The College Board: Connecting Students to College Success

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**Equity Policy Statement**

The College Board and the Advanced Placement Program encourage teachers, AP Coordinators, and school administrators to make equitable access a guiding principle for their AP programs. The College Board is committed to the principle that all students deserve an opportunity to participate in rigorous and academically challenging courses and programs. All students who are willing to accept the challenge of a rigorous academic curriculum should be considered for admission to AP courses. The Board encourages the elimination of barriers that restrict access to AP courses for students from ethnic, racial, and socioeconomic groups that have been traditionally underrepresented in the AP Program. Schools should make every effort to ensure that their AP classes reflect the diversity of their student population.

For more information about equity and access in principle and practice, please send an email to apequity@collegeboard.org.
1. Let $R$ be the region in the first quadrant bounded by the graph of $y = 8 - x^{3/2}$, the $x$-axis, and the $y$-axis.

(a) Find the area of the region $R$.

(b) Find the volume of the solid generated when $R$ is revolved about the $x$-axis.

(c) The vertical line $x = k$ divides the region $R$ into two regions such that when these two regions are revolved about the $x$-axis, they generate solids with equal volumes. Find the value of $k$.

\[ A = \int_{0}^{4} \left( 8 - x^{3/2} \right) \, dx \]
\[ = 8x - \frac{2}{5}x^{5/2} \bigg|_{0}^{4} = 32 - \frac{64}{5} = \frac{96}{5} = 19.2 \]

(b) \[ V = \pi \int_{0}^{4} \left( 8 - x^{3/2} \right)^{2} \, dx \]
\[ = \frac{576\pi}{5} = 115.2\pi \approx 361.911 \]

(c) \[ \pi \int_{0}^{k} \left( 8 - x^{3/2} \right)^{2} \, dx = \frac{115.2\pi}{2} \]
\[ = \left[ \pi \int_{0}^{k} \left( 8 - x^{3/2} \right)^{2} \, dx = \pi \int_{k}^{4} \left( 8 - x^{3/2} \right)^{2} \, dx \right] \]
\[ \int_{0}^{k} \left( 8 - x^{3/2} \right)^{2} \, dx = 57.6 \]
\[ \int_{0}^{k} \left( 64 - 16x^{3/2} + x^3 \right) \, dx = 57.6 \]
\[ 64k - \frac{32}{5}k^{5/2} + \frac{k^4}{4} = 57.6 \]
\[ k \approx 0.995 \text{ or } 0.994 \]

Note: 0/1 for answer in each part if no setup points earned
2. Let \( f \) be the function given by \( f(x) = 2xe^{2x} \).

(a) Find \( \lim_{x \to -\infty} f(x) \) and \( \lim_{x \to \infty} f(x) \).

(b) Find the absolute minimum value of \( f \). Justify that your answer is an absolute minimum.

(c) What is the range of \( f \)?

(d) Consider the family of functions defined by \( y = bxe^{bx} \), where \( b \) is a nonzero constant. Show that the absolute minimum value of \( bxe^{bx} \) is the same for all nonzero values of \( b \).

