Three particles, $A$, $B$, and $C$, have equal positive charges $Q$ and are held in place at the vertices of an equilateral triangle with sides of length $\ell$, as shown in the figures below. The dotted lines represent the bisectors for each side. The base of the triangle lies on the $x$-axis, and the altitude of the triangle lies on the $y$-axis.

(a)

i. Point $P_1$, the intersection of the three bisectors, locates the geometric center of the triangle and is one point where the electric field is zero. On Figure 1 above, draw the electric field vectors $E_A$, $E_B$, and $E_C$ at $P_1$ due to each of the three charges. Be sure your arrows are drawn to reflect the relative magnitude of the fields.

ii. Another point where the electric field is zero is point $P_2$ at $(0, y_2)$. On Figure 2 above, draw electric field vectors $E_A$, $E_B$, and $E_C$ at $P_2$ due to each of the three point charges. Indicate below whether the magnitude of each of these vectors is greater than, less than, or the same as for point $P_1$.

<table>
<thead>
<tr>
<th></th>
<th>Greater than at $P_1$</th>
<th>Less than at $P_1$</th>
<th>The same as at $P_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_A$</td>
<td>$N_0$</td>
<td>$\frac{N_0}{3}$</td>
<td>$N_0$</td>
</tr>
<tr>
<td>$E_B$</td>
<td>$\frac{N_0}{3}$</td>
<td>$N_0$</td>
<td>$N_0$</td>
</tr>
<tr>
<td>$E_C$</td>
<td>$\frac{N_0}{3}$</td>
<td>$N_0$</td>
<td>$N_0$</td>
</tr>
</tbody>
</table>
(b) Explain why the \( x \)-component of the total electric field is zero at any point on the \( y \)-axis.

Because on the \( y \)-axis, the \( x \)-component of the field from \( A \) will always be 0 since it is directly above. The \( x \)-component from \( B \) will cancel each other out since they are equivalent for whatever point on the \( y \)-axis is chosen and they are an oppositely side. The line, thus, has no \( x \)-component of the total electric field on the \( y \)-axis.

(c) Write a general expression for the electric potential \( V \) at any point on the \( y \)-axis inside the triangle in terms of \( Q \), \( \ell \), and \( y \).

\[
V = \frac{1}{4\pi \varepsilon_0} \left( \frac{Q}{(\ell^2 - y)} + \frac{Q}{\sqrt{y^2 + \frac{\ell^2}{4}}} + \frac{Q}{\sqrt{y^2 + \frac{\ell^2}{4}}} \right)
\]

(d) Describe how the answer to part (c) could be used to determine the \( y \)-coordinates of points \( P_1 \) and \( P_2 \) at which the electric field is zero. (You do not need to actually determine these coordinates.)

When the electric field is zero, the negative of the derivative of the voltage is equal to zero \((E = -\frac{dv}{dy})\). Find the derivative of \( V \), set it equal to zero, and solve for \( y \) to get the \( y \)-coordinates at which the electric field is zero.
E & M 2.

Three particles, A, B, and C, have equal positive charges Q and are held in place at the vertices of an equilateral triangle with sides of length $\ell$, as shown in the figures below. The dotted lines represent the bisectors for each side. The base of the triangle lies on the x-axis, and the altitude of the triangle lies on the y-axis.

![Figure 1](image1.png)

![Figure 2](image2.png)

(a)

i. Point $P_1$, the intersection of the three bisectors, locates the geometric center of the triangle and is one point where the electric field is zero. On Figure 1 above, draw the electric field vectors $E_A$, $E_B$, and $E_C$ at $P_1$ due to each of the three charges. Be sure your arrows are drawn to reflect the relative magnitude of the fields.

ii. Another point where the electric field is zero is point $P_2$ at $(0, y_2)$. On Figure 2 above, draw electric field vectors $E_A$, $E_B$, and $E_C$ at $P_2$ due to each of the three point charges. Indicate below whether the magnitude of each of these vectors is greater than, less than, or the same as for point $P_1$.

