



AP Statistics 1999 Free-Response Questions

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Formulas begin on page 3.
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Formulas

(I) Descriptive Statistics

$$\bar{x} = \frac{\sum x_i}{n}$$

$$s_x = \sqrt{\frac{1}{n-1} \sum (x_i - \bar{x})^2}$$

$$s_p = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{(n_1 - 1) + (n_2 - 1)}}$$

$$\hat{y} = b_0 + b_1x$$

$$b_1 = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sum (x_i - \bar{x})^2}$$

$$b_0 = \bar{y} - b_1\bar{x}$$

$$r = \frac{1}{n-1} \sum \left(\frac{x_i - \bar{x}}{s_x} \right) \left(\frac{y_i - \bar{y}}{s_y} \right)$$

$$b_1 = r \frac{s_y}{s_x}$$

$$s_{b_1} = \frac{\sqrt{\frac{\sum (y_i - \hat{y}_i)^2}{n-2}}}{\sqrt{\sum (x_i - \bar{x})^2}}$$

(II) Probability

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$E(X) = \mu_x = \sum x_i p_i$$

$$\text{Var}(X) = s_x^2 = \sum (x_i - \mu_x)^2 p_i$$

If X has a binomial distribution with parameters n and p , then:

$$P(X = k) = \binom{n}{k} p^k (1 - p)^{n-k}$$

$$\mu_x = np$$

$$s_x = \sqrt{np(1 - p)}$$

$$\mu_{\hat{p}} = p$$

$$s_{\hat{p}} = \sqrt{\frac{p(1 - p)}{n}}$$

If \bar{x} is the mean of a random sample of size n from an infinite population with mean μ and standard deviation S , then:

$$\mu_{\bar{x}} = \mu$$

$$s_{\bar{x}} = \frac{S}{\sqrt{n}}$$

(III) Inferential Statistics

Standardized test statistic: $\frac{\text{estimate} - \text{parameter}}{\text{standard deviation of estimate}}$

Confidence interval: estimate \pm (critical value) • (standard deviation of estimate)

Single-Sample

Statistic	Standard Deviation
Mean	$\frac{S}{\sqrt{n}}$
Proportion	$\sqrt{\frac{p(1-p)}{n}}$

Two-Sample

Statistic	Standard Deviation
Difference of means (unequal variances)	$\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}$
Difference of means (equal variances)	$S\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$
Difference of proportions (unequal variances)	$\sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}$
Difference of proportions (equal variances)	$\sqrt{p(1-p)} \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$

$$\text{Chi-square test statistic} = \sum \frac{(\text{observed} - \text{expected})^2}{\text{expected}}$$

STATISTICS

SECTION II

Part A

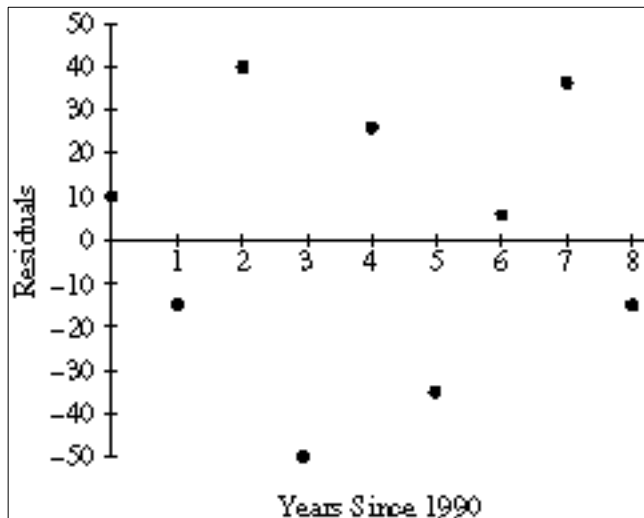
Questions 1-5

Spend about 65 minutes on this part of the exam.

Percent of Section II grade—75

Show all your work. Indicate clearly the methods you use, because you will be graded on the correctness of your methods as well as on the accuracy of your results and explanation.

1. Lydia and Bob were searching the Internet to find information on air travel in the United States. They found data on the number of commercial aircraft flying in the United States during the years 1990-1998. The dates were recorded as years since 1990. Thus, the year 1990 was recorded as year 0. They fit a least squares regression line to the data. The graph of the residuals and part of the computer output for their regression are given below.



