AP® Calculus BC
2002 Sample Student Responses

The materials included in these files are intended for use by AP teachers for course and exam preparation in the classroom; permission for any other use must be sought from the Advanced Placement Program®. Teachers may reproduce them, in whole or in part, in limited quantities, for face-to-face teaching purposes but may not mass distribute the materials, electronically or otherwise. These materials and any copies made of them may not be resold, and the copyright notices must be retained as they appear here. This permission does not apply to any third-party copyrights contained herein.

These materials were produced by Educational Testing Service® (ETS®), which develops and administers the examinations of the Advanced Placement Program for the College Board. The College Board and Educational Testing Service (ETS) are dedicated to the principle of equal opportunity, and their programs, services, and employment policies are guided by that principle.

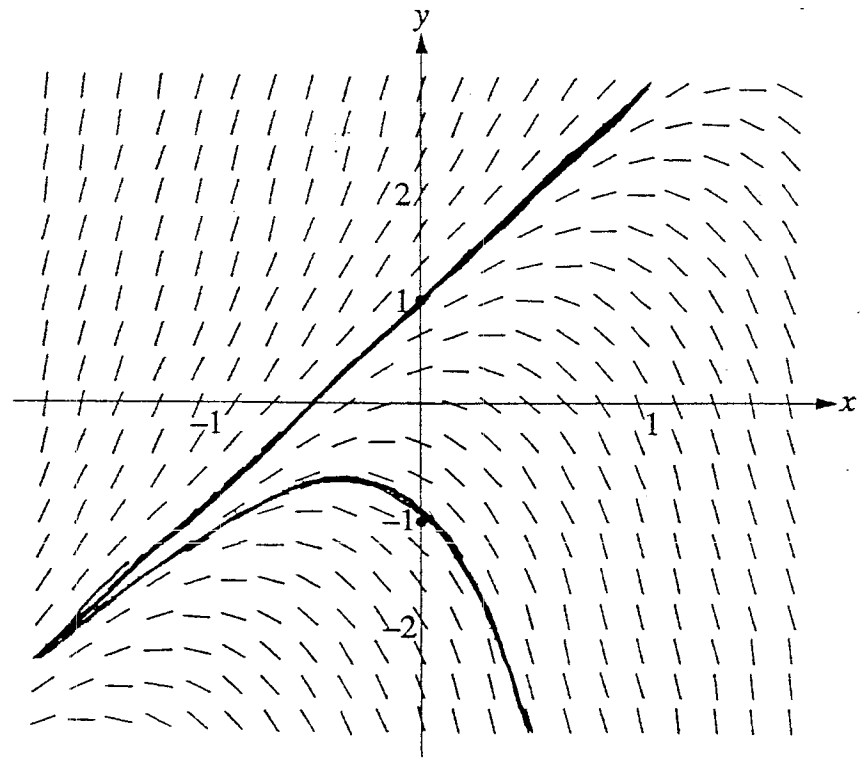
The College Board is a national nonprofit membership association dedicated to preparing, inspiring, and connecting students to college and opportunity. Founded in 1900, the association is composed of more than 4,200 schools, colleges, universities, and other educational organizations. Each year, the College Board serves over three million students and their parents, 22,000 high schools, and 3,500 colleges, through major programs and services in college admission, guidance, assessment, financial aid, enrollment, and teaching and learning. Among its best-known programs are the SAT®, the PSAT/NMSQT®, and the Advanced Placement Program® (AP®). The College Board is committed to the principles of equity and excellence, and that commitment is embodied in all of its programs, services, activities, and concerns.

5 5 5 5 5 5 5 5 5 5

NO CALCULATOR ALLOWED

B1

Work for problem 5(a)



Work for problem 5(b)

$\frac{dy}{dx} = 2y - 4x$

x	f(x)	f'(x)
0	1	2
0.1	1.2	2
0.2	1.4	

$2(1) - 4(0)$ $\quad 1 + (2 \cdot 0.1) = 1.2$

$2(1.2) - 4(0.1)$ $\quad 1.2 + (2 \cdot 0.1) = 1.4$

$f(0.2) \approx 1.4$

NO CALCULATOR ALLOWED

B2

Work for problem 5(c)

$$\frac{dy}{dx} = 2y - 4x = 2$$

~~$dy = (2y - 4x)dx$~~

$$y = \frac{2 + 4x}{2}$$

$$y = 2x + 1$$

$b = 1$

Work for problem 5(d)

