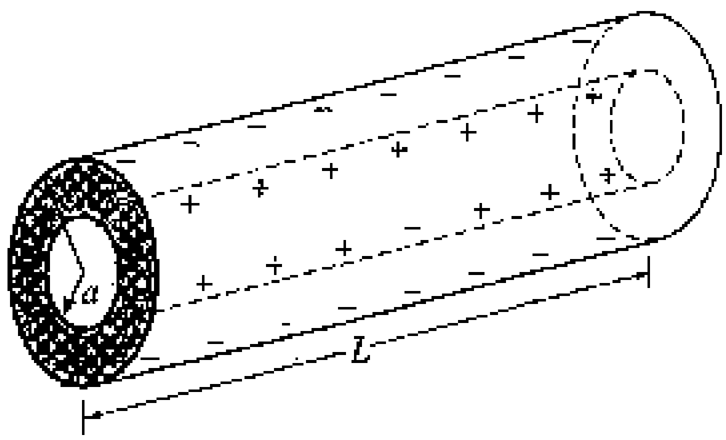
AP Physics C: Electricity and Magnetism
2000 Student Samples

AA1



E & M 3.

A capacitor consists of two conducting, coaxial, cylindrical shells of radius a and b, respectively, and length $L >> b$. The space between the cylinders is filled with oil that has a dielectric constant κ. Initially both cylinders are uncharged, but then a battery is used to charge the capacitor, leaving a charge $+Q$ on the inner cylinder and $-Q$ on the outer cylinder, as shown above. Let r be the radial distance from the axis of the capacitor.

$$\int E \cdot dA = \frac{Q_{in}}{\varepsilon_0}$$

(a) Using Gauss's law, determine the electric field midway along the length of the cylinder for the following values of r, in terms of the given quantities and fundamental constants. Assume end effects are negligible.

i. $a < r < b$

$$E \cdot 2\pi r \ell = \frac{Q}{\varepsilon_0} \cdot \frac{\ell}{L} \cdot \frac{1}{K}$$

$$E = \boxed{\frac{Q}{2\pi\varepsilon_0 \cdot r L K}}$$

ii. $b < r << L$

$$E \cdot 2\pi r \ell = \frac{(Q - Q)}{\varepsilon_0} \cdot \frac{\ell}{L}$$

$$E = \boxed{0 \text{ V/m}}$$

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AA2

(b) Determine the following in terms of the given quantities and fundamental constants.

i. The potential difference across the capacitor $V = \int E \cdot dL$

$$V = \int_a^b E(r)\,dr = \int_a^b \frac{Q}{2\pi\varepsilon_0 r L K}\,dr$$

$$V = \frac{Q\ln(r)}{2\pi\varepsilon_0 L K}\Big|_a^b = \boxed{\frac{Q\ln\left(\frac{b}{a}\right)}{2\pi\varepsilon_0 L K}}$$

ii. The capacitance of this capacitor

$$Q = CV \qquad C = \frac{Q}{V}$$

$$C = \frac{Q}{\frac{Q\ln\left(\frac{b}{a}\right)}{2\pi\varepsilon_0 L K}} = \boxed{\frac{2\pi\varepsilon_0 L K}{\ln\left(\frac{b}{a}\right)}}$$

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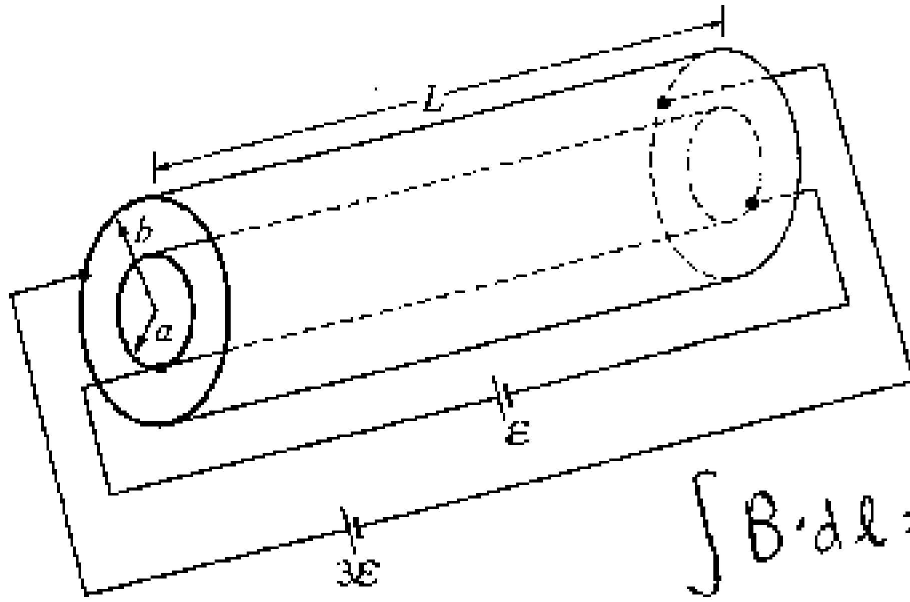
AA3