Predictor	Coef	Stdev	t-ratio	p
Constant	2939.93	20.55	143.09	0.000
Years	233.517	4.316	54.11	0.000

s = 33.43

- Is a line an appropriate model to use for these data? What information tells you this?
- What is the value of the slope of the least squares regression line? Interpret the slope in the context of this situation.
- What is the value of the intercept of the least squares regression line? Interpret the intercept in the context of this situation.
- What is the predicted number of commercial aircraft flying in 1992 ?
- What was the actual number of commercial aircraft flying in 1992 ?



2. The Colorado Rocky Mountain Rescue Service wishes to study the behavior of lost hikers. If more were known about the direction in which lost hikers tend to walk, then more effective search strategies could be devised. Two hundred hikers selected at random from those applying for hiking permits are asked whether they would head uphill, downhill, or remain in the same place if they became lost while hiking. Each hiker in the sample was also classified according to whether he or she was an experienced or novice hiker. The resulting data are summarized in the following table.

	Direction		
	Uphill	Downhill	Remain in Same Place
Novice	20	50	50
Experienced	10	30	40

Do these data provide convincing evidence of an association between the level of hiking expertise and the direction the hiker would head if lost?

Give appropriate statistical evidence to support your conclusion.

3. The dentists in a dental clinic would like to determine if there is a difference between the number of new cavities in people who eat an apple a day and in people who eat less than one apple a week. They are going to conduct a study with 50 people in each group.

Fifty clinic patients who report that they routinely eat an apple a day and 50 clinic patients who report that they eat less than one apple a week will be identified. The dentists will examine the patients and their records to determine the number of new cavities the patients have had over the past two years. They will then compare the number of new cavities in the two groups.

- Why is this an observational study and not an experiment?
- Explain the concept of confounding in the context of this study. Include an example of a possible confounding variable.
- If the mean number of new cavities for those who ate an apple a day was statistically significantly smaller than the mean number of new cavities for those who ate less than one apple a week, could one conclude that the lower number of new cavities can be attributed to eating an apple a day? Explain.

4. A company is considering implementing one of two quality control plans for monitoring the weights of automobile batteries that it manufactures. If the manufacturing process is working properly, the battery weights are approximately normally distributed with a specified mean and standard deviation.

Quality control plan A calls for rejecting a battery as defective if its weight falls more than 2 standard deviations below the specified mean.

Quality control plan B calls for rejecting a battery as defective if its weight falls more than 1.5 interquartile ranges below the lower quartile of the specified population.

Assume the manufacturing process is under control.

- a. What proportion of batteries will be rejected by plan A ?
- b. What is the probability that at least 1 of 2 randomly selected batteries will be rejected by plan A ?
- c. What proportion of batteries will be rejected by plan B ?

5. Die A has four 9's and two 0's on its faces. Die B has four 3's and two 11's on its faces. When either of these dice is rolled, each face has an equal chance of landing on top. Two players are going to play a game. The first player selects a die and rolls it. The second player rolls the remaining die. The winner is the player whose die has the higher number on top.
- Suppose you are the first player and you want to win the game. Which die would you select? Justify your answer.
 - Suppose the player using die A receives 45 tokens each time he or she wins the game. How many tokens must the player using die B receive each time he or she wins in order for this to be a fair game? Explain how you found your answer.
- (A fair game is one in which the player using die A and the player using die B both end up with the same number of tokens in the long run.)

STATISTICS

SECTION II

Part B

Question 6

Spend about 25 minutes on this part of the exam.

Percent of Section II grade—25

6. Researchers want to see whether training increases the capability of people to correctly predict outcomes of coin tosses. Each of twenty people is asked to predict the outcome (heads or tails) of 100 independent tosses of a fair coin. After training, they are retested with a new set of 100 tosses. (All 40 sets of 100 tosses are independently generated.) Since the coin is fair, the probability of a correct guess by chance is 0.5 on each toss. The numbers correct for each of the 20 people were as follows.

<u>Score Before Training (number correct)</u>	<u>Score After Training (number correct)</u>
46	61
48	62
50	53
54	46
54	50
54	52
54	53
54	59
54	60
54	61
55	55
56	59
57	55
58	50
58	56
61	58
61	64
63	57
64	61
65	54
Sum 1,120	Sum 1,126

To answer the following questions, you may want to enter these data into your calculator. As a check that you have entered the data correctly, the sum of the first column is 1,120 and the sum of the second column is 1,126.

