AP® Calculus BC
2001 Sample Student Responses

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NO CALCULATOR ALLOWED

B1

Work for problem 6(a)

Ratio Test

$$L = \lim_{n\to\infty} \left| \frac{(n+2)x^{n+1}}{3^{n+2}} \cdot \frac{3^{n+1}}{(n+1)x^n} \right| = \lim_{n\to\infty} \left| \frac{x}{3} \cdot \frac{n+2}{n+1} \right| = \left|\frac{x}{3}\right| < 1$$

$$-1 < \frac{x}{3} < 1 \quad \therefore -3 < x < 3$$

$$x = -3: \sum_{n=0}^{\infty} \frac{(n+1)(-3)^n}{3^{n+1}} = \sum_{n=0}^{\infty} \frac{(n+1)(-1)^n(3)^n}{3^{n+1}} = \sum_{n=0}^{\infty} \frac{(-1)^n(n+1)}{3}$$ Diverges

$$x = 3: \sum_{n=0}^{\infty} \frac{(n+1)3^n}{3^{n+1}} = \sum_{n=0}^{\infty} \frac{n+1}{3}$$ Diverges

$$\boxed{(-3,3)}$$

Work for problem 6(b)

$$\frac{f(x) - \frac{1}{3}}{x} = \frac{2}{3^2} + \frac{3}{3^3}x + \dots \frac{(n+2)}{3^{n+2}}x^n$$

$$\lim_{x\to 0} \frac{f(x)-\frac{1}{3}}{x} = \frac{2}{3^2} = \boxed{\frac{2}{9}}$$

NO CALCULATOR ALLOWED

B2

Work for problem 6(c)

$$\int_0^1 f(x)\,dx = \left[\frac{1}{3}x + \frac{x^2}{3^2} + \frac{x^3}{3^3} + \frac{x^4}{3^4} + \ldots + \frac{x^n}{3^n}\right]\Bigg|_0^1$$

$$= \boxed{\frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \ldots + \frac{1}{3^n}}$$

Work for problem 6(d)

The series determined in part c is a geometric series with initial term $t_1 = \frac{1}{3}$ and a ratio $r = \frac{1}{3}$.

$$\text{Sum} = \frac{t_1}{1-r} = \frac{\frac{1}{3}}{1-\frac{1}{3}} = \frac{\frac{1}{3}}{\frac{2}{3}} = \boxed{\frac{1}{2}}$$

NO CALCULATOR ALLOWED

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Work for problem 6(a)

$$\lim_{x\to\infty} \frac{\frac{(n+1+1)}{3^{n+1+1}} x^{n+1}}{\frac{(n+1)(x^n)}{3^{n+1}}} \quad \lim_{x\to\infty} \frac{(n+2)(\cancel{x^n}\cdot x^1)}{\cancel{3^n}\cdot 3^2} \cdot \frac{\cancel{3^n}\cdot 3^1}{(n+1)(\cancel{x^n})} \Rightarrow \lim_{n\to\infty} \frac{n+\cancel{2}^{1}}{\cancel{n}+1}\cdot\frac{x}{3}$$

$$\left|\frac{x}{3}\right| < 1 \quad -1 < \frac{x}{3} < 1 \qquad -3 < x < 3$$

Check endpoints

$\sum \frac{n+1}{3^{n+1}}(-3)^n$ converges $\qquad \sum \frac{n+1}{3^{n+1}}(3^n)$ diverges

$$\boxed{-3 \le x < 3}$$

Work for problem 6(b)

$$\lim_{x\to 0} \frac{f(x)-\frac{1}{3}}{x} \Rightarrow \lim_{x\to 0} \frac{\frac{2}{3^2}x + \frac{3}{3^3}x^2 + \dots + \frac{n+1}{3^{n+1}}x^n + \dots}{x}$$

$$\lim_{x\to 0} \frac{2}{3^2} + \cancel{\frac{3}{3^3}x + \dots \frac{n+1}{3^{n+1}}x^{n-1}} \to 0$$

$$= \frac{2}{3^2} = \boxed{\frac{2}{9}}$$

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NO CALCULATOR ALLOWED

D2

Work for problem 6(c)

$g(x) = \frac{1}{3}x + \frac{\cancel{2}x^2}{3^2 \cdot \cancel{2}} + \frac{\cancel{3}x^3}{3^3 \cdot \cancel{3}}$

$g(x) = \frac{1}{3}x\Big|_0^1 + \frac{x^2}{9}\Big|_0^1 + \frac{x^3}{27}\Big|_0^1$

$\boxed{g(x) \quad \frac{1}{3} + \frac{1}{9} + \frac{1}{27}}$

Work for problem 6(d)

$S_n = \frac{a_1}{1-r}$ $\quad a_1 = \frac{1}{3} \quad r = \frac{1}{3}$

$S_n = \frac{\frac{1}{3}}{1-\frac{1}{3}} \quad S_n = \frac{\frac{1}{3}}{\frac{3}{3}-\frac{1}{3}} \quad S_n = \frac{\frac{1}{3}}{\frac{2}{3}} \quad S_n = \frac{1}{\cancel{3}} \cdot \frac{\cancel{3}}{2} = \boxed{\frac{1}{2}}$