You have been hired to determine the internal resistance of 8.0 \( \mu \text{F} \) capacitors for an electronic component manufacturer. (Ideal capacitors have an infinite internal resistance—that is, the material between their plates is a perfect insulator. In practice, however, the material has a very small, but nonzero, conductivity.) You cannot simply connect the capacitors to an ohmmeter, because their resistance is too large for an ohmmeter to measure. Therefore you charge the capacitor to a potential difference of 10 V with a battery, disconnect it from the battery and measure the potential difference across the capacitor every 20 minutes with an ideal voltmeter, obtaining the graph shown above.

(a) Determine the internal resistance of the capacitor.

\[
V = V_0 e^{-\frac{t}{\tau}} ~ \therefore \tau = RC = R \cdot 8 \times 10^{-6}
\]

\[
\tau = 140 \text{ min} \cdot 60 \text{ sec/min} = 8400 \text{ s}
\]

\[
I = I_0 e^{-\frac{8400}{\tau}}
\]

\[
ln \left( \frac{I}{I_0} \right) = -\frac{8400}{\tau}
\]

\[
\tau = \frac{3600}{R \cdot 8 \times 10^{-6}}
\]

\[
R = 4.5 \times 10^8 \Omega
\]

The capacitor can be approximated as a parallel-plate capacitor separated by a 0.10 mm thick dielectric with \( \kappa = 5.6 \).

(b) Determine the approximate surface area of one of the capacitor “plates.”

\[
C = \kappa \varepsilon_0 \frac{A}{\varepsilon}
\]

\[
8 \times 10^{-6} = \frac{8.85 \times 10^{-12} \cdot A}{0.001 \cdot 5.6}
\]

\[
A = 16.1 \text{ m}^2
\]
(c) Determine the resistivity of the dielectric.

\[ R = \rho \frac{L}{A} \quad 4.5 \times 10^8 = \frac{\rho \cdot 0.0001}{16.1} \]

\[ \rho = 7.25 \times 10^{13} \ \Omega \cdot m \]

(d) Determine the magnitude of the charge leaving the positive plate of the capacitor in the first 100 min.

\[ V = IR \quad I = \frac{V}{R} = \frac{V_0 e^{-\frac{t}{RC}}}{R} \]

\[ Q = \int_0^{10} I \, dt \quad Q = \int_0^{6000} \frac{10 \cdot e^{-\frac{t}{4.5 \times 10^8}}}{4.5 \times 10^8} = 2.2 \times 10^{-5} \int_0^{6000} e^{-\frac{t}{4.5 \times 10^8}} = 6.5 \times 10^{-5} \text{ C} \]
E & M 2.

You have been hired to determine the internal resistance of 8.0 \( \mu F \) capacitors for an electronic component manufacturer. (Ideal capacitors have an infinite internal resistance—that is, the material between their plates is a perfect insulator. In practice, however, the material has a very small, but nonzero, conductivity.) You cannot simply connect the capacitors to an ohmmeter, because their resistance is too large for an ohmmeter to measure. Therefore you charge the capacitor to a potential difference of 10 V with a battery, disconnect it from the battery and measure the potential difference across the capacitor every 20 minutes with an ideal voltmeter, obtaining the graph shown above.

(a) Determine the internal resistance of the capacitor.

\[
\text{Step 1:} \quad \frac{\Delta Q}{\Delta t} = \frac{V}{1} \quad Q = CV \quad I = \frac{\Delta Q}{\Delta t}
\]

\[
\text{Step 2:} \quad t = 0 \quad V = 10, \quad C = 8 \times 10^{-6} \quad Q = 9 \times 10^{-5}
\]

\[
\text{Step 3:} \quad I = \frac{\Delta Q}{\Delta t} = \frac{4 \times 10^{-5}}{2400 s} = 1.67 \times 10^{-8} \quad \Rightarrow \quad R = \frac{7.5}{1.67 \times 10^{-8}} = 4.5 \times 10^3 \Omega
\]

The capacitor can be approximated as a parallel-plate capacitor separated by a 0.10 mm thick dielectric with \( \kappa = 5.6 \).

(b) Determine the approximate surface area of one of the capacitor "plates."

\[
\frac{C}{\varepsilon_0} = \frac{K \varepsilon_0 A}{d} = \frac{(5.6)(8.85 \times 10^{-12}) A}{(1 \times 10^{-3})} = 16.14 \text{m}
\]

GO ON TO THE NEXT PAGE.
(c) Determine the resistivity of the dielectric.

\[ \rho = \frac{\rho A}{l} \]

\[ \rho = \frac{RA}{l} \]

\[ = \frac{(4.5 \times 10^8 \text{ N} \cdot \text{m}^2)}{(1.1 \times 10^{-3})} \]

\[ \rho = 7.26 \times 10^{13} \]

(d) Determine the magnitude of the charge leaving the positive plate of the capacitor in the first 100 min.

\[ t = 0 \quad U = 10, \quad C = 9 \times 10^{-6} \Rightarrow Q = 8 \times 10^{-5} \]

\[ t = 100 \quad U = 2, \quad C = 8 \times 10^{-6} \Rightarrow Q = 1.6 \times 10^{-5} \]

\[ \Delta Q = 6.4 \times 10^{-5} \]