- a. Do the data suggest that after training people can correctly predict coin toss outcomes better than the 50 percent expected by chance guessing alone?

Give appropriate statistical evidence to support your conclusion.



- b. Does the statistical test that you completed in part (a) provide evidence that this training is effective in improving a person's ability to predict coin toss outcomes?

If yes, justify your answer. If no, conduct an appropriate analysis that would allow you to determine whether or not the training is effective.

- c. Would knowing a person's score before training be helpful in predicting his or her score after training? Justify your answer.

END OF EXAMINATION

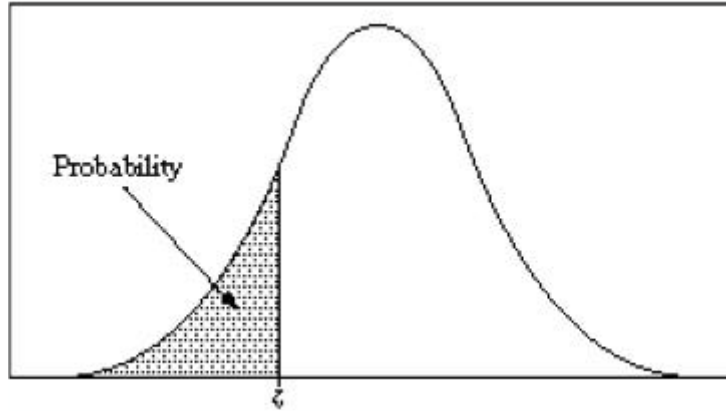


Table entry for z is the probability lying below z .

Table A Standard normal probabilities

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
-3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
-3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
-3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
-3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
-2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
-2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
-2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
-2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
-2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
-2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
-2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
-2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
-2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
-2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
-1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
-1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
-1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
-1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
-1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
-1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
-1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
-1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
-1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
-1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
-0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
-0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
-0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
-0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
-0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
-0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
-0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
-0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
-0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
-0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

Table entry for p and C is the point t^* with probability p lying above it and probability C lying between $-t^*$ and t^* .

