AP Calculus BC
1999 Sample Student Responses

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B

CALCULUS BC

SECTION II

Time—1 hour and 30 minutes

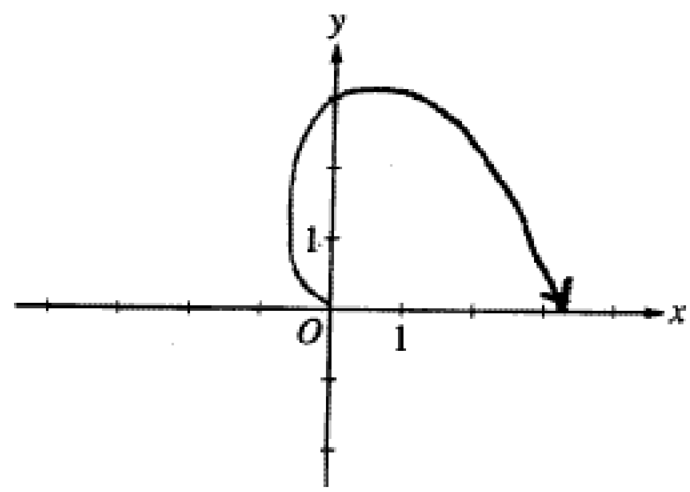
Number of problems — 6

Percent of total grade — 50

REMEMBER TO SHOW YOUR SETUPS AS DESCRIBED IN THE GENERAL INSTRUCTIONS.

1. A particle moves in the xy-plane so that its position at any time t, $0 \le t \le \pi$, is given by $x(t) = \frac{t^2}{2} - \ln(1 + t)$ and $y(t) = 3 \sin t$.

   (a) Sketch the path of the particle in the xy-plane below. Indicate the direction of motion along the path.



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(b) At what time t, $0 \le t \le \pi$, does $x(t)$ attain its minimum value? What is the position $(x(t),\ y(t))$ of the particle at this time?

$$x(t) = \frac{t^2}{2} - \ln(1+t) \qquad 0 = t - \frac{1}{1+t}$$

$$x'(t) = t - \frac{1}{1+t} \qquad t(t+1) - 1 = 0$$

$$t^2 + t - 1 = 0$$

$$\boxed{t = .618}$$

0 —— .618 —— π

$x'(t)$ $-$ $+$

$x(t)$ dec. inc

$\Rightarrow \therefore$ Minimum at $t = .618$

$$P(.618) = \left(\frac{(.618)^2}{2} - \ln(1+.618),\ 3\sin(.618)\right)$$

$$P(.618) = (-.290,\ 1.738)$$

(c) At what time t, $0 < t < \pi$, is the particle on the y-axis? Find the speed and the acceleration vector of the particle at this time.

When $x=0$, the particle is on the y axis

$$\Rightarrow x(t) = 0 \Rightarrow \frac{t^2}{2} - \ln(1+t) = 0$$

$\therefore$ when $t = 1.286$ the particle is on the y-axis

$$\text{Velocity}_x = x'(t) = t - \frac{1}{1+t} \qquad \text{Velocity}_y = 3\cos t$$

$$\text{Speed} = \sqrt{x'(t)^2 + y'(t)^2}$$

$$= \sqrt{\left(t - \frac{1}{1+t}\right)^2 + (3\cos t)^2}$$

Speed at $(t = 1.286) = 1.196$

Acceleration $= (\text{Velocity})'$

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$$A(t) = \left(1 + \frac{1}{(1+t)^2},\ -3\sin t\right)$$

$$A(1.286) = (1.191,\ -2.879)$$

D

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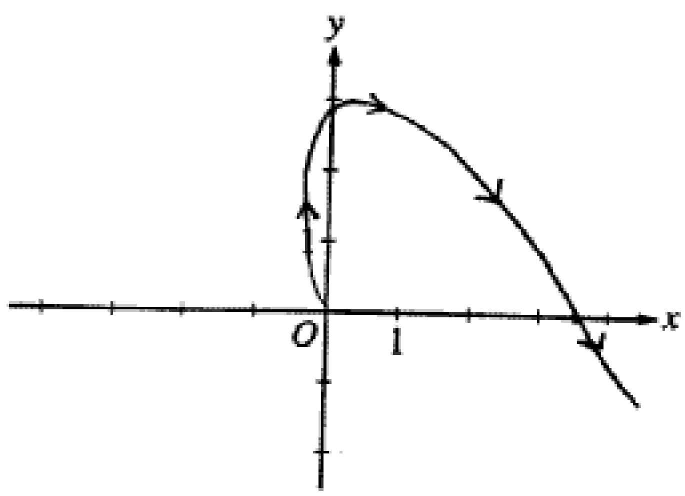

Continue problem 1 on page 5.