$\int B \cdot dl = \mu_0 I$

(c) Now the capacitor is discharged and the oil is drained from it. As shown above, a battery of emf $\mathcal{E}$ is connected to opposite ends of the inner cylinder and a battery of emf $3\mathcal{E}$ is connected to opposite ends of the outer cylinder. Each cylinder has resistance R. Assume that end effects and the contributions to the magnetic field from the wires are negligible. Using Ampere's law, determine the magnitude B of the magnetic field midway along the length of the cylinders due to the current in the cylinders for the following values of r.

$V = IR$

i. $a < r < b$

$B \cdot 2\pi r = \mu_0 \frac{\mathcal{E}}{R}$

$B = \frac{\mu_0 \mathcal{E}}{2\pi r \cdot R}$

ii. $b < r << L$

$B \cdot 2\pi r = \mu_0 \cdot \frac{4\mathcal{E}}{R}$

$B = \frac{2\mu_0 \mathcal{E}}{\pi r \cdot R}$

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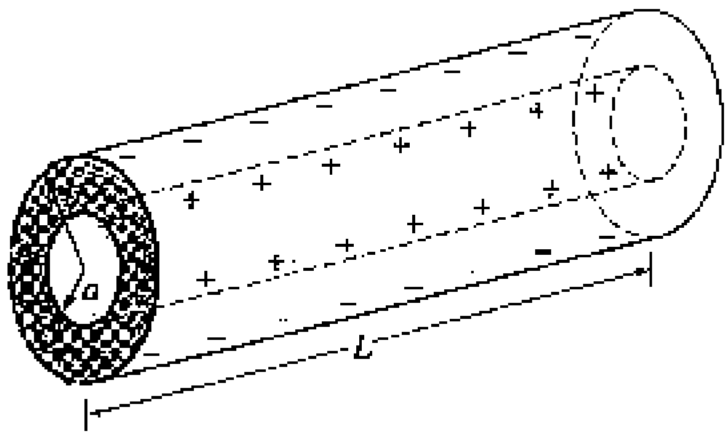
AB1



E & M 3.

A capacitor consists of two conducting, coaxial, cylindrical shells of radius a and b, respectively, and length $L >> b$. The space between the cylinders is filled with oil that has a dielectric constant κ. Initially both cylinders are uncharged, but then a battery is used to charge the capacitor, leaving a charge $+Q$ on the inner cylinder and $-Q$ on the outer cylinder, as shown above. Let r be the radial distance from the axis of the capacitor.

(a) Using Gauss's law, determine the electric field midway along the length of the cylinder for the following values of r, in terms of the given quantities and fundamental constants. Assume end effects are negligible.

i. $a < r < b$

$\int E \cdot dA = \frac{q_{in}}{\varepsilon_0}$ $\quad E\, 2\pi r = \frac{Q}{\varepsilon_0}$

$E = \frac{Q}{2\pi\varepsilon_0 r}$

ii. $b < r << L$

$\int E\, dA = \frac{q_{in}}{\varepsilon_0}$ $\quad$ Q and −Q cancel, $q_{in} = 0$

$\int E\, dA = 0$

$E = 0$

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(b) Determine the following in terms of the given quantities and fundamental constants.

