AP® Calculus BC
2001 Sample Student Responses

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NO CALCULATOR ALLOWED

A1

CALCULUS BC

SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.

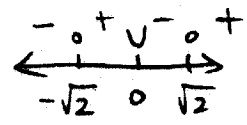
Work for problem 4(a)

$h'(x) = 0 = \frac{x^2-2}{x}$

$0 = x^2-2$

$x = \pm\sqrt{2}$

∴ h has horizontal tangents at $x=\sqrt{2}$ and $x=-\sqrt{2}$

$h'(x)$ 

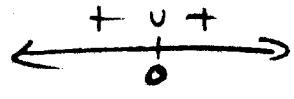
∴ h has a local minimum at $x=\sqrt{2}$ and $x=-\sqrt{2}$

Work for problem 4(b)

$h''(x) = \frac{2x\cdot x - (x^2-2)}{x^2}$

$= \frac{2x^2-x^2+2}{x^2}$

$= \frac{x^2+2}{x^2}$

$h''(x)$ 

∴ h is concave up on the intervals $(-\infty, 0)$ and $(0, \infty)$

NO CALCULATOR ALLOWED

A2

Work for problem 4(c)

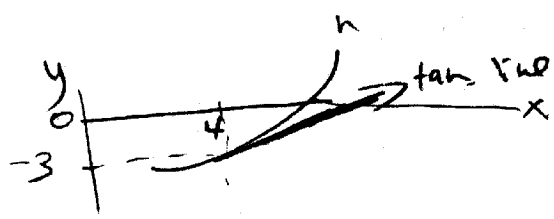
$$h'(4) = \frac{4^2-2}{4} = \frac{14}{4} = \frac{7}{2}$$

$$h(4) = -3$$

$$y+3 = \frac{7}{2}(x-4)$$

Work for problem 4(d)

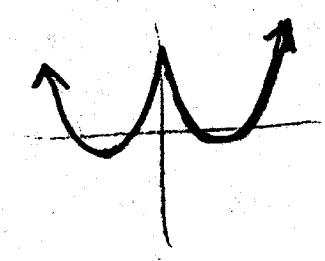
The tangent line to the graph of h at $x = 4$ lies below the graph of h for $x > 4$ because h is concave up on the interval $(0, \infty)$.



4 4 4 4 4 4 4 4 4 4

NO CALCULATOR ALLOWED

D1



CALCULUS AB

SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.

Work for problem 4(a)

$1 - \frac{2}{x^2} : x$

$x - \frac{2}{x}$

$h'(x) = 0$

$x \neq 0$

$\frac{x^2 - 2}{x} = 0$

$x^2 - 2 = 0$

$x^2 = 2$

$x = \pm\sqrt{2}$

$h''(x) = \frac{2x^2 - x^2 + 2}{x^2}$

$h''(+\sqrt{2}) = \frac{4 - 2 + 2}{2}$

$h''(-\sqrt{2}) = \frac{4}{2} = 2$

$h''(\sqrt{2})$ +

$h''(-\sqrt{2})$ +

at $+\sqrt{2}$, minimum

+ at $-\sqrt{2}$, minimum because the second derivative is +, which means the slope is increasing

Work for problem 4(b)

x10 $\frac{2x^2 - x^2 + 2}{x^2} > 0$

~~$2x^2$~~

$x^2 + 2 > 0$

$x^2 > -2$

x^2

(h) is concave up for all values, as x^2 must always be greater than -2

NO CALCULATOR ALLOWED

D2

Work for problem 4(c)

$h(4) = -3$

$$h'(x) = \frac{x^2 - 2}{x}$$

$$h'(4) = \frac{16-2}{4} = \frac{14}{4} = \frac{7}{2}$$

$$y + 3 = \frac{7}{2}(x - 4)$$

$$y = \frac{7}{2}x - 14 - 3$$

$$y = \frac{7}{2}x - 17$$

Work for problem 4(d)

below, because the graph is concaved up