AP® Physics C: Mechanics
2001 Sample Student Responses

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Mech 2.

An explorer plans a mission to place a satellite into a circular orbit around the planet Jupiter, which has mass $M_J = 1.90 \times 10^{27}$ kg and radius $R_J = 7.14 \times 10^7$ m.

(a) If the radius of the planned orbit is R, use Newton's laws to show each of the following.

i. The orbital speed of the planned satellite is given by $v = \sqrt{\frac{GM_J}{R}}$.

$$\frac{\cancel{M_{sat}} V_{sat}^2}{\cancel{R}} = \frac{G M_J \cancel{M_{SAT}}}{R^{\cancel{2}}}$$

$$\boxed{V_{sat} = \sqrt{\frac{GM_J}{R}}}$$

ii. The period of the orbit is given by $T = \sqrt{\frac{4\pi^2 R^3}{GM_J}}$.

$$T = \frac{2\pi}{\omega}$$

$$\omega = \frac{V}{R} = \frac{\sqrt{GM_J/R}}{R} = \frac{\sqrt{GM_J/R}}{\sqrt{R^2}} = \sqrt{\frac{GM_J}{R^3}}$$

$$T = \frac{2\pi}{\sqrt{\frac{GM_J}{R^3}}} = \frac{\sqrt{4\pi^2}}{\sqrt{\frac{GM_J}{R^3}}} = \boxed{\sqrt{\frac{4\pi^2 R^3}{GM_J}}}$$

(b) The explorer wants the satellite's orbit to be synchronized with Jupiter's rotation. This requires an equatorial orbit whose period equals Jupiter's rotation period of 9 hr 51 min = 3.55×10^4 s. Determine the required orbital radius in meters.

$$3.55 \times 10^4\,s = \sqrt{\frac{4\pi^2 R^3}{GM_J}}$$

$$1.26025 \times 10^9 = \frac{4\pi^2 R^3}{GM_J}$$

$$R^3 = 4.0455 \times 10^{24}$$

$$\boxed{R = 1.593 \times 10^8\ m}$$

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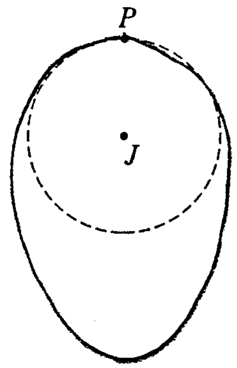
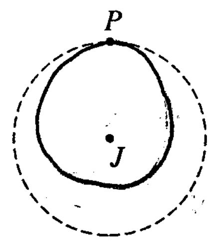
(c) Suppose that the injection of the satellite into orbit is less than perfect. For an injection velocity that differs from the desired value in each of the following ways, sketch the resulting orbit on the figure. (J is the center of Jupiter, the dashed circle is the desired orbit, and P is the injection point.) Also, describe the resulting orbit qualitatively but specifically.

i. When the satellite is at the desired altitude over the equator, its velocity vector has the correct direction, but the speed is slightly faster than the correct speed for a circular orbit of that radius.



It is an ellipse, and pt P is at perogee of the orbit.

ii. When the satellite is at the desired altitude over the equator, its velocity vector has the correct direction, but the speed is slightly slower than the correct speed for a circular orbit of that radius.



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(a) If the radius of the planned orbit is R, use Newton's laws to show each of the following.

i. The orbital speed of the planned satellite is given by $v = \sqrt{\dfrac{GM_J}{R}}$.

$$F_g = F_c$$

$$\frac{GM_J \cancel{m}}{R^2} = \frac{\cancel{m} v^2}{\cancel{R}}$$

$$\sqrt{\frac{GM_J}{R}} = \sqrt{v^2}$$

$$\boxed{V = \sqrt{\frac{GM_J}{R}}}$$

ii. The period of the orbit is given by $T = \sqrt{\dfrac{4\pi^2 R^3}{GM_J}}$.

$$V = \frac{2\pi R}{T} \qquad T = \frac{2\pi R}{V}$$

$$T = \frac{2\pi R}{\sqrt{\frac{GM_J}{R}}} = \frac{\sqrt{4\pi^2 R^2}}{\sqrt{\frac{GM_J}{R}}} = \boxed{\sqrt{\frac{4\pi^2 R^3}{GM_J}}}$$

(b) The explorer wants the satellite's orbit to be synchronized with Jupiter's rotation. This requires an equatorial orbit whose period equals Jupiter's rotation period of 9 hr 51 min = 3.55×10^4 s. Determine the required orbital radius in meters.

$$T = 3.55 \times 10^4 s = \sqrt{\frac{4\pi^2 R^3}{GM_J}}$$

$$1.26 \times 10^9 s^2 = \frac{4\pi^2 R^3}{(6.67 \times 10^{-11} m^3/kg \cdot s^2)(1.9 \times 10^{27} kg)}$$

$$4\pi^2 R^3 = 1.60 \times 10^{26} m^3$$

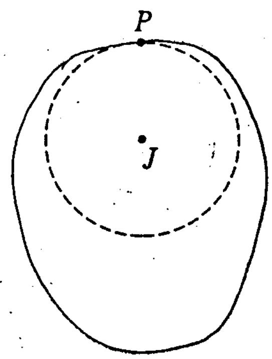
$$\sqrt[3]{R^3} = \sqrt[3]{4.05 \times 10^{24} m^3}$$

$$\boxed{R = 1.59 \times 10^8 m}$$

GO ON TO THE NEXT PAGE.

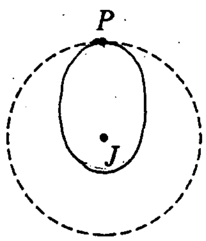
(c) Suppose that the injection of the satellite into orbit is less than perfect. For an injection velocity that differs from the desired value in each of the following ways, sketch the resulting orbit on the figure. (J is the center of Jupiter, the dashed circle is the desired orbit, and P is the injection point.) Also, describe the resulting orbit qualitatively but specifically.

i. When the satellite is at the desired altitude over the equator, its velocity vector has the correct direction, but the speed is slightly <u>faster</u> than the correct speed for a circular orbit of that radius.



The satellite will travel on an elliptical path with the center of Jupiter at one focus, and the radius of the path will always be greater than the desired radius

ii. When the satellite is at the desired altitude over the equator, its velocity vector has the correct direction, but the speed is slightly <u>slower</u> than the correct speed for a circular orbit of that radius.



The satellite will travel on an elliptical path with the center of Jupiter at one focus, and the radius of the path will be smaller than the desired radius,

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