i. The potential difference across the capacitor

$$V_0 = \int_a^b E\,ds = \int_a^b \frac{Q\,dr}{2\pi\varepsilon_0 r} = \frac{Q}{2\pi\varepsilon_0}\int_a^b \frac{dr}{r} = \frac{Q}{2\pi\varepsilon_0}\ln\left(\frac{b}{a}\right)$$

$$V = \frac{V_0}{K} \qquad V = \frac{Q\ln\left(\frac{b}{a}\right)}{2\pi\varepsilon_0 K}$$

ii. The capacitance of this capacitor

$$C = \frac{Q}{V} \text{ or } K\frac{Q}{V_0}$$

assuming $V = \frac{Q}{2\pi\varepsilon_0 K}\ln\left(\frac{b}{a}\right)$

$$C = \frac{2\pi\varepsilon_0 K}{\ln\left(\frac{b}{a}\right)}$$

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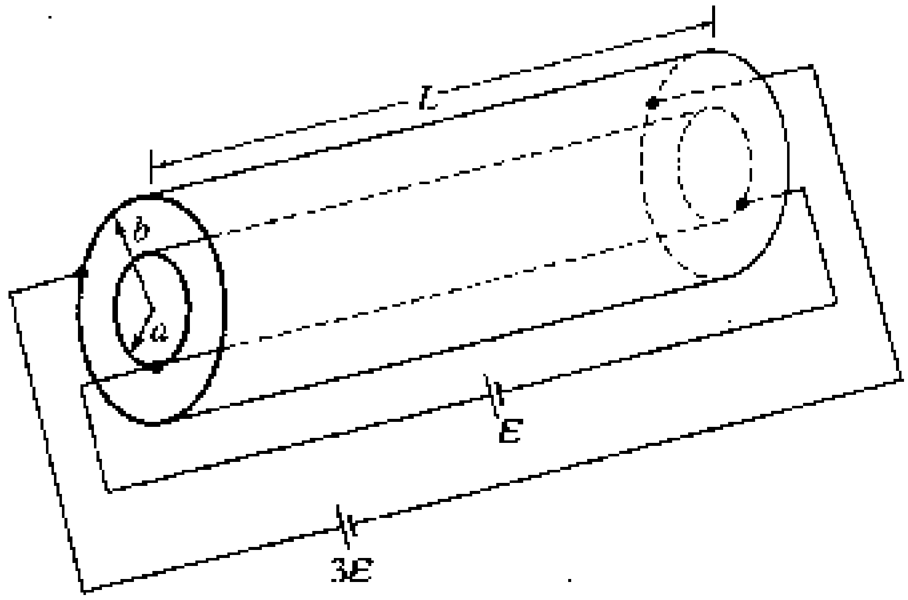

(c) Now the capacitor is discharged and the oil is drained from it. As shown above, a battery of emf $\mathcal{E}$ is connected to opposite ends of the inner cylinder and a battery of emf $3\mathcal{E}$ is connected to opposite ends of the outer cylinder. Each cylinder has resistance R. Assume that end effects and the contributions to the magnetic field from the wires are negligible. Using Ampere's law, determine the magnitude B of the magnetic field midway along the length of the cylinders due to the current in the cylinders for the following values of r.

i. $a < r < b$

$$\int B\,ds = \mu_0 I_{in}$$

$$B\,2\pi r = \mu_0 \frac{\mathcal{E}}{R}$$

$$B = \frac{\mu_0}{2\pi r} \cdot \frac{\mathcal{E}}{R}$$

ii. $b < r << L$

$$\int B\,ds = \mu_0 I_{in}$$

$$B\,2\pi r = \mu_0 \left(\frac{\mathcal{E}}{R} + \frac{3\mathcal{E}}{R}\right)$$

$$B = \frac{2\mu_0}{\pi r} \cdot \frac{\mathcal{E}}{R}$$

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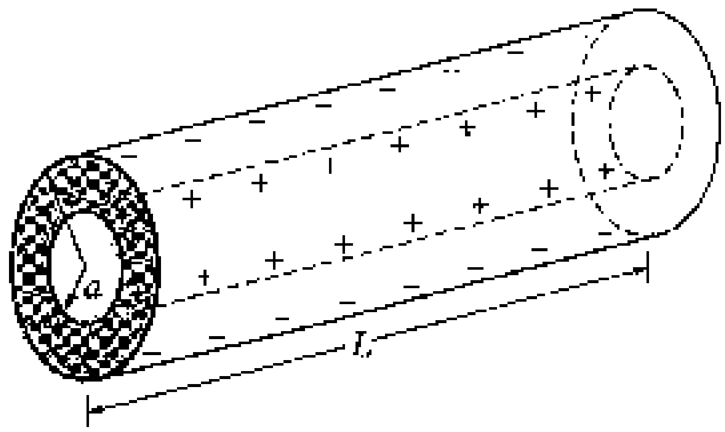
AD 1



E & M 3.

