The materials included in these files are intended for non-commercial use by AP teachers for course and exam preparation; permission for any other use must be sought from the Advanced Placement Program. Teachers may reproduce them, in whole or in part, in limited quantities, for face-to-face teaching purposes but may not mass distribute the materials, electronically or otherwise. These materials and any copies made of them may not be resold, and the copyright notices must be retained as they appear here. This permission does not apply to any third-party copyrights contained herein.
1. A particle moves in the $xy$-plane so that its position at any time $t$, $0 \leq t \leq \pi$, is given by $x(t) = \frac{t^2}{2} - \ln(1 + t)$ and $y(t) = 3 \sin t$.

(a) Sketch the path of the particle in the $xy$-plane below. Indicate the direction of motion along the path.
(b) At what time \( t, 0 \leq t \leq \pi \), does \( x(t) \) attain its minimum value? What is the position \((x(t), y(t))\) of the particle at this time?

\[
\begin{align*}
x(t) &= \frac{t^2}{2} - \ln(1+t) \\
x'(t) &= t - \frac{1}{1+t} \\
y(t) &= 0 - \frac{1}{1+t} \\
y'(t) &= 0 + \frac{1}{1+t} \\
x(0) &= 0 - \frac{1}{1+0} \\
y'(t) &= 0 + \frac{1}{1+t} \\
\Rightarrow &\quad \text{Minimum at } t = 0.618 \\
p(0.618) &= \left( \frac{0.618}{2} - \ln(1+0.618) \right) 3 \sin(-618) \\
p(0.618) &= (-0.290, 1.738)
\end{align*}
\]

(c) At what time \( t, 0 < t < \pi \), is the particle on the y-axis? Find the speed and the acceleration vector of the particle at this time.

\[
\text{When } x = 0, \text{ the particle is on the y-axis} \\
\Rightarrow x(t) = 0 \quad \Rightarrow \frac{t^2}{2} - \ln(1+t) = 0 \\
\Rightarrow \text{when } t = 1.286 \text{ the particle is on the y-axis}
\]

\[
\begin{align*}
\text{Velocity}_x &= x'(t) = t - \frac{1}{1+t} \\
\text{Velocity}_y &= 3 \cos t \\
\text{Speed} &= \sqrt{x'(t)^2 + y'(t)^2} \\
&= \sqrt{\left(\frac{t - \frac{1}{1+t}}{2}\right)^2 + 3 \cos t^2} \\
\text{Speed at } t &= 1.286 = 1.196 \\
\text{Acceleration} &= (\text{Velocity})' \\
A(t) &= \left(1 + \frac{1}{(1+t)^2} - 3 \sin t\right) \\
A(1.286) &= (1.191, -2.879)
\end{align*}
\]
1. A particle moves in the $xy$-plane so that its position at any time $t$, $0 \leq t \leq \pi$, is given by 

$$x(t) = \frac{t^2}{2} - \ln(1 + t)$$ 
and 

$$y(t) = 3 \sin t.$$ 

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\[
x(t) = \frac{t^2}{2} - \ln(1 + t)
\]

\[
\frac{dx}{dt} = t - \frac{1}{1 + t}
\]

0 = \( t - \frac{1}{1 + t} \)

\( t = \frac{1}{1 + t} \)

\[ t^2 + t = 1 \]

\( t = -\frac{1 + \sqrt{5}}{2} \) \( \text{geometric form} \)

\[ x(t) = -2.90 \quad \text{position is} \quad \left( -2.90, 1.73 \right) \]

\[ y(t) = 1.73 \]

(c) At what time \( t \), \( 0 < t < \pi \), is the particle on the y-axis? Find the speed and the acceleration vector of the particle at this time.

\[
x(t) = \frac{t^2}{2} - \ln(1 + t)
\]

\[
0 = \frac{t^2}{2} - \ln(1 + t)
\]

\( t = 0 \) at \( 0 \) and \( 1.285 \)

\( t \) is on the y-axis at

\( 1.285 \leq t < \pi \)

\( \frac{dx}{dt} = t - \frac{1}{1 + t} \quad \frac{dy}{dt} = 3 \cos t \)

\[
V(t) = \left( t - \frac{1}{1 + t}, 3 \cos t \right)
\]

\( \text{speed} = \text{magnitude of velocity} \)

\[
\frac{dx}{dt} = 1.848 \quad \frac{dy}{dt} = 1.843
\]

\[
\sqrt{\left( \frac{dx}{dt} \right)^2 + \left( \frac{dy}{dt} \right)^2} = 1.196 = \text{speed}
\]
1. A particle moves in the $xy$-plane so that its position at any time $t$, $0 \leq t \leq \pi$, is given by $x(t) = \frac{t^2}{2} - \ln(1 + t)$ and $y(t) = 3 \sin t$.

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\[
\frac{dx}{dt} + \frac{1}{t + e} = 0
\]

\[ t = 0.618 \]

\[ (-0.29, 1.738) \]

(c) At what time \( t, \ 0 < t < \pi \), is the particle on the y-axis? Find the speed and the acceleration vector of the particle at this time.

\[ t = 1.286 \]

\[ \mathbf{v} = (0.849, 0.843) \]

\[ \mathbf{a} = (1.191, -2.879) \]