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Mech 2.

The cart shown above is made of a block of mass $m$ and four solid rubber tires each of mass $m/4$ and radius $r$. Each tire may be considered to be a disk. (A disk has rotational inertia $\frac{1}{2} ML^2$, where $M$ is the mass and $L$ is the radius of the disk.) The cart is released from rest and rolls without slipping from the top of an inclined plane of height $h$. Express all algebraic answers in terms of the given quantities and fundamental constants.

(a) Determine the total rotational inertia of all four tires.

\[
4 \left( \frac{1}{2} m r^2 \right) = \frac{1}{2} m r^2
\]

(b) Determine the speed of the cart when it reaches the bottom of the incline.

\[
E_i = E_f
\]

\[
2 m g h = \frac{1}{2} (2 m) v^2 + \frac{1}{2} \left( \frac{1}{2} m r^2 \right) \omega^2
\]

\[
2 m g h = m v^2 + \frac{1}{4} m r^2 \omega^2 = v^2
\]

\[
2 m g h = \frac{S}{4} m v^2
\]

\[
\frac{4}{5} (2 m g h) = m v^2
\]

\[
\frac{8}{5} g h = m v^2 \]

\[
\sqrt{\frac{8}{5} g h} = v
\]
(c) After rolling down the incline and across the horizontal surface, the cart collides with a bumper of negligible mass attached to an ideal spring, which has a spring constant $k$. Determine the distance $x_m$ the spring is compressed before the cart and bumper come to rest.

\[
2 \cdot mgh = \frac{1}{2} kx^2
\]

\[
4mgh = kx^2
\]

\[
\frac{4mgh}{k} = x^2
\]

\[
\sqrt{\frac{4mgh}{k}} = x
\]

(d) Now assume that the bumper has a non-negligible mass. After the collision with the bumper, the spring is compressed to a maximum distance of about 90% of the value of $x_m$ in part (c). Give a reasonable explanation for this decrease.

The collision between the cart and the bumper is inelastic. Therefore

\[mV_0 = MV_f\]

where $m$ is the mass of the cart, $V_0$ is the speed of the cart before the collision, $M$ is the mass of the cart and the bumper, and $V_f$ is the final velocity of the cart-bumper system. If linear momentum is conserved, $V_f$ will be less than $V_0$. A smaller $V_f$ will result in a smaller distance travelled so the spring will only be compressed 90\% of the $x_m$ in part (c).

GO ON TO THE NEXT PAGE.
Mech 2.
The cart shown above is made of a block of mass $m$ and four solid rubber tires each of mass $m/4$ and radius $r$. Each tire may be considered to be a disk. (A disk has rotational inertia $\frac{1}{2}ML^2$, where $M$ is the mass and $L$ is the radius of the disk.) The cart is released from rest and rolls without slipping from the top of an inclined plane of height $h$. Express all algebraic answers in terms of the given quantities and fundamental constants.

(a) Determine the total rotational inertia of all four tires.

$$I = \frac{1}{2}mR^2$$

(b) Determine the speed of the cart when it reaches the bottom of the incline.

$$\delta U = \Delta K_{rot} + \Delta K_{trans}$$

$$2mgh = \frac{1}{2}I\omega^2 + \frac{1}{2}2mV^2$$

$$2mgh = \frac{1}{2}\left(\frac{1}{2}mR^2\right)(\omega^2) + \frac{1}{2}2mV^2$$

$$2gh = \frac{V^2}{4} + \frac{V^2}{2} = \frac{5V^2}{4}$$

$$\frac{8gh}{5} = V$$

GO ON TO THE NEXT PAGE.
(c) After rolling down the incline and across the horizontal surface, the cart collides with a bumper of negligible mass attached to an ideal spring, which has a spring constant $k$. Determine the distance $x_m$ the spring is compressed before the cart and bumper come to rest.

\[ \Delta U_{\text{grav}} + \Delta U_{\text{rot}} + \Delta U_{\text{trans}} = \Delta U_{\text{Elastic}} \]

\[ \Delta U_{g} = \Delta U_{\text{Elastic}} \]

\[ 2mgh = \frac{1}{2} k x^2 \]

\[ \sqrt{\frac{4mgh}{k}} = X_m \]

(d) Now assume that the bumper has a non-negligible mass. After the collision with the bumper, the spring is compressed to a maximum distance of about 90% of the value of $x_m$ in part (c). Give a reasonable explanation for this decrease.

Due to conservation of momentum, the speed ($v$) of the cart would be decreased as it hits the stationary bumper. This would lower the total kinetic energy of the cart-bumper system. Since the total kinetic energy of the cart going in is equal to the final potential energy of the spring, less KE means less compression of the spring.