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(a) Using Gauss's law, determine the electric field midway along the length of the cylinder for the following values of r, in terms of the given quantities and fundamental constants. Assume end effects are negligible.

i. $a < r < b$

$$EA = \frac{Q}{\epsilon_0}$$

$$E = \frac{Q}{A\epsilon_0} = \frac{Q}{(2\pi r^2 + 2\pi r L)\epsilon_0}$$

ii. $b < r << L$

$$EA = \frac{Q}{\epsilon_0}, \quad Q = 0, \quad \therefore E = 0$$

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(b) Determine the following in terms of the given quantities and fundamental constants.

i. The potential difference across the capacitor

$$V = -\frac{dE}{dr} = \frac{Q}{\epsilon_0}\frac{d}{dr}\left[(2\pi r^2 + 2\pi r L)^{-1}\right]$$

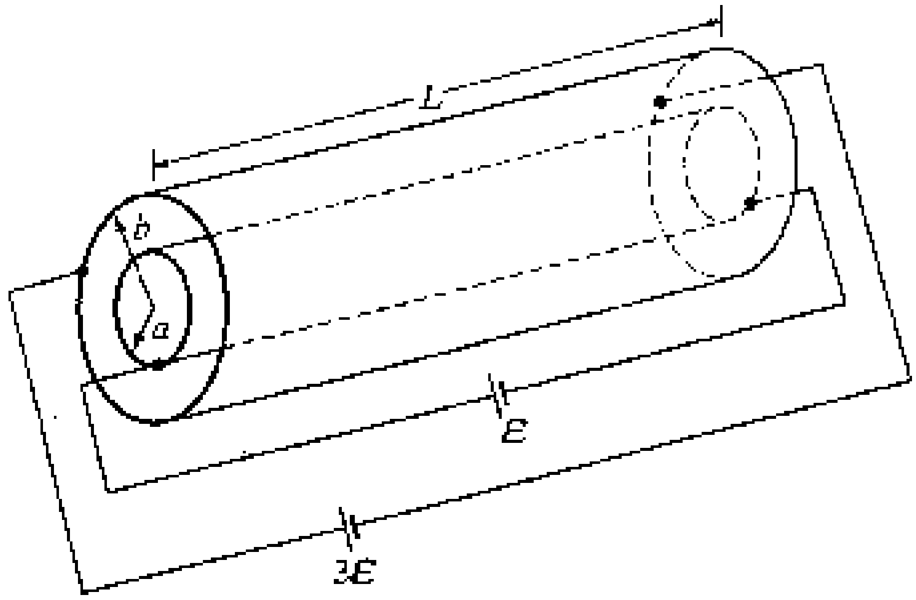
$$= -\frac{Q}{\epsilon_0}\left(-(2\pi r^2 + 2\pi r L)^{-2}(4\pi r + 2\pi L)\right)$$

$$= \frac{Q(4\pi r + 2\pi L)}{\epsilon_0 (2\pi r^2 + 2\pi r L)^2}$$

ii. The capacitance of this capacitor

$$C = \frac{KQ}{V} = \frac{KQ(\epsilon_0(2\pi r^2 + 2\pi r L)^2}{Q(4\pi r + 2\pi L)}$$

$$= \frac{K\epsilon_0 (2\pi r^2 + 2\pi r L)^2}{4\pi r + 2\pi L}$$



(c) Now the capacitor is discharged and the oil is drained from it. As shown above, a battery of emf $\mathcal{E}$ is connected to opposite ends of the inner cylinder and a battery of emf $3\mathcal{E}$ is connected to opposite ends of the outer cylinder. Each cylinder has resistance R. Assume that end effects and the contributions to the magnetic field from the wires are negligible. Using Ampere's law, determine the magnitude B of the magnetic field midway along the length of the cylinders due to the current in the cylinders for the following values of r.

i. $a < r < b$

$$\int B\,dl = \mu_0 I_c \qquad I_c = \frac{V}{R}$$

$$B = \frac{\mu_0 I_c}{l} = \frac{\mu_0 \mathcal{E}}{R(2\pi r)}$$

ii. $b < r << L$

$$\int B\,dl = \mu_0 I_c$$

$$B = \frac{\mu_0 I_c}{l} \qquad I_c : \frac{\mathcal{E}}{R} - \frac{3\mathcal{E}}{R} = \frac{-2}{3}\frac{\mathcal{E}}{R}$$

$$= \frac{\mu_0}{2\pi r}\left(\frac{2}{3}\frac{\mathcal{E}}{R}\right) = \frac{2\mu_0\mathcal{E}}{(3R)(2\pi r)}$$

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