AP Physics C: Electricity & Magnetism
1999 Scoring Guidelines

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E & M 1 (15 points)

(a) 4 points

For using the relationship between potential and charge
\[ V = \frac{1}{4\pi \varepsilon_0} \frac{Q}{r} \]
1 point

Solving for \( Q \):
\[ Q = 4\pi \varepsilon_0 V r \]
1 point

For correct substitutions for the potential and radius
\[ Q_0 = 4\pi \varepsilon_0 (-2000 \text{ V})(0.20 \text{ m}) \quad \text{or} \quad (-2000 \text{ V})(0.20 \text{ m})/(9 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \]
1 point

\[ Q_0 = -1600\pi \varepsilon_0 \text{ C} \quad \text{or} \quad -4.4 \times 10^{-8} \text{ C} \]
1 point

For the correct magnitude of \( Q_0 \)
For the negative sign
1 point

(b) 5 points

i. For indicating that the electric field is zero
1 point

ii. The charge on the sphere can be treated as a point charge at its center
\[ E = \frac{1}{4\pi \varepsilon_0} \frac{Q_0}{r^2} \]

\[ E = (9 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \left( \frac{4.4 \times 10^{-8} \text{ C}}{r^2} \right) \]

\[ E = \frac{396}{r^2} \text{ N/C} \quad \text{or} \quad \frac{400}{r^2} \text{ N/C} \quad \text{where } r \text{ is in meters} \]
1 point

iii. For indicating that the electric field is zero
1 point

iv. For indicating that the electric field is zero
1 point

For having all four answers correct OR for some mention of using the enclosed charge OR for some mention of Gauss’ law
1 point
1999 Physics C Solutions

E & M 1 (continued)

(c) 3 points

\[ \Delta V = V_b - V_a = - \int_a^b E \, dr \]

For recognition of the need to take the difference of the potentials at radii \( a \) and \( b \), or for writing the definite integral (with limits)

\[ |\Delta V| = \frac{Q_0}{4\pi\varepsilon_0} \int_a^b \frac{dr}{r^2} \]

\[ = \frac{Q_0}{4\pi\varepsilon_0} \left( \frac{1}{r} \right)_a^b \]

\[ |\Delta V| = \frac{Q_0}{4\pi\varepsilon_0} \left( \frac{1}{b} - \frac{1}{a} \right) \]

For correct substitution of variables or numerical values for \( Q_0, a, \) and \( b \) 1 point

For the correct answer 1 point

\[ |\Delta V| = \frac{5Q_0}{8\pi\varepsilon_0} \quad \text{or} \quad 1000 \, \text{V} \]

(Alternate solution)

\[ \Delta V = V_b - V_a \]

\[ \Delta V = \frac{Q_0}{4\pi\varepsilon_0} \left( \frac{1}{r_b} \right) - \frac{Q_0}{4\pi\varepsilon_0} \left( \frac{1}{r_a} \right) \]

For correct substitution of \( Q_0, a, \) and \( b \) 1 point

For the correct answer 1 point

\[ |\Delta V| = \frac{5Q_0}{8\pi\varepsilon_0} \quad \text{or} \quad 1000 \, \text{V} \]

(Alternate solution)

\[ V = \frac{Q}{C} \]

For using the above relationship 1 point

For substituting \( Q_0 \) from part (a) and \( C \) from part (d) alternate solution 1 point

For the correct answer 1 point

\[ |\Delta V| = \frac{5Q_0}{8\pi\varepsilon_0} \quad \text{or} \quad 1000 \, \text{V} \]
E & M 1 (continued)

(d) 2 points

\[ C = \frac{Q_0}{V} \]

For using the above relationship
For substituting \( Q_0 \) from part (a) and \( \Delta V \) from part (c)

\[ C = \frac{4.4 \times 10^{-8} \text{ C}}{1000 \text{ V}} \]
\[ C = 4.4 \times 10^{-11} \text{ F} \]

(Alternate solution)
For writing the equation for the capacitance of the spherical capacitor

\[ C = \frac{4\pi\varepsilon_0 ab}{b-a} \]

\[ C = \frac{(0.02 \text{ m})(0.04 \text{ m})}{(9 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(0.04 \text{ m} - 0.02 \text{ m})} \]

For the correct answer
\[ C = 4.4 \times 10^{-11} \text{ F} \]

For correct units on two answers and no incorrect units 1 point
E & M 2 (15 points)

(a) 5 points

For using Faraday’s law for a loop
\[ \mathcal{E} = -\frac{d\phi}{dt} \quad \text{or} \quad \mathcal{E} = -\Delta\phi / \Delta t \]

1 point

For relating magnetic flux to magnetic field and area
\[ \frac{d\phi}{dt} = A \frac{dB}{dt} \quad \text{or} \quad \frac{\Delta\phi}{\Delta t} = A \frac{\Delta B}{\Delta t} \]

1 point

For using the proper expression for the area of a loop
\[ A = \pi r^2 \]

1 point

\[ \mathcal{E} = \pi r^2 \frac{dB}{dt} \quad \text{or} \quad \mathcal{E} = \pi r^2 \frac{\Delta B}{\Delta t} \]

1 point

For using the correct radius, i.e. the radius of the field
\[ \mathcal{E} = \pi(0.6 \text{ m})^2 (0.40 \text{ T/s}) \]

1 point

For computing the correct answer
\[ \mathcal{E} = 0.45 \text{ V} \]

1 point

(b) 3 points

For any statement of Ohm’s law
\[ V = IR \]

1 point

Solving for the current:
\[ I = \frac{V}{R} = \frac{E}{R} \]

For computing the correct answer
\[ I = 0.090 \text{ A} \]

