AP® Calculus BC: Syllabus 1

Syllabus 1058782v1

Scoring Components		Page(s)
SC1	The course teaches all topics associated with Functions, Graphs, and Limits as delineated in the Calculus BC Topic Outline in the AP Calculus Course Description.	2, 4
SC2	The course teaches all topics associated with Derivatives as delineated in the Calculus BC Topic Outline in the AP Calculus Course Description.	2
SC3	The course teaches all topics associated with Integrals as delineated in the Calculus BC Topic Outline in the AP Calculus Course Description.	3
SC4	The course teaches all topics associated with Polynomial Approximations and Infinite Series as delineated in the Calculus BC Topic Outline in the AP Calculus Course Description.	4
SC5	The course provides students the opportunity to work with functions represented graphically.	2–3
SC6	The course provides provides students with the opportunity to work with functions represented numerically.	5
SC7	The course provides students with the opportunity to work with functions represented analytically.	2–4
SC8	The course provides students with the opportunity to work with functions represented verbally.	3, 5
SC9	The course teaches students how to explain solutions to problems orally.	6
SC10	The course teaches students how to explain solutions to problems in written sentences.	4
SC11	The course teaches students how to use graphing calculators to help solve problems.	6
SC12	The course teaches students how to use graphing calculators to experiment.	5
SC13	The course teaches students how to use graphing calculators to interpret results and support conclusions.	5–6

Number crunching and symbol manipulation are only small parts of learning calculus. I want my students to feel they understand each concept we study. One of the major goals is for the students to learn how to use precise language to describe these concepts and the relationships between ideas.

Course Planner

Primary Textbook

Finney, Ross L., Franklin Demana, Bert Waits, and Daniel Kennedy. *Calculus: Graphical, Numerical, Algebraic*. Reading, Mass.: Addison-Wesley, 2007.

The chapter numbers follow the textbook.

Chapter 1: Prerequisites for Calculus (7 days) [SC1]

- Elementary functions:
  - Linear, power, exponential/logarithmic, trigonometric / inverse trigonometric
- Parametric equations
- Getting familiar with the graphing calculator

Chapter 2: Limits and Continuity (10 days) [SC1]

- Limits:
  - Limit at a point, limit at infinity, infinite limits
  - Properties of limits
- Continuity
- Tangent line to a curve
- Slope of a curve at a point

Chapter 3: Derivatives (15 days) [SC2]

- Definition of f' **[SC7]**
- Derivative at a point
- Relating the graphs of f and f' **[SC5]**
- When does $f'(a)$ fail to exist?
- Rules for differentiation:
  - Sum, product, quotient **[SC7]**
- Chain rule
- Implicit differentiation
- Derivatives of trigonometric, inverse trigonometric, exponential, and logarithmic functions

Chapter 4: Applications of Derivatives (17 days) [SC2]

- Mean value theorem
- Using the derivative to find:
  - Critical point(s) and extreme values

SC1—The course teaches all topics associated with Functions, Graphs, and Limits as delineated in the Calculus BC Topic Outline in the AP Calculus Course Description.

SC2—The course teaches all topics associated with Derivatives as delineated in the Calculus BC Topic Outline in the AP Calculus Course Description.

SC7—The course provides students with the opportunity to work with functions represented analytically.

SC5—The course provides students the opportunity to work with functions represented graphically.

  - When the function is increasing or decreasing
  - Point(s) of inflection
  - When the function is concave up or concave down
- Optimization problems
- Using the tangent line to approximate function values
- Newton's method
- Differentials and change
- Related rates

Chapter 5: The Definite Integral (14 days) [SC3]

- RAM (rectangle approximation method)
- Riemann sums
- Finding a formula for an antiderivative **[SC7]**
- Using a definite integral to find area, volume, average value of a function
- Fundamental theorem of calculus
- Approximating the definite integral:
  - Trapezoidal rule, Simpson's rule, error analysis

SC3—The course teaches all topics associated with Integrals as delineated in the Calculus BC Topic Outline in the AP Calculus Course Description.