\[
\begin{align*}
&\text{(a)} \quad \lim_{x \to -\infty} 2xe^{2x} = 0 \\
&\quad \lim_{x \to \infty} 2xe^{2x} = \infty \text{ or DNE} \\
&\text{(b)} \quad f'(x) = 2e^{2x} + 2x \cdot 2e^{2x} = 2e^{2x}(1 + 2x) = 0 \\
&\quad \text{if } x = -1/2 \\
&\quad f(-1/2) = -1/e \text{ or -0.368 or -0.367} \\
&\quad -1/e \text{ is an absolute minimum value because:} \\
&\quad (i) \quad f'(x) < 0 \text{ for all } x < -1/2 \text{ and} \\
&\quad f'(x) > 0 \text{ for all } x > -1/2 \\
&\quad \text{or} -1/2 \\
&\quad (ii) \quad f'(x) \quad \begin{array}{c}
- \\
+ \\
\end{array} \\
&\quad \begin{array}{c}
-1/2 \\
\end{array} \\
&\quad \text{and } x = -1/2 \text{ is the only critical number} \\
&\text{(c)} \quad \text{Range of } f = [-1/e, \infty) \\
&\quad \text{or } [-0.368, \infty) \\
&\text{(d)} \quad y' = be^{bx} + b^2 xe^{bx} = be^{bx}(1 + bx) = 0 \\
&\quad \text{if } x = -1/b \\
&\quad \text{At } x = -1/b, y = -1/e \\
&\quad y \text{ has an absolute minimum value of } -1/e \text{ for all nonzero } b
\end{align*}
\]
3. Let $f$ be a function that has derivatives of all orders for all real numbers. Assume $f(0) = 5$, $f'(0) = -3$, $f''(0) = 1$, and $f'''(0) = 4$.

(a) Write the third-degree Taylor polynomial for $f$ about $x = 0$ and use it to approximate $f(0.2)$.

(b) Write the fourth-degree Taylor polynomial for $g$, where $g(x) = f(x^2)$, about $x = 0$.

(c) Write the third-degree Taylor polynomial for $h$, where $h(x) = \int_{0}^{x} f(t) \, dt$, about $x = 0$.

(d) Let $h$ be defined as in part (c). Given that $f(1) = 3$, either find the exact value of $h(1)$ or explain why it cannot be determined.

\[
P_3(f)(x) = 5 - 3x + \frac{1}{2}x^2 + \frac{2}{3}x^3
\]

\[
f(0.2) \approx P_3(f)(0.2) = 5 - 3(0.2) + \frac{1}{2}(0.04) + \frac{2}{3}(0.008) = 4.425
\]

\[
P_3(g)(x) = P_3(f)(x^2) = 5 - 3x^2 + \frac{1}{2}x^4
\]

\[
P_3(h)(x) = \int_{0}^{x} \left(5 - 3t + \frac{1}{2}t^2\right) \, dt
\]

\[
= \left[5t - \frac{3}{2}t^2 + \frac{1}{6}t^3\right]_{0}^{x}
\]

\[
= 5x - \frac{3}{2}x^2 + \frac{1}{6}x^3
\]

\[
h(1) = \int_{0}^{1} f(t) \, dt
\]

cannot be determined because $f(t)$ is known only for $t = 0$ and $t = 1$
4. Consider the differential equation given by \( \frac{dy}{dx} = \frac{xy}{2} \).

(a) On the axes provided below, sketch a slope field for the given differential equation at the nine points indicated.

(b) Let \( y = f(x) \) be the particular solution to the given differential equation with the initial condition \( f(0) = 3 \). Use Euler's method starting at \( x = 0 \), with a step size of 0.1, to approximate \( f(0.2) \). Show the work that leads to your answer.

(c) Find the particular solution \( y = f(x) \) to the given differential equation with the initial condition \( f(0) = 3 \). Use your solution to find \( f(0.2) \).

(a)

\[
\begin{align*}
\text{(a)} & \quad \frac{dy}{dx} = \frac{xy}{2} \\
1: & \quad \text{line segments at nine points with negative - zero - positive slope left to right and increasing steepness bottom to top at } x = 1 \text{ and } x = -1
\end{align*}
\]

(b) \( f(0.1) \approx f(0) + f'(0)(0.1) \)
\[
= 3 + \frac{1}{2} \cdot 0(3)(0.1) = 3
\]
\( f(0.2) \approx f(0.1) + f'(0.1)(0.1) \)
\[
= 3 + \frac{1}{2} \cdot (0.1)(3)(0.1)
= 3 + \frac{0.3}{2} = 3.015
\]

