These materials were produced by Educational Testing Service (ETS), which develops and administers the examinations of the Advanced Placement Program for the College Board. The College Board and Educational Testing Service (ETS) are dedicated to the principle of equal opportunity, and their programs, services, and employment policies are guided by that principle.

The College Board is a national nonprofit membership association dedicated to preparing, inspiring, and connecting students to college and opportunity. Founded in 1900, the association is composed of more than 3,900 schools, colleges, universities, and other educational organizations. Each year, the College Board serves over three million students and their parents, 22,000 high schools, and 3,500 colleges, through major programs and services in college admission, guidance, assessment, financial aid, enrollment, and teaching and learning. Among its best-known programs are the SAT®, the PSAT/NMSQT™, the Advanced Placement Program® (AP®), and Pacesetter®. The College Board is committed to the principles of equity and excellence, and that commitment is embodied in all of its programs, services, activities, and concerns.

Copyright © 2001 by College Entrance Examination Board. All rights reserved. College Board, Advanced Placement Program, AP, and the acorn logo are registered trademarks of the College Entrance Examination Board.
Work for problem 6(a)

\[
L = \lim_{n \to \infty} \left| \frac{(n+2)x^{n+1}}{3^{n+2}} \cdot \frac{3^{n+1}}{(n+1)x^n} \right| = \lim_{n \to \infty} \left| \frac{x}{3} \cdot \frac{n+2}{n+1} \right| = \left| \frac{x}{3} \right| < 1
\]

\[-1 < \frac{x}{3} < 1 \quad \Rightarrow \quad -3 < x < 3\]

\[
X = -3: \sum_{n=0}^{\infty} \frac{(n+1)(-3)^n}{3^{n+1}} = \sum_{n=0}^{\infty} \frac{(-1)^n(3)^n}{3^{n+1}} = \sum_{n=0}^{\infty} \frac{(-1)^n}{3} = \frac{3}{3} = 1
\]

\[
X = 3: \sum_{n=0}^{\infty} \frac{(n+1)3^n}{3^{n+1}} = \sum_{n=0}^{\infty} \frac{n+1}{3} = \infty
\]

\[\boxed{(-3, 3)}\]

Work for problem 6(b)

\[
\frac{f(x) - \frac{1}{3}}{x} = \frac{2}{x^2} + \frac{3}{3^2}x + \ldots \frac{(n+2)}{3^{n+2}}x^n
\]

\[
\lim_{x \to 0} \frac{f(x) - \frac{1}{3}}{x} = \frac{2}{3^2} = \frac{2}{9}
\]
Work for problem 6(c)

\[
\int_0^1 f(x) \, dx = \left[ \frac{1}{3} x + \frac{x^2}{3^2} + \frac{x^3}{3^3} + \frac{x^4}{3^4} + \ldots + \frac{x^n}{3^n} \right]_0^1
\]

\[
= \frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \ldots + \frac{1}{3^n}
\]

Work for problem 6(d)

The series determined in part c is a geometric series with initial term \( t_1 = \frac{1}{3} \) and a ratio \( r = \frac{1}{3} \).

\[ \text{Sum} = \frac{t_1}{1-r} = \frac{\frac{1}{3}}{1-\frac{1}{3}} = \frac{\frac{1}{3}}{\frac{2}{3}} = \frac{1}{2} \]
Work for problem 6(a)

\[
\lim_{x \to 0} \frac{(n+1+1) x^{n+1}}{3^{n+1} + 1} = \lim_{x \to 0} \frac{(n+1) x^n}{3^{n+1}}
\]

\[
= \lim_{x \to 0} \frac{(n+2)(x^n x^1)}{3^1 \cdot 3^2} = \lim_{x \to 0} \frac{n+2}{(n+1)(x^2)} \cdot \frac{x}{3}
\]

\[
|\frac{x}{3}| < 1 \quad -1 < \frac{x}{3} < 1 \quad -3 < x < 3
\]

Check endpoints

\[
\sum_{n=1}^{\infty} \frac{(n+1)(3)^n}{3^{n+1}} \text{ converges} \quad \sum_{n=1}^{\infty} \frac{(n+1)(3)^n}{3^{n+1}} \text{ diverges}
\]

\[
-3 \leq x \leq 3
\]

Work for problem 6(b)

\[
\lim_{x \to 0} \frac{f(x) - \frac{1}{3}}{x} = \lim_{x \to 0} \frac{2}{3} x + \frac{3}{3^2} x^2 + \ldots + \frac{n+1}{3^{n+1}} x^n + \ldots
\]

\[
= \lim_{x \to 0} \frac{n+1}{3^{n+1}} x^{n-1}
\]

\[
= \frac{2}{3^2} = \frac{2}{9}
\]
Work for problem 6(c)

\[ g(x) = \frac{1}{3} + \frac{x^2}{3^2} + \frac{x^3}{3^3} \]

\[ g(x) = \frac{1}{3} x^1 + \frac{x^2}{9} + \frac{x^3}{27} \]

\[ g(x) = \frac{1}{3} + \frac{1}{9} + \frac{1}{27} \]

Work for problem 6(d)

\[ S_n = \frac{a_1}{1-r} \]

\[ a_1 = \frac{1}{3}, \quad r = \frac{1}{3} \]

\[ S_n = \frac{1}{3} \]

\[ S_n = \frac{1}{3} \]

\[ S_n = \frac{1}{3} \]

\[ S_n = \frac{1}{3} \times \frac{2}{3} = \frac{1}{2} \]