PHYSICS C
SECTION II, ELECTRICITY AND MAGNETISM

Time—45 minutes
3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part, NOT in the green insert.

E&M 1. An isolated conducting sphere of radius \( a = 0.20 \text{ m} \) is at a potential of \(-2000 \text{ V}\).

(a) Determine the charge \( Q_0 \) on the sphere.

\[ -2000 = \frac{KQ_0}{a} \]
\[ \phi_0 = -4.44 \times 10^6 \text{ C} \]

The charged sphere is then concentrically surrounded by two uncharged conducting hemispheres of inner radius \( b = 0.40 \text{ m} \) and outer radius \( c = 0.50 \text{ m} \), which are joined together as shown above, forming a spherical capacitor. A wire is connected from the outer sphere to ground, and then removed.
(b) Determine the magnitude of the electric field in the following regions as a function of the distance $r$ from the center of the inner sphere.

i. $r < a$
\[ \oint E \cdot dA = \frac{Q_{enc}}{\varepsilon_0} \]
\[ E = 0 \text{ inside a conductor} \]

ii. $a < r < b$
\[ E_r(r) = \frac{Q_0}{4\pi \varepsilon_0 r^2} = \frac{4.44 \times 10^8}{4\pi \varepsilon_0 r^2} \]

iii. $b < r < c$
\[ E = 0 \text{ inside conductor} \]

iv. $r > c$
\[ E_r(r) = \frac{0}{\varepsilon_0} \]
\[ E = 0 \]
(c) Determine the magnitude of the potential difference between the sphere and the conducting shell.

\[ V_a - V_b = \int_a^b E_r \, dr = \frac{Q_0}{4\pi\varepsilon_0} \left[ b^2 - a^2 \right] \]

\[ = \frac{Q_0}{4\pi\varepsilon_0} \cdot 999 \checkmark \]

(d) Determine the capacitance of the spherical capacitor.

\[ C = \frac{Q}{V} = \frac{4.44 \times 10^{-8}}{999} \approx 4.44 \times 10^{-11} \, \text{F} \]
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E&M 1. An isolated conducting sphere of radius \( a = 0.20 \text{ m} \) is at a potential of \(-2,000 \text{ V}\).

(a) Determine the charge \( Q_0 \) on the sphere.

\[
\left( \frac{1}{4\pi \varepsilon_0} \right) \frac{Q_0}{(0.20)} = -2000
\]

\[
\frac{1}{4\pi \varepsilon_0} Q_0 = -400
\]

\[
Q_0 = -4.45 \times 10^{-8} \text{ C} = -44.5 \text{ nC}
\]

The charged sphere is then concentrically surrounded by two uncharged conducting hemispheres of inner radius \( b = 0.40 \text{ m} \) and outer radius \( c = 0.50 \text{ m} \), which are joined together as shown above, forming a spherical capacitor. A wire is connected from the outer sphere to ground, and then removed.
(b) Determine the magnitude of the electric field in the following regions as a function of the distance $r$ from the center of the inner sphere.

i. $r < a$
   
   \[ E = 0 \]

ii. $a < r < b$

   \[ E = \frac{1}{4\pi \varepsilon_0} \left( \frac{-44.5 \times 10^{-9}}{r - a} \right) = -1000.14 \text{ N/C} \]

iii. $b < r < c$
   
   \[ E = 0 \]

iv. $r > c$

   \[ E = \frac{1}{4\pi \varepsilon_0} \left( \frac{-44.5 \times 10^{-9}}{r} \right) = -800.11 \text{ N/C} \]
(c) Determine the magnitude of the potential difference between the sphere and the conducting shell.

\[
\frac{\Phi}{4\pi \varepsilon_0} \left[ \frac{1}{b} - \frac{1}{a} \right]
\]

(d) Determine the capacitance of the spherical capacitor.

\[
C = \frac{\varepsilon}{\sqrt{\varepsilon}} = \frac{\varepsilon}{\frac{4\pi \varepsilon_0}{\sqrt{\frac{1}{b} - \frac{1}{a}}}} = \frac{4\pi \varepsilon_0}{\sqrt{\frac{1}{b} - \frac{1}{a}}}
\]
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E&M 1. An isolated conducting sphere of radius $a = 0.20 \text{ m}$ is at a potential of $-2,000 \text{ V}$.

(a) Determine the charge $Q_0$ on the sphere.

$$E_0 = \frac{V}{a} = \frac{-2,000}{0.20} = -10,000 \text{ N/C}$$

$$-2,000 = \frac{Q_0}{4\pi \varepsilon_0 a^2}, \quad Q_0 = -4.48 \times 10^{-8} \text{ C}$$

The charged sphere is then concentrically surrounded by two uncharged conducting hemispheres of inner radius $b = 0.40 \text{ m}$ and outer radius $c = 0.50 \text{ m}$, which are joined together as shown above, forming a spherical capacitor. A wire is connected from the outer sphere to ground, and then removed.
(b) Determine the magnitude of the electric field in the following regions as a function of the distance \( r \) from the center of the inner sphere.

i. \( r < a \)

\[ E = 0, \text{ because of it being inside a conductor.} \]

ii. \( a < r < b \)

\[ E = \frac{Q_0}{4 \pi \varepsilon_0 r^2} \]

iii. \( b < r < c \)

\[ E = 0, \text{ because of it being inside a conductor.} \]

iv. \( r > c \)

\[ E = \frac{Q_0}{4 \pi \varepsilon_0 r^2} \]
(c) Determine the magnitude of the potential difference between the sphere and the conducting shell.

\[ \Delta V = - \int_a^b \text{E} \, dr = - \int_a^b \frac{Q_o}{4\pi\epsilon_0 r^2} \, dr \]

\[ \Delta V = - \frac{Q_o}{4\pi\epsilon_0} \left[ \frac{1}{r} \right]_a^b \]

\[ \Delta V = \frac{Q_o}{4\pi\epsilon_0} \left( \frac{1}{b} - \frac{1}{a} \right) \]

(d) Determine the capacitance of the spherical capacitor.

\[ C = \epsilon_0 \frac{Q_o}{V} = \frac{Q_o}{-4.48 \times 10^{-8} \text{C}} \]

\[ C = 2.224 \times 10^{-11} \text{ Farads} \]