$

$$U_G = -\frac{G m_1 m_2}{r}$$

GEOMETRY AND TRIGONOMETRY

Rectangle
$A = bh$

Triangle
$A = \frac{1}{2} bh$

Circle
$A = \pi r^2$
$C = 2\pi r$

Rectangular solid
$V = \ell w h$

Cylinder
$V = \pi r^2 \ell$
$S = 2\pi r \ell + 2\pi r^2$

Sphere
$V = \frac{4}{3}\pi r^3$
$S = 4\pi r^2$

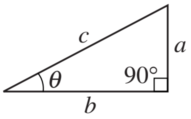
A = area
C = circumference
V = volume
S = surface area
b = base
h = height
ℓ = length
w = width
r = radius

Right triangle
$c^2 = a^2 + b^2$

$\sin\theta = \frac{a}{c}$

$\cos\theta = \frac{b}{c}$

$\tan\theta = \frac{a}{b}$



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ADVANCED PLACEMENT PHYSICS 2 TABLE OF INFORMATION

CONSTANTS AND CONVERSION FACTORS

Proton mass,	$m_p = 1.67 \times 10^{-27}$ kg	Electron charge magnitude,	$e = 1.60 \times 10^{-19}$ C
Neutron mass,	$m_n = 1.67 \times 10^{-27}$ kg	1 electron volt,	$1 \text{ eV} = 1.60 \times 10^{-19}$ J
Electron mass,	$m_e = 9.11 \times 10^{-31}$ kg	Speed of light,	$c = 3.00 \times 10^{8}$ m/s
Avogadro's number,	$N_0 = 6.02 \times 10^{23}$ mol^{-1}	Universal gravitational constant,	$G = 6.67 \times 10^{-11}$ $\text{m}^3/\text{kg}{\cdot}\text{s}^2$
Universal gas constant,	$R = 8.31$ J/(mol•K)	Acceleration due to gravity at Earth's surface,	$g = 9.8$ m/s^2
Boltzmann's constant,	$k_B = 1.38 \times 10^{-23}$ J/K		

1 unified atomic mass unit,	$1 \text{ u} = 1.66 \times 10^{-27} \text{ kg} = 931 \text{ MeV}/c^2$
Planck's constant,	$h = 6.63 \times 10^{-34} \text{ J}{\cdot}\text{s} = 4.14 \times 10^{-15} \text{ eV}{\cdot}\text{s}$
	$hc = 1.99 \times 10^{-25} \text{ J}{\cdot}\text{m} = 1.24 \times 10^{3} \text{ eV}{\cdot}\text{nm}$
Vacuum permittivity,	$\varepsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/\text{N}{\cdot}\text{m}^2$
Coulomb's law constant,	$k = 1/4\pi\varepsilon_0 = 9.0 \times 10^{9} \text{ N}{\cdot}\text{m}^2/\text{C}^2$
Vacuum permeability,	$\mu_0 = 4\pi \times 10^{-7} \text{ (T}{\cdot}\text{m)/A}$
Magnetic constant,	$k' = \mu_0/4\pi = 1 \times 10^{-7} \text{ (T}{\cdot}\text{m)/A}$
1 atmosphere pressure,	$1 \text{ atm} = 1.0 \times 10^{5} \text{ N/m}^2 = 1.0 \times 10^{5} \text{ Pa}$

UNIT SYMBOLS	meter,	m	mole,	mol	watt,	W	farad,	F
	kilogram,	kg	hertz,	Hz	coulomb,	C	tesla,	T
	second,	s	newton,	N	volt,	V	degree Celsius,	°C
	ampere,	A	pascal,	Pa	ohm,	Ω	electron volt,	eV
	kelvin,	K	joule,	J	henry,	H		

PREFIXES		
Factor	Prefix	Symbol
10^{12}	tera	T
10^{9}	giga	G
10^{6}	mega	M
10^{3}	kilo	k
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p

VALUES OF TRIGONOMETRIC FUNCTIONS FOR COMMON ANGLES

θ	0°	30°	37°	45°	53°	60°	90°
$\sin\theta$	0	$1/2$	$3/5$	$\sqrt{2}/2$	$4/5$	$\sqrt{3}/2$	1
$\cos\theta$	1	$\sqrt{3}/2$	$4/5$	$\sqrt{2}/2$	$3/5$	$1/2$	0
$\tan\theta$	0	$\sqrt{3}/3$	$3/4$	1	$4/3$	$\sqrt{3}$	∞

The following conventions are used in this exam.

