



## AP Calculus AB 2001 Scoring Guidelines

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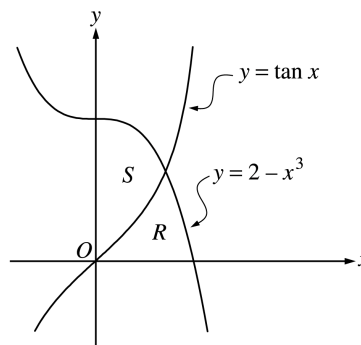
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**Question 1**

Let  $R$  and  $S$  be the regions in the first quadrant shown in the figure above. The region  $R$  is bounded by the  $x$ -axis and the graphs of  $y = 2 - x^3$  and  $y = \tan x$ . The region  $S$  is bounded by the  $y$ -axis and the graphs of  $y = 2 - x^3$  and  $y = \tan x$ .



- (a) Find the area of  $R$ .  
 (b) Find the area of  $S$ .  
 (c) Find the volume of the solid generated when  $S$  is revolved about the  $x$ -axis.

Point of intersection

$$2 - x^3 = \tan x \text{ at } (A, B) = (0.902155, 1.265751)$$

(a) Area  $R = \int_0^A \tan x \, dx + \int_A^{\sqrt[3]{2}} (2 - x^3) \, dx = 0.729$

or

$$\text{Area } R = \int_0^B ((2 - y)^{1/3} - \tan^{-1} y) \, dy = 0.729$$

or

$$\text{Area } R = \int_0^{\sqrt[3]{2}} (2 - x^3) \, dx - \int_0^A (2 - x^3 - \tan x) \, dx = 0.729$$

(b) Area  $S = \int_0^A (2 - x^3 - \tan x) \, dx = 1.160$  or  $1.161$

or

$$\text{Area } S = \int_0^B \tan^{-1} y \, dy + \int_B^2 (2 - y)^{1/3} \, dy = 1.160 \text{ or } 1.161$$

or

$$\begin{aligned} \text{Area } S &= \int_0^2 (2 - y)^{1/3} \, dy - \int_0^B ((2 - y)^{1/3} - \tan^{-1} y) \, dy \\ &= 1.160 \text{ or } 1.161 \end{aligned}$$

(c) Volume  $= \pi \int_0^A ((2 - x^3)^2 - \tan^2 x) \, dx$   
 $= 2.652\pi$  or  $8.331$  or  $8.332$

3 : { 1 : limits  
1 : integrand  
1 : answer

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1 : integrand  
1 : answer

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**Question 2**

The temperature, in degrees Celsius ( $^{\circ}\text{C}$ ), of the water in a pond is a differentiable function  $W$  of time  $t$ . The table above shows the water temperature as recorded every 3 days over a 15-day period.

$t$ (days)	$W(t)$ ( $^{\circ}\text{C}$ )
0	20
3	31
6	28
9	24
12	22
15	21

- (a) Use data from the table to find an approximation for  $W'(12)$ . Show the computations that lead to your answer. Indicate units of measure.
- (b) Approximate the average temperature, in degrees Celsius, of the water over the time interval  $0 \leq t \leq 15$  days by using a trapezoidal approximation with subintervals of length  $\Delta t = 3$  days.
- (c) A student proposes the function  $P$ , given by  $P(t) = 20 + 10te^{(-t/3)}$ , as a model for the temperature of the water in the pond at time  $t$ , where  $t$  is measured in days and  $P(t)$  is measured in degrees Celsius. Find  $P'(12)$ . Using appropriate units, explain the meaning of your answer in terms of water temperature.
- (d) Use the function  $P$  defined in part (c) to find the average value, in degrees Celsius, of  $P(t)$  over the time interval  $0 \leq t \leq 15$  days.

- (a) Difference quotient; e.g.

$$W'(12) \approx \frac{W(15) - W(12)}{15 - 12} = -\frac{1}{3} \text{ }^{\circ}\text{C/day or}$$

$$W'(12) \approx \frac{W(12) - W(9)}{12 - 9} = -\frac{2}{3} \text{ }^{\circ}\text{C/day or}$$

$$W'(12) \approx \frac{W(15) - W(9)}{15 - 9} = -\frac{1}{2} \text{ }^{\circ}\text{C/day}$$

- (b)  $\frac{3}{2}(20 + 2(31) + 2(28) + 2(24) + 2(22) + 21) = 376.5$

$$\text{Average temperature} \approx \frac{1}{15}(376.5) = 25.1 \text{ }^{\circ}\text{C}$$

- (c)  $P'(12) = 10e^{-t/3} - \frac{10}{3}te^{-t/3} \Big|_{t=12}$   
 $= -30e^{-4} = -0.549 \text{ }^{\circ}\text{C/day}$

This means that the temperature is decreasing at the rate of  $0.549 \text{ }^{\circ}\text{C/day}$  when  $t = 12$  days.

