The College Board AP® ADVANCED PLACEMENT PROGRAM®

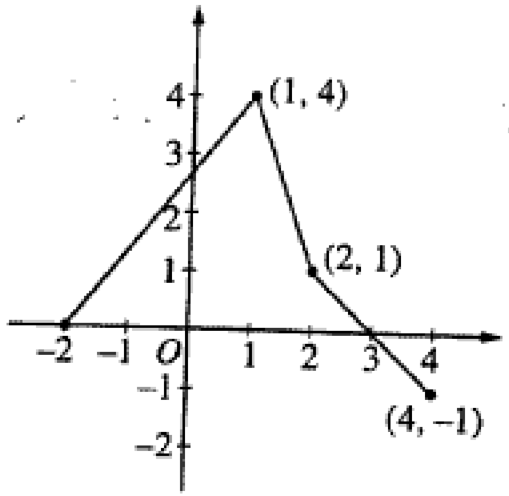
AP Calculus BC

1999 Sample Student Responses

These materials were produced by Educational Testing Service (ETS), which develops and administers the examinations of the Advanced Placement Program for the College Board. The College Board and Educational Testing Service (ETS) are dedicated to the principle of equal opportunity, and their programs, services, and employment policies are guided by that principle.

The College Board is a national nonprofit membership association dedicated to preparing, inspiring, and connecting students to college and opportunity. Founded in 1900, the association is composed of more than 3,900 schools, colleges, universities, and other educational organizations. Each year, the College Board serves over three million students and their parents, 22,000 high schools, and 3,500 colleges, through major programs and services in college admission, guidance, assessment, financial aid, enrollment, and teaching and learning. Among its best-known programs are the SAT®, the PSAT/NMSQT™, the Advanced Placement Program® (AP®), and Pacesetter®. The College Board is committed to the principles of equity and excellence, and that commitment is embodied in all of its programs, services, activities, and concerns.

B



5. The graph of the function f, consisting of three line segments, is given above. Let $g(x) = \int_1^x f(t)dt$.

(a) Compute $g(4)$ and $g(-2)$.

$$g(4) = \int_1^4 f(t)dt = \int_1^2 f(t)\,dt + \int_2^4 f(t)dt$$

$$= \frac{5}{2} + 0 = \frac{5}{2}$$

$$g(-2) = \int_1^{-2} f(t)dt = -\int_{-2}^{1} f(t)\,dt = -(\tfrac{1}{2}(3)(4)) = -6$$

(b) Find the instantaneous rate of change of g, with respect to x, at $x = 1$.

$$g'(x) = f(x)$$

$$g'(1) = f(1) = 4$$

Continue problem 5 on page 13.

(c) Find the absolute minimum value of g on the closed interval $[-2, 4]$. Justify your answer.

absolute minimum could occur at endpoint or when $g'(x)=0$

	x	$g(x)$
endpt/$g'(x)=0$	-2	-6
$g'(x)=0$	3	3
endpt	4	$\frac{5}{2}$

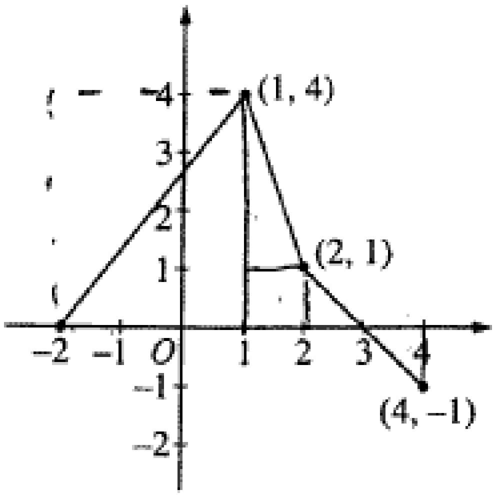
since $g(-2) < g(3)$ and $g(-2) < g(4)$ the absolute minimum occurs at -2 and is $g(-2) = -6$.

(d) The second derivative of g is not defined at $x = 1$ and $x = 2$. How many of these values are x-coordinates of points of inflection of the graph of g? Justify your answer.

$x=1$ is an inflection point because $g''(x) > 0$ for $x < 1$ and $g''(x) < 0$ for $x > 1$. $x=2$ is not an inflection point because $g''(x) < 0$ for $x < 2$ and $x > 2$.

GO ON TO THE NEXT PAGE

D



5. The graph of the function f, consisting of three line segments, is given above. Let $g(x) = \int_1^x f(t)dt$.

(a) Compute $g(4)$ and $g(-2)$.

$g(x) = \int_1^x f(t)\,dt$

$g(4) = \int_1^4 f(t)\,dt = 1 + \frac{3}{2} + \frac{1}{2} - \frac{1}{2} \Rightarrow$

$\boxed{g(4) = \frac{5}{2}}$

$g(-2) = \int_1^{-2} f(t)\,dt = -\int_{-2}^{1} f(t)\,dt = -\left[\frac{3 \times 4}{2}\right] = -6$

$\boxed{g(-2) = -6}$

(b) Find the instantaneous rate of change of g, with respect to x, at $x = 1$.

