AP Calculus BC
1999 Sample Student Responses

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B

t (hours)	$R(t)$ (gallons per hour)
0	9.6
3	10.4
→ 6	10.8
9	11.2
→ 12	11.4
15	11.3
→ 18	10.7
21	10.2
→ 24	9.6

3. The rate at which water flows out of a pipe, in gallons per hour, is given by a differentiable function R of time t. The table above shows the rate as measured every 3 hours for a 24-hour period.

(a) Use a midpoint Riemann sum with 4 subdivisions of equal length to approximate $\int_0^{24} R(t)\,dt$. Using correct units, explain the meaning of your answer in terms of water flow.

$\sum_{i=1}^{4} R(c_i)6 \quad 10.4\cdot 6 + 11.2\cdot 6 + 11.3\cdot 6 + 10.2\cdot 6$

where c_i = midpoint of interval

$(t = 3, 9, 15, 21)$

$= 258.6$ gallons

= # of gallons of water to flow out of a pipe from $t=0$ to $t=24$

(b) Is there some time t, $0 < t < 24$, such that $R'(t) = 0$? Justify your answer.

Yes $\rightarrow \dfrac{R(24) - R(0)}{24 - 0} = 0$, therefore, by the Mean Value Theorem, there is some t in $(0, 24)$ such that $R'(t) = 0$

Continue problem 3 on page 9.

B

(c) The rate of water flow $R(t)$ can be approximated by $Q(t) = \frac{1}{79}\left(768 + 23t - t^2\right)$.
Use $Q(t)$ to approximate the average rate of water flow during the 24-hour time period.
Indicate units of measure.

$$\frac{\frac{1}{79}\int_0^{24}(768+23t-t^2)\,dt}{24}$$

≈ 10.7848 gallons / hour

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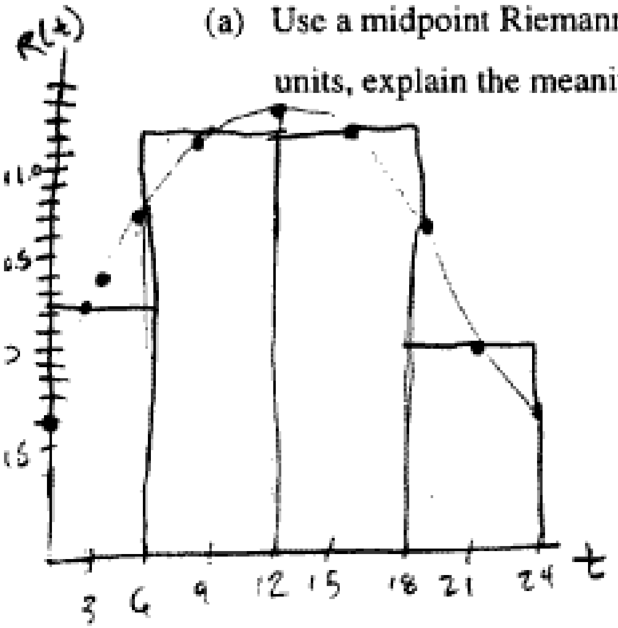
15	11.3
18	10.7
21	10.2
24	9.6

3. The rate at which water flows out of a pipe, in gallons per hour, is given by a differentiable function R of time t. The table above shows the rate as measured every 3 hours for a 24-hour period.

(a) Use a midpoint Riemann sum with 4 subdivisions of equal length to approximate $\int_0^{24} R(t)\,dt$. Using correct units, explain the meaning of your answer in terms of water flow.



b·h

$(2\cdot 10.4)+(2\cdot 11.2)+(2\cdot 11.3)+(2\cdot 10.2)$

$\int_0^{24} R(t)\,dt \approx 86.2$ gallons

It means that 86.2 gallons of water flowed out of the pipe for that 24 hour period.

(b) Is there some time t, $0 < t < 24$, such that $R'(t) = 0$? Justify your answer.

Yes there is. At approx. $t \approx 12$ the slope of a tangent line to that point is 0. $\therefore$ $R'(t) = 0$ at that point.

Continue problem 3 on page 9.

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(c) The rate of water flow $R(t)$ can be approximated by $Q(t) = \frac{1}{79}\left(768 + 23t - t^2\right)$.
Use $Q(t)$ to approximate the average rate of water flow during the 24-hour time period.
Indicate units of measure.

$$\text{Total Flow} = \int_0^{24} Q(t)\,dt$$

$$= 258.83544 \text{ gallons}$$

$$\text{average rate} = \frac{258.83544 \text{ gallons}}{24 \text{ hours}}$$

$$= 10.785 \text{ gallons/hour}$$

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E

t (hours)	$R(t)$ (gallons per hour)
0	9.6
3	10.4
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9	11.2
12	11.4
15	11.3
18	10.7
21	10.2
24	9.6

3. The rate at which water flows out of a pipe, in gallons per hour, is given by a differentiable function R of time t. The table above shows the rate as measured every 3 hours for a 24-hour period.

(a) Use a midpoint Riemann sum with 4 subdivisions of equal length to approximate $\int_0^{24} R(t)dt$. Using correct units, explain the meaning of your answer in terms of water flow.

$RS = \frac{b-a}{n}\left[f(x_1) + f(x_2) + f(x_3) + \dots f(x_n)\right]$

$RS = \frac{24}{4}\left[f(3) + f(9) + f(15) + f(21)\right]$

$RS = 6\ [10.4 + 11.2 + 11.3 + 10.2]$

$RS = 258.600$ gallons

after 24 hours 258.600 gallons of water have flowed from the pipe

(b) Is there some time t, $0 < t < 24$, such that $R'(t) = 0$? Justify your answer.

$R'(t)$ is the slope of $R(t)$
if $R(t)$ is a velocity then $R'(t)$ is the acceleration or change in velocity between time $t=12$ and time $t=15$ the change in velocity changes from positive to negative so $R'(t)$ must $=0$ at some time t $12 \le t \le 15$ which is between 0 and 24

Continue problem 3 on page 9.

E

(c) The rate of water flow $R(t)$ can be approximated by $Q(t) = \frac{1}{79}\left(768 + 23t - t^2\right)$. Use $Q(t)$ to approximate the average rate of water flow during the 24-hour time period. Indicate units of measure.

$$\text{Avg Rate} \doteq \frac{Q(24) - Q(0)}{24 - 0}$$

$$Q(24) = \frac{1}{79}(768 + 23(24) - (24)^2)$$
$$Q(24) = 9.418$$
$$Q(0) = \frac{1}{79}(768 + 23(0) - (0)^2)$$
$$Q(0) = 9.722$$
$$\text{Avg Rate} \doteq \frac{9.418 - 9.722}{24}$$

$\text{Avg Rate} \doteq -.013$ gallons per hour