SC7—The course provides students with the opportunity to work with functions represented analytically.

Chapter 6: Differential Equations and Mathematical Modeling (15 days)

- Slope fields **[SC5]**
- Antiderivatives and the indefinite integral
- Techniques of integration:
  - Substitution, integration by parts, trigonometric substitution, partial fractions **[SC7]**
- Separable differential equations
- Euler's method
- Exponential growth and decay
- Logistic growth

SC5—The course provides students the opportunity to work with functions represented graphically.

Chapter 7: Applications of Definite Integrals (12 days) [SC3]

- Using the definite integral to discuss:
  - Net change—motion on a line, consumption over time **[SC8]**
  - Area, volume, length of a curve, surface area of a solid of revolution
  - Work, fluid force **[SC8]**

SC8—The course provides students with the opportunity to work with functions represented verbally.

Chapter 8: L'Hôpital's Rule, Improper Integrals, Partial Fractions (13 days) [SC3]

- Indeterminate forms $\left(\frac{0}{0}, \frac{\infty}{\infty}, \infty - \infty, 1^{\infty}, 0^{0}, \infty^{0}\right)$ and L'Hôpital's rule
- Relative rates of growth
- Improper integrals (partial fractions and trig substitutions—done with Chapter 6)

Chapter 10: Parametric, Vector, and Polar Functions (14 days) [SC1]

- Parametric functions:
  - Derivative at a point
  - $\frac{d^2y}{dx^2}$
  - Length of a curve, surface area of a solid of revolution
- Vectors:
  - Angle between vectors
  - Scalar product
  - Using vectors to describe motion in the plane
- Polar coordinates and pole graphs:
  - Slope, horizontal and vertical tangent lines
  - Area, length of a curve

SC1—The course teaches all topics associated with Functions, Graphs, and Limits as delineated in the Calculus AB Topic Outline in the AP Calculus course description.

Chapter 9: Infinite Series (15 days) [SC4]

- Geometric series
- Power series:
  - Term-by-term differentiation and integration to find power series of new functions **[SC7]**
- Taylor's series / Maclaurin series
- Lagrange form of the remainder
- Tests for convergence/divergence:
  - *n*th term test
  - Direct comparison
  - Ratio test
  - Integral test
  - Limit comparison test
  - Alternating series test (Leibniz's theorem)
- Radius and interval of convergence

SC4—The course teaches all topics associated with Polynomial Approximations and Infinite Series as delineated in the Calculus BC Topic Outline in the AP Calculus Course Description.

SC7— The course provides students with the opportunity to work with functions represented analytically.

Review for AP Exam

We go through a few free-response questions together in class and discuss the scoring guidelines so students understand the need for "complete" solutions. Over the next two weeks, students work on free-response sections from three previous AP Calculus Exams (all six questions in each exam) at home. They usually have two nights or a weekend to do each test. Students are encouraged to talk to each other but are supposed to prepare their own solutions in well-written sentences. **[SC10]** They also take a couple

SC10—The course teaches students how to explain solutions to problems in written sentences.

of free-response "mini-tests" in class. I choose four of the six questions (two from the calculator section and two from the noncalculator section), and they work on these under AP Exam–like conditions—that is, 20 minutes with the calculator for the first two questions and the remainder of the 42-minute period without the calculator to finish the test.

Teaching Strategies

AP Calculus Course Description

The topic outline shown above is the skeleton of our course. We study every area mentioned in the *AP Calculus Course Description*. We also include some other topics—Newton's method, Simpson's rule, integration by trigonometric substitution, volume by cylindrical shells—to provide alternate methods of solution or to give an additional look at how calculus zooms into a function to inspect what might be happening at a particular instant, or puts many small pieces of information together to say something about the whole function.

Functions from Multiple Representations

Functions represented analytically make up one segment of our study of calculus. There are so many situations that use functions described verbally, graphically, or numerically, and we work with those along the way.