D

(b) At what time t, $0 \le t \le \pi$, does $x(t)$ attain its minimum value? What is the position $(x(t),\ y(t))$ of the particle at this time?

$x(t) = \frac{t^2}{2} - \ln(1+t)$

$\frac{dx}{dt} = t - \frac{1}{1+t}$

$0 = t - \frac{1}{1+t}$

$t = \frac{1}{1+t}$

$t^2 + t = 1$

$t^2 + t - 1 = 0$

$t = \frac{-1+\sqrt{5}}{2}$ by quadratic formula

$x(t) = -.290$

$y(t) = 1.73$

position is $(-.290, 1.73)$

(c) At what time t, $0 < t < \pi$, is the particle on the y-axis? Find the speed and the acceleration vector of the particle at this time.

$x(t) = \frac{t^2}{2} - \ln(1+t)$

$0 = \frac{t^2}{2} - \ln(1+t)$

$\ln(1+t) = \frac{t^2}{2}$

$t = 0$ at 0 and 1.285

t is on the y axes at 1.285 for $0 < t < \pi$

$\frac{dx}{dt} = t - \frac{1}{1+t}$ $\quad \frac{dy}{dt} = 3\cos t$

$v(t) = \left(t - \frac{1}{1+t},\ 3\cos t\right)$

speed = magnitude of velocity

$\frac{dx}{dt}$ .848 $\quad \frac{dy}{dt} = .843$

$\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} = 1.196$ = speed

$v(t) = \left(t - \frac{1}{1+t},\ 3\cos t\right)$

$a(t) = \left(1 + \frac{1}{(1+t)^2},\ -3\sin t\right)$

$a(t) = (1.191,\ -2.879)$

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F

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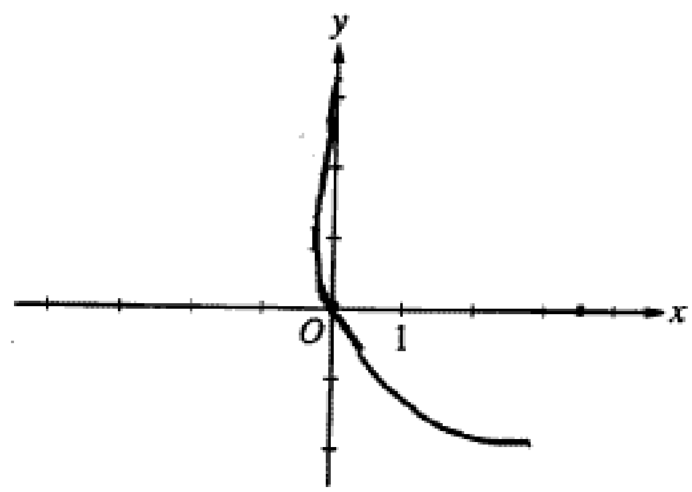
   (a) Sketch the path of the particle in the xy-plane below. Indicate the direction of motion along the path.

x

$\frac{e}{e^{\frac{t^2}{2}}} = 1+t$



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F

(b) At what time t, $0 \le t \le \pi$, does $x(t)$ attain its minimum value? What is the position $(x(t),\ y(t))$ of the particle at this time?

$\frac{dx}{dt} = t - \frac{1}{1+t} = 0$

$t = 0.618$

$(-0.29, 1.738)$

(c) At what time t, $0 < t < \pi$, is the particle on the y-axis? Find the speed and the acceleration vector of the particle at this time.

$t = 1.286$

$\vec{v} = (0.849,\ 0.843)$

$\vec{a} = (1.191, -2.879)$

$1 + \frac{1}{(1+t)^2}$

$-3\sin t$

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