<table>
<thead>
<tr>
<th></th>
<th>Greater than at $P_1$</th>
<th>Less than at $P_1$</th>
<th>The same as at $P_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_A$</td>
<td></td>
<td>$\checkmark$</td>
<td></td>
</tr>
<tr>
<td>$E_B$</td>
<td></td>
<td>$\checkmark$</td>
<td></td>
</tr>
<tr>
<td>$E_C$</td>
<td></td>
<td>$\checkmark$</td>
<td></td>
</tr>
</tbody>
</table>
(b) Explain why the x-component of the total electric field is zero at any point on the y-axis.

The x-components of the \( \mathbf{E}_B \) and \( \mathbf{E}_C \) vectors will always be equal and opposite, and the x-component of \( \mathbf{E}_A \) vector is zero, so the vector sum of \( \mathbf{E}_A \) will always be zero.

(c) Write a general expression for the electric potential \( V \) at any point on the y-axis inside the triangle in terms of \( Q, \ell, \) and \( y. \)

\[
\text{distance from } A = \frac{\ell^2}{2} - y
\]
\[
\text{distance from } B = \text{distance from } C = \sqrt{\frac{x^2}{4} + y^2}
\]

\[
\text{voltage } = \frac{1}{4\pi \varepsilon_0} \frac{Q}{R}, \quad \text{and voltages are additive, so}
\]

\[
V = \frac{1}{4\pi \varepsilon_0} Q \left[ \frac{1}{\frac{\ell^2}{2} - y} + \frac{1}{\sqrt{\frac{x^2}{4} + y^2}} \right]
\]

(d) Describe how the answer to part (c) could be used to determine the y-coordinates of points \( P_1 \) and \( P_2 \) at which the electric field is zero. (You do not need to actually determine these coordinates.)

\[
\vec{E} = -\nabla V; \quad \text{on the x-axis this means}
\]

\[
\vec{E} = -\frac{dV}{dt}. \quad \text{if } |\vec{E}| = 0,
\]

\[
|\frac{dV}{dt}| = 0, \quad \text{so}
\]

we can take \( \frac{dV}{dt} \), set it \( = 0 \),

and solve.
E & M 2.

Three particles, $A$, $B$, and $C$, have equal positive charges $Q$ and are held in place at the vertices of an equilateral triangle with sides of length $\ell$, as shown in the figures below. The dotted lines represent the bisectors for each side. The base of the triangle lies on the $x$-axis, and the altitude of the triangle lies on the $y$-axis.

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<table>
<thead>
<tr>
<th>Vector</th>
<th>Greater than at $P_1$</th>
<th>Less than at $P_1$</th>
<th>The same as at $P_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_A$</td>
<td>$E_C$, $E_B$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E_B$</td>
<td>$E_A$</td>
<td>$E_C$</td>
<td></td>
</tr>
<tr>
<td>$E_C$</td>
<td>$E_A$</td>
<td></td>
<td>$E_B$</td>
</tr>
</tbody>
</table>
(b) Explain why the $x$-component of the total electric field is zero at any point on the $y$-axis.

Because the magnitude of the $x$-component of $E_y$ and the $x$-component $E_c$ are equal all along the $y$-axis, but in opposite directions, $E_{ex} + E_{cx} = 0$. Also, along the $y$-axis, the $x$-component of $E_p$ is always equal to zero.

(c) Write a general expression for the electric potential $V$ at any point on the $y$-axis inside the triangle in terms of $Q$, $\ell$, and $y$.

$$V = k \left( \frac{Q}{y - \frac{\ell}{2}} + \frac{Q}{\sqrt{y^2 + \left(\frac{\ell}{2}\right)^2}} \right)$$

$$(V = k \left( \frac{Q}{y - \frac{\ell}{2}} + \frac{2Q}{\sqrt{y^2 + \left(\frac{\ell}{2}\right)^2}} \right))$$

(d) Describe how the answer to part (c) could be used to determine the $y$-coordinates of points $P_1$ and $P_2$ at which the electric field is zero. (You do not need to actually determine these coordinates.)

$$E = -\frac{dV}{dr} \quad \text{Take the derivative of } V(y) \text{ in part (c) with respect to } y$$

$$\text{and set it equal to zero. Then solve for } y.$$

GO ON TO THE NEXT PAGE.