1 point

For indicating a clockwise direction for the current

1 point

(c) 3 points

For relating the energy dissipated to the power in the resistor
\[ E = \int P \, dt \quad \text{or} \quad E = Pt \]

1 point

For an expression for electric power
\[ P = I^2 R \quad \text{or} \quad \frac{V^2}{R} \quad \text{or} \quad IV \]

1 point

Example using \( P = I^2 R \):
\[ E = I^2 R t \]

For computing the correct answer
\[ E = (0.090 \text{ A})^2 (5.0 \text{ } \Omega)(15 \text{ s}) \]

1 point

\[ E = 0.61 \text{ J} \]
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E & M 2 (continued)

(d) 3 points

For stating that the brightness of the bulb will be less 1 point
For indicating that the reduction in brightness is due to a decrease in current or a decrease in the emf 1 point
For indicating that the decrease in current or emf, or the reduction in brightness, is due to a decrease in the area of the loop or a decrease in the changing flux 1 point

For using correct units with three numerical answers 1 point
E & M 3 (15 points)

(a) 3 points

The charge on any section of the ring is equidistant from a point on the x-axis, so one can write an equation in terms of the single distance $r$

For a correct expression of the potential

$$\frac{1}{4\pi\epsilon_0} \frac{dq}{r} \quad \text{or} \quad \frac{1}{4\pi\epsilon_0} \frac{Q}{r}$$

For a correct expression for the distance of the charge from location $x$

$$r = \sqrt{x^2 + R^2}$$

For the correct answer

$$\frac{1}{4\pi\epsilon_0} \frac{Q}{\sqrt{x^2 + R^2}}$$

Alternate solution

For correctly expressing the potential as an integral of the electric field

$$dV = -\int E \, dr$$

For a correct expression for the field

$$dV = -\int \frac{1}{4\pi\epsilon_0} \frac{Qx}{(x^2 + R^2)^{3/2}} \, dx$$

For correctly integrating to get the final answer

$$\frac{1}{4\pi\epsilon_0} \frac{Q}{\sqrt{x^2 + R^2}}$$

(b)

i. 3 points

$$E = -\frac{dV}{dr}$$

For using the above relationship

For taking the derivative with respect to $x$ 1 point

For using the expression for $V$ obtained in part (a)

$$E_x = -\frac{d}{dx} \left( \frac{1}{4\pi\epsilon_0} \frac{Q}{\sqrt{x^2 + R^2}} \right)$$

$$E_x = \frac{1}{4\pi\epsilon_0} \frac{Qx}{(x^2 + R^2)^{3/2}}$$
Alternate solution

Calculating the field by integration:

$$E = \int dE_x = \int dE \cos \theta,$$
where $\theta$ is the angle between the $x$-axis and the distance vector $\mathbf{r}$.

For using the horizontal component of the field

For using a correct expression of Coulomb's law

$$E = \frac{1}{4\pi \varepsilon_0} \frac{Q}{r^2}$$

For indicating that the integral is over the charge

$$E_x = \int \frac{1}{4\pi \varepsilon_0} \frac{dq}{r^2} \cos \theta$$

Substituting $\cos \theta = x/r$ and $r = \sqrt{x^2 + R^2}$

$$E_x = \frac{1}{4\pi \varepsilon_0} \frac{Q}{(x^2 + R^2)^{3/2}} \int dq$$

$$E_x = \frac{1}{4\pi \varepsilon_0} \frac{Qx}{(x^2 + R^2)^{3/2}}$$

ii. 1 point

For any indication that the $y$- and $z$-components are zero or cancel 1 point
(c)

i. 2 points

For taking the derivative of $E$ with respect to $x$ and setting it equal to zero

$$\frac{dE_x}{dx} = \frac{d}{dx} \left( \frac{1}{4\pi \varepsilon_0} \frac{Qx}{(x^2 + R^2)^{\frac{3}{2}}} \right) = 0$$

$$\frac{Q}{4\pi \varepsilon_0} \left( \frac{1}{(x^2 + R^2)^{\frac{3}{2}}} + \left( -\frac{3}{2} \right) \frac{2x^2}{(x^2 + R^2)^{\frac{5}{2}}} \right) = 0$$

$$\frac{1}{(x^2 + R^2)^{\frac{3}{2}}} - \frac{3x^2}{(x^2 + R^2)^{\frac{5}{2}}} = 0$$

$$\frac{1}{(x^2 + R^2)^{\frac{3}{2}}} = \frac{3x^2}{(x^2 + R^2)^{\frac{5}{2}}}$$

$$1 = \frac{3x^2}{x^2 + R^2}$$

$$3x^2 = x^2 + R^2$$

$$x = \pm \frac{R}{\sqrt{2}}$$ and the maximum occurs at the positive value of $x$

For the correct answer

$$x = + \frac{R}{\sqrt{2}}$$

1 point

ii. 1 point

For substituting the answer from part (c)i into the given expression for the electric field

$$E_{x\ max} = \frac{1}{4\pi \varepsilon_0} \frac{Q(R/\sqrt{2})}{((R/\sqrt{2})^2 + R^2)^{\frac{3}{2}}}$$

$$E_{x\ max} = \frac{1}{4\pi \varepsilon_0} \frac{2Q}{3\sqrt{2}R^2}$$

1 point
E & M 3 (continued)

(d) 3 points

For a curve in the first quadrant displaying a single positive maximum 1 point
For a curve passing through the origin 1 point
For the negative reflection of the first quadrant curve in the third quadrant 1 point

(e) 2 points

For any statement that describes the subsequent motion as oscillating, periodic etc. 2 points

One point was awarded for a statement that only described the electron as moving toward the ring or along the x-axis.