I. The frame of reference of any problem is assumed to be inertial unless otherwise stated.

II. In all situations, positive work is defined as work done on a system.

III. The direction of current is conventional current: the direction in which positive charge would drift.

IV. Assume all batteries and meters are ideal unless otherwise stated.

V. Assume edge effects for the electric field of a parallel plate capacitor unless otherwise stated.

VI. For any isolated electrically charged object, the electric potential is defined as zero at infinite distance from the charged object.

ADVANCED PLACEMENT PHYSICS 2 EQUATIONS

MECHANICS

$$v_x = v_{x0} + a_x t$$

$$x = x_0 + v_{x0}t + \frac{1}{2}a_x t^2$$

$$v_x^2 = v_{x0}^2 + 2a_x(x - x_0)$$

$$\vec{a} = \frac{\sum \vec{F}}{m} = \frac{\vec{F}_{net}}{m}$$

$$|\vec{F}_f| \le \mu |\vec{F}_n|$$

$$a_c = \frac{v^2}{r}$$

$$\vec{p} = m\vec{v}$$

$$\Delta\vec{p} = \vec{F}\,\Delta t$$

$$K = \frac{1}{2}mv^2$$

$$\Delta E = W = F_{\parallel}d = Fd\cos\theta$$

$$P = \frac{\Delta E}{\Delta t}$$

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\omega = \omega_0 + \alpha t$$

$$x = A\cos(\omega t) = A\cos(2\pi f t)$$

$$x_{cm} = \frac{\sum m_i x_i}{\sum m_i}$$

$$\vec{\alpha} = \frac{\sum \vec{\tau}}{I} = \frac{\vec{\tau}_{net}}{I}$$

$$\tau = r_{\perp}F = rF\sin\theta$$

$$L = I\omega$$

$$\Delta L = \tau\,\Delta t$$

$$K = \frac{1}{2}I\omega^2$$

$$|\vec{F}_s| = k|\vec{x}|$$

a = acceleration
A = amplitude
d = distance
E = energy
F = force
f = frequency
I = rotational inertia
K = kinetic energy
k = spring constant
L = angular momentum
ℓ = length
m = mass
P = power
p = momentum
r = radius or separation
T = period
t = time
U = potential energy
v = speed
W = work done on a system
x = position
y = height
α = angular acceleration
μ = coefficient of friction
θ = angle
τ = torque
ω = angular speed

$$U_s = \frac{1}{2}kx^2$$

$$\Delta U_g = mg\,\Delta y$$

$$T = \frac{2\pi}{\omega} = \frac{1}{f}$$

$$T_s = 2\pi\sqrt{\frac{m}{k}}$$

$$T_p = 2\pi\sqrt{\frac{\ell}{g}}$$

$$|\vec{F}_g| = G\frac{m_1 m_2}{r^2}$$

$$\vec{g} = \frac{\vec{F}_g}{m}$$

$$U_G = -\frac{Gm_1 m_2}{r}$$

ELECTRICITY AND MAGNETISM

$$|\vec{F}_E| = \frac{1}{4\pi\varepsilon_0}\frac{|q_1 q_2|}{r^2}$$

$$\vec{E} = \frac{\vec{F}_E}{q}$$

$$|\vec{E}| = \frac{1}{4\pi\varepsilon_0}\frac{|q|}{r^2}$$

$$\Delta U_E = q\Delta V$$

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

$$|\vec{E}| = \left|\frac{\Delta V}{\Delta r}\right|$$

$$\Delta V = \frac{Q}{C}$$

$$C = \kappa\varepsilon_0\frac{A}{d}$$

$$E = \frac{Q}{\varepsilon_0 A}$$

$$U_C = \frac{1}{2}Q\Delta V = \frac{1}{2}C(\Delta V)^2$$

$$I = \frac{\Delta Q}{\Delta t}$$

$$R = \frac{\rho\ell}{A}$$

$$P = I\,\Delta V$$

$$I = \frac{\Delta V}{R}$$

$$R_s = \sum_i R_i$$

$$\frac{1}{R_p} = \sum_i \frac{1}{R_i}$$

$$C_p = \sum_i C_i$$

$$\frac{1}{C_s} = \sum_i \frac{1}{C_i}$$

$$B = \frac{\mu_0}{2\pi}\frac{I}{r}$$

A = area
B = magnetic field
C = capacitance
d = distance
E = electric field
$\mathcal{E}$ = emf
F = force
I = current
ℓ = length
P = power
Q = charge
q = point charge
R = resistance
r = separation
t = time
U = potential (stored) energy
V = electric potential
v = speed
κ = dielectric constant
ρ = resistivity
θ = angle
Φ = flux