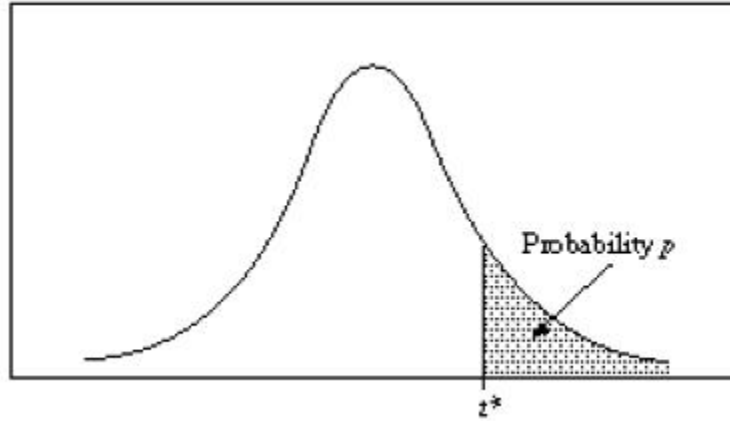


Table B t distribution critical values

df	Tail probability p											
	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.000	1.376	1.963	3.078	6.314	12.71	15.89	31.82	63.66	127.3	318.3	636.6
2	.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.09	22.33	31.60
3	.765	.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.21	12.92
4	.741	.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610
5	.727	.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773	5.893	6.869
6	.718	.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317	5.208	5.959
7	.711	.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029	4.785	5.408
8	.706	.889	1.108	1.397	1.860	2.306	2.449	2.896	3.355	3.833	4.501	5.041
9	.703	.883	1.100	1.383	1.833	2.262	2.398	2.821	3.250	3.690	4.297	4.781
10	.700	.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	3.581	4.144	4.587
11	.697	.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497	4.025	4.437
12	.695	.873	1.083	1.356	1.782	2.179	2.303	2.681	3.055	3.428	3.930	4.318
13	.694	.870	1.079	1.350	1.771	2.160	2.282	2.650	3.012	3.372	3.852	4.221
14	.692	.868	1.076	1.345	1.761	2.145	2.264	2.624	2.977	3.326	3.787	4.140
15	.691	.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.286	3.733	4.073
16	.690	.865	1.071	1.337	1.746	2.120	2.235	2.583	2.921	3.252	3.686	4.015
17	.689	.863	1.069	1.333	1.740	2.110	2.224	2.567	2.898	3.222	3.646	3.965
18	.688	.862	1.067	1.330	1.734	2.101	2.214	2.552	2.878	3.197	3.611	3.922
19	.688	.861	1.066	1.328	1.729	2.093	2.205	2.539	2.861	3.174	3.579	3.883
20	.687	.860	1.064	1.325	1.725	2.086	2.197	2.528	2.845	3.153	3.552	3.850
21	.686	.859	1.063	1.323	1.721	2.080	2.189	2.518	2.831	3.135	3.527	3.819
22	.686	.858	1.061	1.321	1.717	2.074	2.183	2.508	2.819	3.119	3.505	3.792
23	.685	.858	1.060	1.319	1.714	2.069	2.177	2.500	2.807	3.104	3.485	3.768
24	.685	.857	1.059	1.318	1.711	2.064	2.172	2.492	2.797	3.091	3.467	3.745
25	.684	.856	1.058	1.316	1.708	2.060	2.167	2.485	2.787	3.078	3.450	3.725
26	.684	.856	1.058	1.315	1.706	2.056	2.162	2.479	2.779	3.067	3.435	3.707
27	.684	.855	1.057	1.314	1.703	2.052	2.158	2.473	2.771	3.057	3.421	3.690
28	.683	.855	1.056	1.313	1.701	2.048	2.154	2.467	2.763	3.047	3.408	3.674
29	.683	.854	1.055	1.311	1.699	2.045	2.150	2.462	2.756	3.038	3.396	3.659
30	.683	.854	1.055	1.310	1.697	2.042	2.147	2.457	2.750	3.030	3.385	3.646
40	.681	.851	1.050	1.303	1.684	2.021	2.123	2.423	2.704	2.971	3.307	3.551
50	.679	.849	1.047	1.299	1.676	2.009	2.109	2.403	2.678	2.937	3.261	3.496
60	.679	.848	1.045	1.296	1.671	2.000	2.099	2.390	2.660	2.915	3.232	3.460
80	.678	.846	1.043	1.292	1.664	1.990	2.088	2.374	2.639	2.887	3.195	3.416
100	.677	.845	1.042	1.290	1.660	1.984	2.081	2.364	2.626	2.871	3.174	3.390
1000	.675	.842	1.037	1.282	1.646	1.962	2.056	2.330	2.581	2.813	3.098	3.300
	.674	.841	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807	3.091	3.291
	50%	60%	70%	80%	90%	95%	96%	98%	99%	99.5%	99.8%	99.9%
Confidence level C												

Table entry for p is the point (c^2) with probability p lying above it.