When a function is presented graphically, we can describe its behavior in broad strokes, and sometimes we can describe the details of its behavior at some specific points. On the other hand, when a function described analytically is graphed, we are careful to check that technological "features" do not mask some behavior.

We work very early in the year with functions described numerically in tables of data, but at that time we use the regression equations given by the TI-83's STAT menu. **[SC6]** As we learn more calculus, we use that data (without a regression equation) to approximate the rate of change at a particular time or the accumulated change over a specified period of time. Students need constant reminders that the regression formula is *not* necessarily the analytic description of the function described by the data. Students use the graphing calculator to experiment with the zoom feature to discover the concept of local linearity and to confirm results that were first obtained analytically, such as the locations of extrema and inflection points. **[SC12 & SC13]**

Functions described verbally have been part of the course for a long time. Verbal descriptions in real-life word problems about the rate of change of a function are often used to create differential equations, and we solve those equations, either analytically or numerically. **[SC8]**

SC6—The course provides students with the opportunity to work with functions represented numerically.

SC12—The course teaches students how to use graphing calculators to experiment.

SC13—The course teaches students how to use graphing calculators to interpret results and support conclusions.

SC8—The course provides students with the opportunity to work with functions represented verbally.

Graphing Calculator

The students come to AP Calculus BC already knowing the basics of operating a graphing calculator. They can graph a function in a variable-sized window, and they know how to "read" that graph. The first chapter of our textbook reviews the various elementary functions with examples from the real world, so this is a good time to introduce regression equations that the STAT menu gives us. We use these regression equations

to make projections about data and to address the very important question about the reasonableness of the projection. The TABLE feature builds on the graph in our initial discussions of the limit of a function. Our slope-field program provides a view of a family of antiderivatives, especially those that cannot be found analytically. **[SC11]** Many calculator-dependent solutions are based on manipulating graphs or on understanding how several graphs are related to each other. Students adapt quickly to using their calculators in clever ways, experimenting to find which line is tangent to a curve at a given point and confirming results that were obtained analytically, such as the numerical values of derivatives or definite integrals. The instruction "support graphically" has become an often-used tool because it is so quick to do and the graph the calculator produces gives students confidence in their conjectures and answers. **[SC13]**

SC11—The course teaches students how to use graphing calculators to help solve problems.

SC13—The course teaches students how to use graphing calculators to interpret results and support conclusions.

My goal for each class period is to develop an interesting discussion. I tend to get into the lecturer mode too easily, so I always welcome questions or comments. In fact, some of the best classes occur when a student asks "Why?" or "Why not?" and the class proposes answers that need to be examined.

Students discuss homework questions in small groups (two to four members) for a few minutes, and unresolved questions are saved for discussion with the entire class. Individual students serve for several days or several weeks as the "homework boss." In that role, the student leads the homework review for the entire class and solicits volunteers who share their work. The "homework boss" also ensures that everyone else is following along.

I preach a positive attitude and encourage hard work, a lot of practice, and organized thinking and speaking.

Student Evaluation

Students meet in small groups of two to four for a few minutes at the beginning of the class period to discuss immediate questions from the assignment. Questions that are not resolved are passed on for discussion with the entire class. **[SC9]**

SC9—The course teaches students how to explain solutions to problems orally.

Quizzes are unannounced and cover small pieces of material, usually work from the prior two or three days. Tests cover larger amounts of material and are almost always cumulative. In-class tests are divided into calculator/noncalculator sections similar to the free-response section of the AP Exam. I occasionally include some multiple-choice questions, but most of the questions require students to show the work that leads to their answers or are short-answer questions.

There is one take-home test each of the first three quarters. Students are encouraged to talk about their ideas with their classmates but are told they should write their own solutions to turn in.

Tests are not scheduled at the conclusion of chapters or units, but rather when they fit into the schedule. The tests tend to include material that is covered up to a few days before each test is given.

Web Resources

- AP Central (apcentral.collegeboard.com)
- AP Calculus Electronic Discussion Group (EDG)