$g'(x) = \frac{dy}{dx}$ @ $(0,0) = 2(0) - 4(0) = 0$ critical point

$g''(x) = 2(2y - 4x) - 4 = 4y - 8x - 4$

$g''(x)$ @ $(0,0) = 4(0) - 8(0) - 4 = -4$ negative; g is concave down

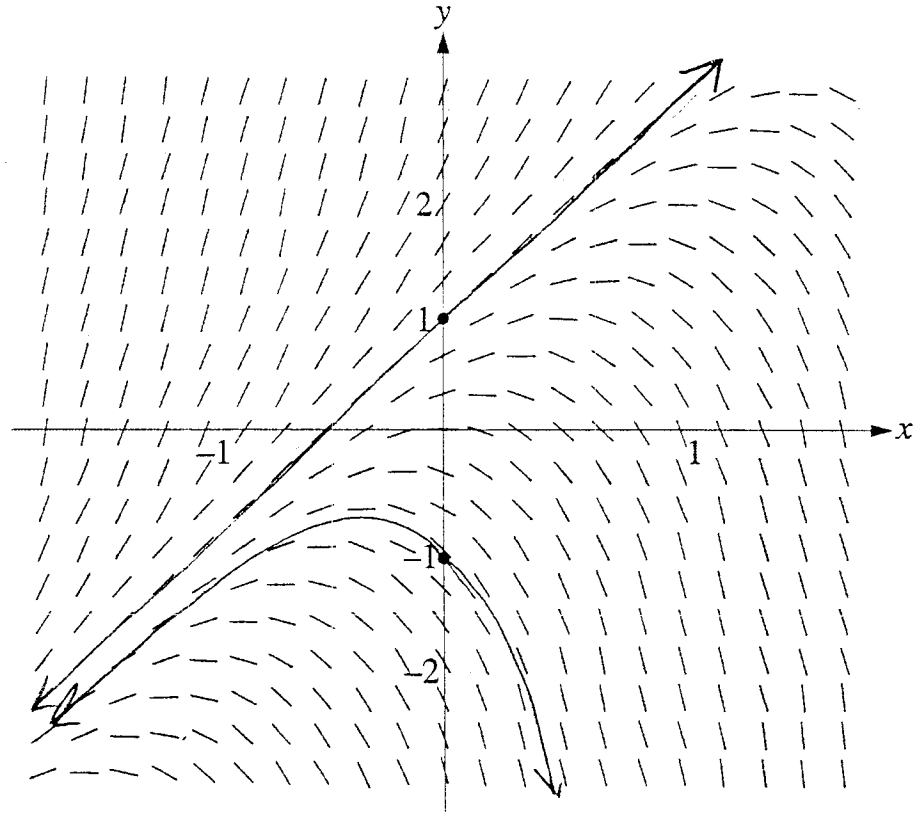
g has a local maximum at (0,0)

NO CALCULATOR ALLOWED

1D1

Work for problem 5(a)

$\frac{dy}{dx} = 2y - 4x$



Work for problem 5(b)

$f(0) = 1$

x	y	dx	f'(x)	dy
0	1	0.1	2	0.2
0.1	1.2	0.1	2	0.2
0.2	1.4			

$\left.\frac{dy}{dx}\right]_{x=0, y=1} = 2(1) - 4(0) = 2$

$dy = f'(x)\,dx$

$\left.\frac{dy}{dx}\right]_{x=0.1, y=1.2} = 2(1.2) - 4(0.1)$

$= 2.4 - 0.4 = 2$

$f(0.2) \approx 1.4$ using Euler's method.

NO CALCULATOR ALLOWED

ID 2

Work for problem 5(c)

$y = 2x + b$

When the solution to $\frac{dy}{dx}$ passes through $(0,1)$, the resulting line has the equation $y = 2x + 1$. Therefore, $b = 1$ when $y = 2x + b$ is a solution to this diff. equation.

Work for problem 5(d)

$g(0) = 0$

$g'(x) = \frac{dy}{dx} = 2y - 4x$

$g'(x)$: + 0 − (0)

$\left.\frac{dy}{dx}\right]_{x=0, y=0} = 0$

$\left.\frac{dy}{dx}\right]_{x=-1, y=0} = -4(-1) = 4$

$\left.\frac{dy}{dx}\right]_{x=1, y=0} = -4(1) = -4$

There is a local maximum at $(0,0)$ on the graph of g, since $g'(x)$ is increasing then decreasing on either side of $x=0$ (see sign chart).