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Work for problem 6(a)

\[ \begin{align*}
(0,0) &= 0 \\
(0,1) &= 0 \\
(0,-1) &= 0 \\
(1,0) &= 1 \\
(1,1) &= 0 \\
(1,-1) &= 4 \\
(2,1) &= 0 \\
(-1,1) &= 0 \\
(-2,1) &= 0 \\
(-1,0) &= -1 \\
(-1,-1) &= -4
\end{align*} \]

Work for problem 6(b)

This graph passes through the points \((-1,1)\) and \((1,1)\) and has a slope whose absolute value is greater than 0. However, in the slope field, the slope at these points is 0. Therefore, this graph cannot match the differential equation.

Continue problem 6 on page 15.
Work for problem 6(c)

\[
\frac{dy}{(y-1)^2} = x \, dx
\]

Integrate:

\[
\frac{-1}{y-1} = \frac{1}{2} x^2 + C
\]

\[
\frac{-1}{-1-1} = \frac{1}{2} (0)^2 + C
\]

\[
C = \frac{1}{2}
\]

Work for problem 6(d)

When \(x = 0\), \(y = -1\)

As \(x\) approaches \(-\infty\) or \(\infty\), \(y\) goes to 1

\([-1, 1]\)

END OF EXAMINATION

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Work for problem 6(a)

Work for problem 6(b)

This graph can not be a solution because at the points (-1, 1) and (1, 1) the slope of the function must be zero. In this particular graph there is a slope not equaling 0 at these two points.

Continue problem 6 on page 15.
Work for problem 6(c)

\[ \frac{dy}{(y-1)^2} = \frac{x}{x^2 + 1} \, dx \]

\[ u = y-1 \]

\[ \int \frac{du}{u^2} = \int \frac{x}{x^2 + 1} \, dx \]

\[ -\frac{1}{u} = \frac{1}{2} \ln |u| + C \]

\[ \frac{1}{(y-1)^2} = \frac{1}{2} \ln |x^2 + 1| + C \]

\[ -\frac{1}{y-1} = \frac{1}{2} \ln |x^2 + 1| + C \]

\[ y-1 = e^{\frac{1}{2} \ln |x^2 + 1| + C} \]

\[ y-1 = e^{\frac{1}{2} \ln |x^2 + 1|} e^{C} \]

\[ y-1 = e^{C} \sqrt{x^2 + 1} \]

\[ c = \frac{1}{2} \]

Work for problem 6(d)

\[ \text{range} = -1 < y < 1 \]

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\[
\frac{dy}{dx} = x(y-1)^2 \rightarrow y^2x-2xy+x
\]

Parabolas such as the one shown above have 2 values of \( x \) for each unique value \( y \). However, the differential equation from above shows that there are more values for \( y \) than \( x \), so the shape should be lie sideways.

Continue problem 6 on page 15.
Work for problem 6(c)

\[ f(0) = -1 \]

\[ \int \frac{dy}{(y-1)^2} = \int x \, dx \rightarrow \int \frac{dy}{y^2 - 2y + 1} = \int x \, dx \]

\[ \frac{1}{2}x^2 = -(y-1)^{-1} + C \]

\[ \frac{1}{3}x^2 + C = -\frac{1}{y-1} \rightarrow (0, -1): \quad 0 + C = -\frac{1}{1-1} \rightarrow C = -\frac{1}{(-2)} = \frac{1}{2} \]

\[ \frac{1}{2}x^2 + \frac{1}{2} = -\frac{1}{y-1} \rightarrow -\frac{1}{2}x^2 - \frac{1}{2} = \frac{1}{y-1} \rightarrow \frac{-x^2 - 1}{2} = \frac{1}{y - 1} \]

\[ \frac{2}{-(x^2 - 1)} \rightarrow \frac{-2}{x^2 + 1} + 1 = y \]

\[ \frac{-2 + x^2 + 1}{x^2 + 1} \Rightarrow \frac{x^2 - 1}{x^2 + 1} = y \]

Work for problem 6(d)

Range:

**every value of \( y \) allowed because \( x^2 + 1 \) can never equal 0, thus \( y \) will never be \( \infty \)**

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