$$\vec{F}_M = q\vec{v} \times \vec{B}$$

$$|\vec{F}_M| = |q\vec{v}||\sin\theta||\vec{B}|$$

$$\vec{F}_M = I\vec{\ell} \times \vec{B}$$

$$|\vec{F}_M| = |I\vec{\ell}||\sin\theta||\vec{B}|$$

$$\Phi_B = \vec{B}\cdot\vec{A}$$

$$\Phi_B = |\vec{B}|\cos\theta|\vec{A}|$$

$$\mathcal{E} = -\frac{\Delta\Phi_B}{\Delta t}$$

$$\mathcal{E} = B\ell v$$

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ADVANCED PLACEMENT PHYSICS 2 EQUATIONS

FLUID MECHANICS AND THERMAL PHYSICS

$$\rho = \frac{m}{V}$$

$$P = \frac{F}{A}$$

$$P = P_0 + \rho g h$$

$$F_b = \rho V g$$

$$A_1 v_1 = A_2 v_2$$

$$P_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$$

$$\frac{Q}{\Delta t} = \frac{kA\,\Delta T}{L}$$

$$PV = nRT = Nk_BT$$

$$K = \frac{3}{2}k_BT$$

$$W = -P\,\Delta V$$

$$\Delta U = Q + W$$

A = area
F = force
h = depth
k = thermal conductivity
K = kinetic energy
L = thickness
m = mass
n = number of moles
N = number of molecules
P = pressure
Q = energy transferred to a system by heating
T = temperature
t = time
U = internal energy
V = volume
v = speed
W = work done on a system
y = height
ρ = density

MODERN PHYSICS

$$E = hf$$

$$K_{\max} = hf - \phi$$

$$\lambda = \frac{h}{p}$$

$$E = mc^2$$

E = energy
f = frequency
K = kinetic energy
m = mass
p = momentum
λ = wavelength
ϕ = work function

WAVES AND OPTICS

$$\lambda = \frac{v}{f}$$

$$n = \frac{c}{v}$$

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$$\frac{1}{s_i} + \frac{1}{s_o} = \frac{1}{f}$$

$$|M| = \left|\frac{h_i}{h_o}\right| = \left|\frac{s_i}{s_o}\right|$$

$$\Delta L = m\lambda$$

$$d\sin\theta = m\lambda$$

d = separation
f = frequency or focal length
h = height
L = distance
M = magnification
m = an integer
n = index of refraction
s = distance
v = speed
λ = wavelength
θ = angle

GEOMETRY AND TRIGONOMETRY

Rectangle
$A = bh$

Triangle
$A = \frac{1}{2}bh$

Circle
$A = \pi r^2$
$C = 2\pi r$

Rectangular solid
$V = \ell w h$

Cylinder
$V = \pi r^2 \ell$
$S = 2\pi r\ell + {}^{2}\pi^{r2}$

Sphere
$V = \frac{4}{3}\pi r^3$
$S = 4\pi r^2$

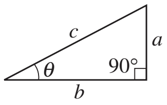
A = area
C = circumference
V = volume
S = surface area
b = base
h = height
ℓ = length
w = width
r = radius

Right triangle
$c^2 = a^2 + b^2$

$$\sin\theta = \frac{a}{c}$$

$$\cos\theta = \frac{b}{c}$$

$$\tan\theta = \frac{a}{b}$$



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AP® Physics 1 and 2 Inquiry-Based Lab Investigations

Aligned with best practices in science instruction as proposed by the National Science Foundation and America's Lab Report, *AP® Physics 1 and 2 Inquiry-Based Lab Investigations: A Teacher's Manual* serves to guide teachers through inquiry-based lab experiments and procedures that are easily tailored to diverse needs and are appropriate for small and large classes.

- Features 15 student-directed, inquiry-based lab investigations (7 for AP Physics 1 and 8 for AP Physics 2)
- Emphasizes scientific inquiry, reasoning, and critical thinking
- Aligns with the learning objectives in the *AP Physics 1: Algebra-Based and AP Physics 2: Algebra-Based Curriculum Framework*
- Enables students to plan, direct, and integrate a range of science practices, such as designing experiments, collecting data, and applying quantitative skills
- Includes lists of supplemental resources

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