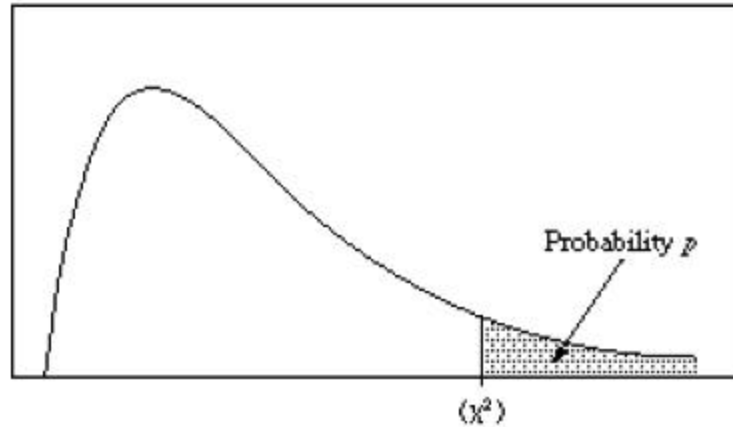


Table C c^2 critical values

df	Tail probability p											
	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.32	1.64	2.07	2.71	3.84	5.02	5.41	6.63	7.88	9.14	10.83	12.12
2	2.77	3.22	3.79	4.61	5.99	7.38	7.82	9.21	10.60	11.98	13.82	15.20
3	4.11	4.64	5.32	6.25	7.81	9.35	9.84	11.34	12.84	14.32	16.27	17.73
4	5.39	5.99	6.74	7.78	9.49	11.14	11.67	13.28	14.86	16.42	18.47	20.00
5	6.63	7.29	8.12	9.24	11.07	12.83	13.39	15.09	16.75	18.39	20.51	22.11
6	7.84	8.56	9.45	10.64	12.59	14.45	15.03	16.81	18.55	20.25	22.4	24.10
7	9.04	9.80	10.75	12.02	14.07	16.01	16.62	18.48	20.28	22.04	24.3	26.02
8	10.22	11.03	12.03	13.36	15.51	17.53	18.17	20.09	21.95	23.77	26.1	27.87
9	11.39	12.24	13.29	14.68	16.92	19.02	19.68	21.67	23.59	25.46	27.8	29.67
10	12.55	13.44	14.53	15.99	18.31	20.48	21.16	23.21	25.19	27.11	29.5	31.42
11	13.70	14.63	15.77	17.28	19.68	21.92	22.62	24.72	26.76	28.73	31.26	33.14
12	14.85	15.81	16.99	18.55	21.03	23.34	24.05	26.22	28.30	30.32	32.91	34.82
13	15.98	16.98	18.20	19.81	22.36	24.74	25.47	27.69	29.82	31.88	34.53	36.48
14	17.12	18.15	19.41	21.06	23.68	26.12	26.87	29.14	31.32	33.43	36.12	38.11
15	18.25	19.31	20.60	22.31	25.00	27.49	28.26	30.58	32.80	34.95	37.70	39.72
16	19.37	20.47	21.79	23.54	26.30	28.85	29.63	32.00	34.27	36.46	39.2	41.31
17	20.49	21.61	22.98	24.77	27.59	30.19	31.00	33.41	35.72	37.95	40.7	42.88
18	21.60	22.76	24.16	25.99	28.87	31.53	32.35	34.81	37.16	39.42	42.3	44.43
19	22.72	23.90	25.33	27.20	30.14	32.85	33.69	36.19	38.58	40.88	43.8	45.97
20	23.83	25.04	26.50	28.41	31.41	34.17	35.02	37.57	40.00	42.34	45.3	47.50
21	24.93	26.17	27.66	29.62	32.67	35.48	36.34	38.93	41.40	43.78	46.80	49.01
22	26.04	27.30	28.82	30.81	33.92	36.78	37.66	40.29	42.80	45.20	48.27	50.51
23	27.14	28.43	29.98	32.01	35.17	38.08	38.97	41.64	44.18	46.62	49.73	52.00
24	28.24	29.55	31.13	33.20	36.42	39.36	40.27	42.98	45.56	48.03	51.18	53.48
25	29.34	30.68	32.28	34.38	37.65	40.65	41.57	44.31	46.93	49.44	52.62	54.95
26	30.43	31.79	33.43	35.56	38.89	41.92	42.86	45.64	48.29	50.82	54.0	56.41
27	31.53	32.91	34.57	36.74	40.11	43.19	44.14	46.96	49.64	52.22	55.4	57.86
28	32.62	34.03	35.71	37.92	41.34	44.46	45.42	48.28	50.99	53.55	56.8	59.30
29	33.71	35.14	36.85	39.09	42.56	45.72	46.69	49.59	52.34	54.97	58.3	60.73
30	34.80	36.25	37.99	40.26	43.77	46.98	47.96	50.89	53.67	56.33	59.7	62.16
40	45.62	47.27	49.24	51.81	55.76	59.34	60.44	63.69	66.77	69.70	73.40	76.09
50	56.33	58.16	60.35	63.17	67.50	71.42	72.61	76.15	79.49	82.66	86.66	89.56
60	66.98	68.97	71.34	74.40	79.08	83.30	84.58	88.38	91.95	95.34	99.61	102.7
80	88.13	90.41	93.11	96.58	101.9	106.6	108.1	112.3	116.3	120.1	124.8	128.3
100	109.1	111.7	114.7	118.5	124.3	129.6	131.1	135.8	140.2	144.3	149.4	153.2

NO TEST MATERIAL ON THIS PAGE