- (d)  $\frac{1}{15} \int_0^{15} (20 + 10te^{-t/3}) dt = 25.757 \text{ }^{\circ}\text{C}$

$$2 : \begin{cases} 1 : \text{difference quotient} \\ 1 : \text{answer (with units)} \end{cases}$$

$$2 : \begin{cases} 1 : \text{trapezoidal method} \\ 1 : \text{answer} \end{cases}$$

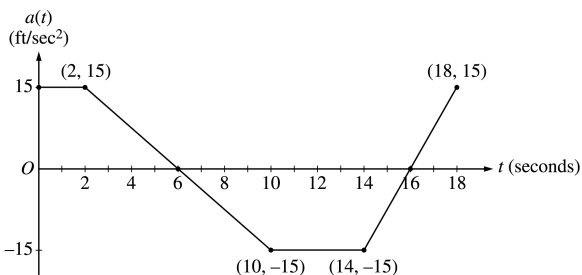
$$2 : \begin{cases} 1 : P'(12) \text{ (with or without units)} \\ 1 : \text{interpretation} \end{cases}$$

$$3 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{limits and} \\ \quad \text{average value constant} \\ 1 : \text{answer} \end{cases}$$

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**Question 3**

A car is traveling on a straight road with velocity 55 ft/sec at time  $t = 0$ . For  $0 \leq t \leq 18$  seconds, the car's acceleration  $a(t)$ , in ft/sec<sup>2</sup>, is the piecewise linear function defined by the graph above.



- (a) Is the velocity of the car increasing at  $t = 2$  seconds? Why or why not?
- (b) At what time in the interval  $0 \leq t \leq 18$ , other than  $t = 0$ , is the velocity of the car 55 ft/sec? Why?
- (c) On the time interval  $0 \leq t \leq 18$ , what is the car's absolute maximum velocity, in ft/sec, and at what time does it occur? Justify your answer.
- (d) At what times in the interval  $0 \leq t \leq 18$ , if any, is the car's velocity equal to zero? Justify your answer.

(a) Since  $v'(2) = a(2)$  and  $a(2) = 15 > 0$ , the velocity is increasing at  $t = 2$ .

1 : answer and reason

(b) At time  $t = 12$  because  

$$v(12) - v(0) = \int_0^{12} a(t) dt = 0.$$

2 :  $\left\{ \begin{array}{l} 1 : t = 12 \\ 1 : \text{reason} \end{array} \right.$

(c) The absolute maximum velocity is 115 ft/sec at  $t = 6$ .

The absolute maximum must occur at  $t = 6$  or at an endpoint.

$$\begin{aligned} v(6) &= 55 + \int_0^6 a(t) dt \\ &= 55 + 2(15) + \frac{1}{2}(4)(15) = 115 > v(0) \\ \int_6^{18} a(t) dt &< 0 \text{ so } v(18) < v(6) \end{aligned}$$

4 :  $\left\{ \begin{array}{l} 1 : t = 6 \\ 1 : \text{absolute maximum velocity} \\ 1 : \text{identifies } t = 6 \text{ and } \\ \quad t = 18 \text{ as candidates} \\ \text{or} \\ \text{indicates that } v \text{ increases,} \\ \quad \text{decreases, then increases} \\ 1 : \text{eliminates } t = 18 \end{array} \right.$

(d) The car's velocity is never equal to 0. The absolute minimum occurs at  $t = 16$  where

$$v(16) = 115 + \int_6^{16} a(t) dt = 115 - 105 = 10 > 0.$$

2 :  $\left\{ \begin{array}{l} 1 : \text{answer} \\ 1 : \text{reason} \end{array} \right.$

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**Question 4**

Let  $h$  be a function defined for all  $x \neq 0$  such that  $h(4) = -3$  and the derivative of  $h$  is given by  $h'(x) = \frac{x^2 - 2}{x}$  for all  $x \neq 0$ .

- (a) Find all values of  $x$  for which the graph of  $h$  has a horizontal tangent, and determine whether  $h$  has a local maximum, a local minimum, or neither at each of these values. Justify your answers.
- (b) On what intervals, if any, is the graph of  $h$  concave up? Justify your answer.
- (c) Write an equation for the line tangent to the graph of  $h$  at  $x = 4$ .
- (d) Does the line tangent to the graph of  $h$  at  $x = 4$  lie above or below the graph of  $h$  for  $x > 4$ ? Why?

