AP® Calculus BC
2001 Sample Student Responses

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t (days)	$W(t)$ (°C)
0	20
3	31
6	28
9	24
12	22
15	21

A_1

Work for problem 2(a)

$$\frac{dW}{dt}\bigg|_{t=12} \approx \frac{\Delta W}{\Delta t}\bigg|_{9<t<15} = \frac{24-21}{9-15} = -.5\,°C/day$$

Work for problem 2(b)

$$\text{Average } W = \frac{\int_0^{15} W\,dt}{\Delta t} = \frac{\int_0^{15} W\,dt}{15} \approx \frac{\left(\frac{20+31}{2} + \frac{31+28}{2} + \frac{28+24}{2} + \frac{24+22}{2} + \frac{22+21}{2}\right)3}{15}$$

(bases; height)

$$A_{trap} = \frac{b_1+b_2}{2}\cdot h \qquad h=3$$

$$A_{ave_W} = 25.1\,°C$$

Work for problem 2(c)

A_2

$$P'(t) = 10e^{-\frac{t}{3}} + 10t \cdot -\frac{1}{3} e^{-\frac{t}{3}}$$

$$P'(12) = 10e^{-\frac{12}{3}}\left(1 + -\frac{1}{3}t\right)$$

$$-30e^{-4} = -.5495\,°C/day$$

On the 12th day in the 15 day time period, the temperature of the pond will be decreasing at such a rate that it will be getting about .5495 degrees C colder per day.

Work for problem 2(d)

$$\text{Average } P = \frac{\int_0^{15} P\,dt}{\Delta t} = \frac{\int_0^{15} 20 + 10te^{-\frac{1}{3}t}\,dt}{15}$$

$$= \frac{\Big|_0^{15} 20t + 10\int_0^{15} te^{-\frac{1}{3}t}\,dt}{15}$$

$$= \frac{300 + 10\Big|_0^{15} -3e^{-\frac{1}{3}t}(t+3)}{15}$$

$$= \frac{300 + 86.3616}{15} = 25.757\,°C$$

$$\int te^{-\frac{1}{3}t}\,dt \qquad u = t \quad v = -3e^{-\frac{1}{3}t}$$

$$du = dt \quad dv = e^{-\frac{1}{3}t}dt$$

$$= -3te^{-\frac{1}{3}t} + 3\int e^{-\frac{1}{3}t}\,dt$$

$$= -3te^{-\frac{1}{3}t} - 9e^{-\frac{1}{3}t} + C$$

$$= -3e^{-\frac{1}{3}t}(t+3) + C$$

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C1

t (days)	$W(t)$ (°C)
0	20
3	31
6	28
9	24
12	22
15	21

Work for problem 2(a)

$$W'(12) \approx \frac{W(9) - W(15)}{9 - 15}$$

$$\approx \frac{24 - 21}{9 - 15}$$

$$W'(12) \approx -\frac{1}{2}$$

Work for problem 2(b)

$$\text{Ave } W(t) = \frac{1}{2} \cdot (3) \left(\frac{W(0) + 2W(3) + 2W(6) + 2W(9) + 2W(12) + W(15)}{15 - 0} \right)$$

$$\text{Ave } W(t) = \frac{1}{10} \left[20 + (2 \cdot 31) + (2 \cdot 28) + (2 \cdot 24) + (2 \cdot 22) + 21 \right]$$

$$\text{Ave } W(t) = \frac{1}{10}(251)$$

$$\text{Ave } W(t) = 25.1 \text{ °C}$$

C_2

Work for problem 2(c)

$$P'(t) = 10e^{(-t/3)} - \frac{10}{3}te^{(-t/3)}$$

$$P'(12) = 10e^{-12/3} - (10/3)(12)\left(e^{\left(\frac{-12}{3}\right)}\right)$$

$$P'(12) = -7.5495 \text{ degrees/day}$$

$P'(12)$ is the instantaneous rate of change of degrees Celcius per day, when $t = 12$ days.

Work for problem 2(d)

$$\text{Ave } P(t) = \frac{1}{15-0}\int_0^{15}\left[20 + 10 + e^{(-t/3)}\right]dt$$

$$\text{Ave } P(t) = \frac{1}{15}(386.362)$$

$$\text{Ave } P(t) = 25.757\ ^\circ C$$