(c) \[ \frac{dy}{dx} = \frac{xy}{2} \]
\[
\int \frac{dy}{y} = \int \frac{x}{2} \, dx
\]
\[
\ln |y| = \frac{1}{4} x^2 + C_1
\]
\[
y = Ce^{x^2/4}
\]
\[
3 = Ce^0 \implies C = 3
\]
\[
y = 3e^{x^2/4}
\]
\[
f(0.2) = 3e^{0.04/4} = 3e^{0.01} = 3.030
\]

1: Euler's Method equations or equivalent table
2
1: answer (not eligible without first point)
Special Case: 1/2 for first iteration 3.015 and second iteration 3.045

0
1: separates variables
1: antiderivative of \( dy \) term
1: antiderivative of \( dx \) term
1: solves for \( y \)
1: solves for constant of integration
1: evaluates \( f(0.2) \)

Note: max 4/6 [1-1-1-0-0-1] if no constant of integration
5. The temperature outside a house during a 24-hour period is given by

\[ F(t) = 80 - 10 \cos \left( \frac{\pi t}{12} \right), \quad 0 \leq t \leq 24, \]

where \( F(t) \) is measured in degrees Fahrenheit and \( t \) is measured in hours.

(a) Sketch the graph of \( F \) on the grid below.

(b) Find the average temperature, to the nearest degree Fahrenheit, between \( t = 6 \) and \( t = 14 \).

(c) An air conditioner cooled the house whenever the outside temperature was at or above 78 degrees Fahrenheit. For what values of \( t \) was the air conditioner cooling the house?

(d) The cost of cooling the house accumulates at the rate of $0.05 per hour for each degree the outside temperature exceeds 78 degrees Fahrenheit. What was the total cost, to the nearest cent, to cool the house for this 24-hour period?

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1: bell-shaped graph
minimum 70 at \( t = 0 \), \( t = 24 \) only
maximum 90 at \( t = 12 \) only

2: integral
1: limits and \( 1/(14 - 6) \)
1: integrand

3: 1: answer
0/1 if integral not of the form
\[
\frac{1}{b - a} \int_a^b F(t) \, dt
\]

1: inequality or equation
1: solutions with interval

---

2: integral
1: limits and 0.05
1: integrand

3: 1: answer
0/1 if integral not of the form
\[
k \int_a^b (F(t) - 78) \, dt
\]
6. A particle moves along the curve defined by the equation \( y = x^3 - 3x \). The \( x \)-coordinate of the particle, \( x(t) \), satisfies the equation \( \frac{dx}{dt} = \frac{1}{\sqrt{2t + 1}} \), for \( t \geq 0 \) with initial condition \( x(0) = -4 \).

(a) Find \( x(t) \) in terms of \( t \).

(b) Find \( \frac{dy}{dt} \) in terms of \( t \).

(c) Find the location and speed of the particle at time \( t = 4 \).

(a) \( x(t) = \int \frac{1}{\sqrt{2t + 1}} \, dt \)

\( x(t) = \sqrt{2t + 1} + C \)

\( x(0) = -4 = 1 + C \Rightarrow C = -5 \)

\( x(t) = \sqrt{2t + 1} - 5 \)

(b) \( y = x^3 - 3x \)

\( \frac{dy}{dt} = 3x^2 \frac{dx}{dt} - 3 \frac{dx}{dt} \)

\( = (3x^2 - 3) \frac{dx}{dt} \)

\( = \left[ 3 \left( \sqrt{2t + 1} - 5 \right)^2 - 3 \right] \frac{1}{\sqrt{2t + 1}} \)

(c) \( x(4) = \sqrt{9} - 5 = -2 \)

\( y(4) = (-2)^3 - 3(-2) = -2 \)

Location at \( t = 4 \) is \((-2,-2)\)

\( \frac{dx}{dt} \bigg|_{t=4} = \frac{1}{3} \)

\( \frac{dy}{dt} \bigg|_{t=4} = 3(3-5)^2 - 3 = 3 \)

Speed = \( \sqrt{\left(\frac{1}{3}\right)^2 + 3^2} = \sqrt{\frac{82}{9}} \approx 3.018 \)