$\frac{dg}{dt} = \frac{d}{dt}\int_1^x f(t)\,dt$

$\frac{dg}{dt} = f(t)\Big|_x$

$\boxed{\frac{d}{dt}\,g(1) = 4}$

Continue problem 5 on page 13.

D

(c) Find the absolute minimum value of g on the closed interval $[-2, 4]$. Justify your answer.

$g' = f(t) = 0$

$t = 3$

f 3

+ + + + 0 - - -

pos | neg

$t > 3$, g increasing

$t < 3$, g decreasing

$\therefore$ $t = 3$ is rel <u>max</u>

⇨ absolute min must be one of the endpoints

OR $x = 4$

$x = -2$

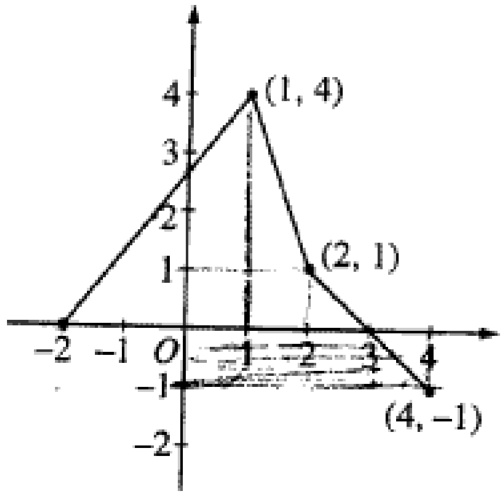
$$-\int_{-2}^{1} f(t)\,dt < \int_{1}^{4} f(t)\,dt$$

$x = -2$ is absolute minimum because the area between $f(t)$ and the x axis in the interval $[-2, 1]$ by -1 is clearly less than the area between $f(t)$ and the x axis on the interval $[1, 4]$.

(d) The second derivative of g is not defined at $x = 1$ and $x = 2$. How many of these values are x-coordinates of points of inflection of the graph of g ? Justify your answer.

$x = 1$ is a point of inflection because the slope of $f(t)$ [which equals $g'(x)$] changes from positive to negative at $x = 1$. At $x = 2$, the slope of $f(t)$ stays positive for $1 < x < 2$ and $2 < x < 4$.

E



5. The graph of the function f,$^{-5}$ consisting of^{-4} three line segments, is given above. Let $g(x) = \int_1^x f(t)dt$.

(a) Compute $g(4)$ and $g(-2)$.

$g(4) = \int_1^4 f(t)dt$

~~$A_{⏢} = \frac{1}{2}(b_1+b_2)h$~~

~~$A_{⏢} = \frac{1}{2}($~~

~~$g(1) = \int_0^1 (\frac{1}{3}x$~~

4 $A_{⏢_1} = \frac{1}{2}(b_1+b_2)h$

$A_{⏢_1} = \frac{1}{2}(6.5)\times 1$

$A_{⏢_1} = 3.25$

$A_{⏢_2} = \frac{1}{2}(4+1)(1)$

$A_{⏢_2} = 2.5$

$g(4) = 3.25 + 2.5 + 1$

$\boxed{g(4) = 6.75}$

$g(-2) = \int_1^{-2} f(t)dt$

4 $A_{\triangle} = \frac{1}{2}bh$

$A_{\triangle} = \frac{1}{2}(2 \cdot 2.5)$

$A_{\triangle} = 5$

$\boxed{g(-2) = 5}$

(b) Find the instantaneous rate of change of g, with respect to x, at $x = 1$.

$g'(x) = f(x)$

$g'(1) = f(1)$

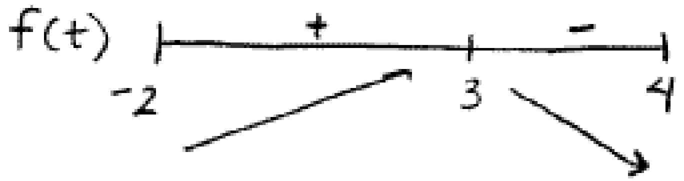
$f(1) = 4$

$\boxed{\text{Instantaneous rate of change} = 4}$

Continue problem 5 on page 13.

(c) Find the absolute minimum value of g on the closed interval $[-2, 4]$. Justify your answer.

$g'(x) = f(t)$



$f(-2) = 0$

$f(4) = -1$

∴ There is an absolute minimum for $g(x)$ over $[-2,4]$ at $x = 4$

(d) The second derivative of g is not defined at $x = 1$ and $x = 2$. How many of these values are x-coordinates of points of inflection of the graph of g? Justify your answer.

$g'(x) = f(x)$

$f'(x) = g''(x)$

$x = 1$ is a pt of inflection for $g(x)$

GO ON TO THE NEXT PAGE