(a)  $h'(x) = 0$  at  $x = \pm\sqrt{2}$

$$h'(x) \quad \begin{array}{ccccccc} & - & 0 & + & \text{und} & - & 0 & + \\ & & | & & | & & | & \\ x & & -\sqrt{2} & & 0 & & \sqrt{2} & \end{array}$$

Local minima at  $x = -\sqrt{2}$  and at  $x = \sqrt{2}$

(b)  $h''(x) = 1 + \frac{2}{x^2} > 0$  for all  $x \neq 0$ . Therefore, the graph of  $h$  is concave up for all  $x \neq 0$ .

(c)  $h'(4) = \frac{16 - 2}{4} = \frac{7}{2}$

$$y + 3 = \frac{7}{2}(x - 4)$$

(d) The tangent line is below the graph because the graph of  $h$  is concave up for  $x > 4$ .

$$4 : \left\{ \begin{array}{l} 1 : x = \pm\sqrt{2} \\ 1 : \text{analysis} \\ 2 : \text{conclusions} \\ \quad < -1 > \text{not dealing with} \\ \quad \text{discontinuity at } 0 \end{array} \right.$$

$$3 : \left\{ \begin{array}{l} 1 : h''(x) \\ 1 : h''(x) > 0 \\ 1 : \text{answer} \end{array} \right.$$

1 : tangent line equation

1 : answer with reason

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**Question 5**

A cubic polynomial function  $f$  is defined by

$$f(x) = 4x^3 + ax^2 + bx + k$$

where  $a$ ,  $b$ , and  $k$  are constants. The function  $f$  has a local minimum at  $x = -1$ , and the graph of  $f$  has a point of inflection at  $x = -2$ .

(a) Find the values of  $a$  and  $b$ .

(b) If  $\int_0^1 f(x) dx = 32$ , what is the value of  $k$ ?

(a)  $f'(x) = 12x^2 + 2ax + b$

$$f''(x) = 24x + 2a$$

$$f'(-1) = 12 - 2a + b = 0$$

$$f''(-2) = -48 + 2a = 0$$

$$a = 24$$

$$b = -12 + 2a = 36$$

$$5 : \left\{ \begin{array}{l} 1 : f'(x) \\ 1 : f''(x) \\ 1 : f'(-1) = 0 \\ 1 : f''(-2) = 0 \\ 1 : a, b \end{array} \right.$$

(b)  $\int_0^1 (4x^3 + 24x^2 + 36x + k) dx$

$$= x^4 + 8x^3 + 18x^2 + kx \Big|_{x=0}^{x=1} = 27 + k$$

$$27 + k = 32$$

$$k = 5$$

$$4 : \left\{ \begin{array}{l} 2 : \text{antidifferentiation} \\ \quad < -1 > \text{ each error} \\ 1 : \text{expression in } k \\ 1 : k \end{array} \right.$$

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**Question 6**

The function  $f$  is differentiable for all real numbers. The point  $\left(3, \frac{1}{4}\right)$  is on the graph of  $y = f(x)$ , and the slope at each point  $(x, y)$  on the graph is given by  $\frac{dy}{dx} = y^2(6 - 2x)$ .

- (a) Find  $\frac{d^2y}{dx^2}$  and evaluate it at the point  $\left(3, \frac{1}{4}\right)$ .
- (b) Find  $y = f(x)$  by solving the differential equation  $\frac{dy}{dx} = y^2(6 - 2x)$  with the initial condition  $f(3) = \frac{1}{4}$ .

(a) 
$$\begin{aligned} \frac{d^2y}{dx^2} &= 2y \frac{dy}{dx} (6 - 2x) - 2y^2 \\ &= 2y^3(6 - 2x)^2 - 2y^2 \end{aligned}$$

$$\left. \frac{d^2y}{dx^2} \right|_{\left(3, \frac{1}{4}\right)} = 0 - 2\left(\frac{1}{4}\right)^2 = -\frac{1}{8}$$

(b) 
$$\frac{1}{y^2} dy = (6 - 2x) dx$$

$$-\frac{1}{y} = 6x - x^2 + C$$

$$-4 = 18 - 9 + C = 9 + C$$

$$C = -13$$

$$y = \frac{1}{x^2 - 6x + 13}$$

$$3 : \left\{ \begin{array}{l} 2 : \frac{d^2y}{dx^2} \\ < -2 > \text{product rule or} \\ & \text{chain rule error} \\ 1 : \text{value at } \left(3, \frac{1}{4}\right) \end{array} \right.$$

$$6 : \left\{ \begin{array}{l} 1 : \text{separates variables} \\ 1 : \text{antiderivative of } dy \text{ term} \\ 1 : \text{antiderivative of } dx \text{ term} \\ 1 : \text{constant of integration} \\ 1 : \text{uses initial condition } f(3) = \frac{1}{4} \\ 1 : \text{solves for } y \end{array} \right.$$

Note: max 3/6 [1-1-1-0-0-0] if no constant of integration

Note: 0/6 if no separation of variables